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ENGINEERING SCIENCE

Books by H. B. Brown and A. J. Bryant

ENGINEERING SCIENCE

VOL. I.—APPLIED MECHANICS AND HYDRAULICS.

VOL. II.—HEAT AND HEAT ENGINES, AND ELECTROTECHNICS.

ENGINEERING SCIENCE

BY

H. B. BROWN
B.Sc. (Eng.) London; Wh.Ex.: A.M.I.Mech.E.
HEADMASTER, BORDESLEY GREEN COUNTY TECHNICAL SCHOOL AND
PRINCIPAL, BORDESLEY GREEN TECHNICAL INSTITUTE BIRMINGHAM

AND

A. J. BRYANT
B.Sc. (Eng. Hons.) London; A.M.I.Mech.E.
HEADMASTER HANDSWORTH COUNTY TECHNICAL SCHOOL, BIRMINGHAM

VOLUME III

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PREFACE

THE three volumes which form *Engineering Science* have been planned to cover the full study of this subject in the Secondary Technical School, with the requirements of any sixth form or other senior work falling within the scope of the future development of this type of School. The work also meets fully the whole of the Ordinary National Certificate course where Machines, Strength of Materials, Structures, Hydraulics and Heat Engines appear in the approved scheme.

More precisely, its scope is that of the Engineering Science Course of the Secondary Technical School, the Ordinary National Certificate Course in Mechanical Engineering and the Engineering Science of the Ordinary National Certificate in Electrical Engineering. The work also meets the requirements of much of the syllabuses of the Institution of Mechanical Engineers examinations, and those of the Scottish Education Authorities affiliated to the Royal Technical College, Glasgow. Consideration has also been given to the subject-matter of Service and pre-Service courses in engineering subjects, and to those professional examinations which demand engineering science as an integral part of their syllabuses. It is also anticipated that the three volumes will prove of considerable use to designers, draughtsmen and industrialists when questions of principle and application arise in their practice.

In Volumes I and II the presentation has been made with a full appreciation of the mathematical limitations of the students, but Volume III assumes a parity of mathematical training with the demands of the work. Experimental work, testing and full-scale trials of prime movers take an important place in Volume III, and the examples and exercises have been selected to give a practical bias associated with the problems of design and operation. The extent to which graphical work is used has been determined by the need for this type of solution, and wherever possible both the practical and analytical solution to a problem have been envisaged.

This preface is supplementary to that of Volumes I and II, where certain acknowledgements have been made to firms and examination bodies, but the authors would like to add to this the following firms who have given assistance in the loan of blocks and supply of data for Volume III : Messrs. W. & T. Avery Ltd., Birmingham ; Messrs. Bellis and Morcom Ltd., Birmingham ; Messrs. T. C. Howden & Co., Birmingham ; Messrs. Tensometer Ltd., Croydon ; Messrs. Vickers Ltd.

Particular appreciation is due to Sir Richard Gregory and Mr. A. J. V. Gale, M.A., who have ungrudgingly lent their wide experience to the general project, and given great help in their suggestions for the final arrangement of the work and its contents.

It is too much to hope that errors will not be found in this work, and the authors will be grateful to teachers and students who are good enough to point out errors which have escaped detection.

H. B. BROWN
A. J. BRYANT

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CHAPTER I

MOTION AND POWER AND THEIR TRANSMISSION. MACHINES AND THEIR VELOCITY RATIOS. MOMENTUM AND ENERGY

It is possible to divide part of the study of Engineering Science into two sections, namely, (1) Machines and (2) Structures ; and for the study of these sections to include the strength and properties of materials used in construction. Under the heading Machines, arrangements of links and components which transmit forces and motion are studied, while under Structures, arrangements of members at rest which are firmly coupled together and capable of resisting the application of external forces are considered. The members or parts of both a machine and a structure receive external forces and suffer internal stresses and strains ; and the material from which they are made exhibits elastic resistance to such stresses. The primary difference between a machine and a structure is shown in the following statements :

In the case of a machine : (1) work is done by the applied force or forces which produce motion, and (2) energy is absorbed in operating the machine and in overcoming friction between the moving parts.

In the case of a structure : (1) work is only done in deflecting and straining the members, and (2) energy is stored in the strained material, to be mostly released on the removal of the load and *not* used to produce motion.

The two sections of the study are related in many ways and it is important for the student to realise that certain fundamental principles can be applied to both sections. For convenience of presentation, the effects of a force in producing motion will receive first consideration. Force and motion with velocity and acceleration are closely associated with work, power, momentum and energy, and a brief summary of the meaning of these terms, which were dealt with in Vol. I, will here serve as an introduction to Machines.

Rest and motion. These terms have been mentioned in the preceding paragraphs and it is necessary to obtain a clear conception of their meaning. Consider two points P and Q situated in space, and the possible ways of altering their positions. One point P is said to be at rest, or fixed, in relation to Q if the straight line joining P to Q does not alter in length or direction during the period of time under review. P is said to move in relation to Q if the straight line PQ alters in length or direction. This motion is called *rectilinear motion* if the straight line PQ changes in length but not in direction, and *angular or rotary motion* if PQ changes in direction but not in length. The motion is regarded as plane motion when all the changes of position occur in one plane, and when this plane is regarded as fixed or its motion neglected. Most of the problems on motion which confront the engineer are those of plane motion. *Motion is thus change of position with regard to some point or plane deemed to be, but not usually, fixed.* Hence all motion is relative motion.

To illustrate this point, consider a motor-car moving in a straight line on a level road. Actually, due to the earth's motion, the road has combined rectilinear and angular motion in relation to the fixed stars. If the motion and curvature of the earth are ignored, then the plane motion of the body of the motor-car will be rectilinear, while any point on the tyres will have combined rectilinear and angular motion. Now, if the car is regarded as at rest, the motion of the points in the car's engine and transmission will be plane motion; points on the pistons, valves and gudgeon pins will have rectilinear motion; while points on the cranks, crank shafts and flywheel will have angular motion. Points on the connecting rods will have combined linear and angular motion.

Speed and velocity. When the direction of motion of a point or body is not a matter of concern and the time factor becomes a direct consideration, the rate of motion is generally termed the **speed** of the point or body. **Velocity** is the name given to the rate of motion when the direction, sense and magnitude of the motion are completely specified. Thus, the velocity of an aeroplane might be given as 250 knots on a true bearing of 270° at a height of 5000 ft. above sea-level. Velocity and speed are said to be uniform when the changes in position are the same in equal intervals of time no matter how short those intervals of time may be. Otherwise, the velocity is termed non-uniform or variable velocity.

Rectilinear, or more briefly **linear velocity,** is measured in feet per second, feet per minute, miles per hour (m.p.h.) or knots. A knot is equal to 1 nautical mile (6080 ft.) per hour. The metric equivalent would be centimetres or metres per second, or kilometres per hour.

Angular or **rotary velocity** is usually measured in radians per second, revolutions per second or revolutions per minute (r.p.m.). Since a radian is the angle subtended at the centre of a circle by an arc equal in length to the radius, a point moving with linear velocity v ft. per sec. in a circle of radius r ft. will have an angular velocity ω equal to $\frac{v}{r}$ radians per sec.

$$\therefore\quad \omega=\frac{v}{r}=\frac{2\pi N}{60}\quad\text{and}\quad v=\omega r,$$

where N is the angular velocity in revolutions per minute.

A person swinging in an ordinary garden swing or a child on a see-saw may be regarded as moving either with variable linear velocity through the arc of a circle, or with variable angular velocity about the centre of oscillation or rotation.

Example. *An aeroplane is turning in a horizontal plane through an arc of a circle of* 5000 *ft. radius at a uniform speed of* 240 *m.p.h. Calculate its linear velocity in ft. per sec. and its angular velocity in radians per sec.*

60 m.p.h. = 88 ft. per sec., therefore 240 m.p.h. = 4 × 88 or **352 ft. per sec.**

$$\text{Angular velocity}=\frac{v}{r}=\frac{352}{5000}\text{ or }\mathbf{0{\cdot}0704\ radian\ per\ sec.}$$

Acceleration. When the velocity is variable, the rate of change of velocity is called **acceleration,** and is termed positive or negative according as the velocity is increasing or decreasing. Negative acceleration is often called **retardation.** Linear acceleration is the rate of change of linear velocity and is generally measured in feet per second per second, while angular acceleration is the rate of change of angular velocity and is measured as a rule in radians per second per second.

Example. *In the previous example, if the speed of the aeroplane increases uniformly from* 240 *to* 300 *m.p.h. while turning at the same radius in* 40 *sec., calculate the tangential linear and the angular acceleration.*

$$\text{Linear acceleration} = \frac{\text{Change of velocity}}{\text{Change of time}} = 60 \text{ m.p.h. or } \frac{88 \text{ ft. per sec.}}{40 \text{ sec.}}$$

$$= \mathbf{2{\cdot}2 \text{ ft. per sec. per sec.}}$$

$$\text{Change of velocity} = \frac{88 \text{ ft. per sec.}}{5000 \text{ ft.}} = 0{\cdot}0176 \text{ radian per sec.}$$

$$\text{Angular acceleration} = \frac{\text{Change in angular velocity}}{\text{Change of time}}$$

$$= \frac{0{\cdot}0176 \text{ radian per sec.}}{40 \text{ sec.}}$$

$$= \mathbf{0{\cdot}00044 \text{ rad. per sec. per sec.}}$$

Mathematical aspect of motion. Suppose a body to move in a straight line through a distance s to a point P in time t. Now imagine the body to move a further distance δs to a point Q in time δt, where δt is small. Over the distance PQ, the average velocity is $\delta s/\delta t$, and if δt is taken infinitely small it can be said that the limiting value of $\delta s/\delta t$, as δt approaches zero, is the instantaneous value of the velocity at P and is written ds/dt.

Uniform velocity is expressed by $ds/dt = \text{constant}$. If variable velocity can be expressed in the form $ds/dt = f(t)$, continuously through a range from $t = a$ to $t = b$, then

$$s = \int_{t=a}^{t=b} f(t)\, dt.$$

Similarly, if the velocity changes from v to $v + \delta v$ in the distance PQ in the time δt, then,

$$\text{Acceleration} = \frac{\text{Change of velocity}}{\text{Change of time}} = \frac{\delta v}{\delta t}.$$

In this case, $\delta v/\delta t$ is positive or negative according as δv is positive or negative. As δt approaches zero, the acceleration at P is given by the limiting value of $\delta v/\delta t$, that is, dv/dt.

Uniform acceleration is expressed by $dv/dt = \text{a constant}$. Should $dv/dt = f(t)$ represent variable acceleration continuously from $t = a$ to $t = b$, then

$$v = \int_{t=a}^{t=b} f(t)\, dt.$$

Transmission of motion. The problem of transmitting motion initiated at one point to other points is obviously important, and to solve it the engineer uses devices known as mechanisms. A **mechanism** consists of a link or body, or an assemblage of links or bodies, one of which is regarded as fixed. These links are so connected that a definite movement of any point on them must lead to a definite movement of all other points in the system except those on the fixed link. The links may consist of steel rods, chains, springs, leather belting or even fluids, as water and oil in hydraulic mechanisms. All these varying types of links or bodies can transmit motion in certain controlled circumstances, and all consist of what is called matter.

Matter is the name given to any existing substance or material irrespective of its constitution, state, condition or shape. A quantity of matter bounded by known limits is called a **body.** The **mass** of a body is the quantity of matter it contains.

When a body changes its state of rest or of uniform motion in a straight line, it is said to be under the action of a **force,** that is, a force always produces or tends to produce motion, or alteration of motion, either in direction or speed.

The **weight** of a body is the measure of the attractive force of the earth on the body. A mass can be weighed, and it is upon this fact that statements and calculations relating to mass are based. Thus the mass of a body remains unchanged, but its weight will vary with the distance of the body from the earth's centre. For British engineers, the standard of mass is the Imperial Standard Pound, a piece of platinum kept at the Standards Office in London. Thus, the body which weighs 10 lb. in London will have a mass 10 times the standard mass, because at any particular place at a fixed distance from the centre of the earth the mass of the body is proportional to its weight.

The **unit of force** is the weight of the standard mass in London. Imagine a mass of 10 lb. supported by a spring balance at sea-level in London ; then the force extending the spring, if the mass of the spring itself is neglected, is a force of 10 lb. weight, because the force stretching the spring is the attractive force of the earth acting on the mass of 10 lb. Thus, 10 lb. weight is a *force,* while 10 lb. mass is

10 times the quantity of matter contained in the standard pound. Although it is customary for engineers to omit the word weight as being understood and to refer to the magnitude of a force as being so many pounds or tons, it must be realised that pounds weight or tons weight is meant.

When the point of application of a force moves in the direction of the force against a resistance, the force is said to do **work.** The quantity of work done is measured by the product of the force and the distance measured in the direction in which the force acts. The units generally employed are the foot pound (ft. lb.), inch pound (in. lb.) and ft. ton.

Power is the time-rate of doing work. For example, if a load of 2200 lb. is raised 4 ft. against gravity in 2 sec., the work done is 8800 ft. lb. and the power developed will be

$$\frac{2200 \text{ lb.} \times 4 \text{ ft.}}{2 \text{ sec.}} = 2200 \text{ lb.} \times \frac{4 \text{ ft.}}{2 \text{ sec.}}$$

$$= \text{Force} \times \text{velocity} = 4400 \text{ ft. lb. per sec.}$$

It will be noticed that the units of power indicate that it is equivalent to force times velocity. The unit of power is the **horse power** (abbreviated to H.P.), which is equivalent to the rate of doing work at 33,000 ft. lb. per min. or 550 ft. lb. per sec. This rate of doing work was defined by James Watt in 1783. In the above illustration, 4400 ft. lb. per sec. is equal to $\frac{60 \times 4400}{33,000}$ or 8 H.P. The electrical unit of power is the **watt,** which is the power developed in a circuit when a current of 1 ampere flows under a pressure of 1 volt. One kilowatt (kW.) = 1000 watts and one H.P. = 746 watts.

Sources of power. The forces of Nature have been harnessed by man to supplement his own power, which does not exceed about $\frac{1}{8}$ H.P. Up to about the year 1750, the only natural power was obtained by wind on ship's sails and windmills and by running water on water wheels. Both these sources, as then used, were too weak and intermittent to supply power economically. Modern methods of using large quantities of water and large natural falls to generate electricity by hydro-electric plant and to convey the electric power so generated wherever it is required, are, of course, much more efficient. Unfortunately, only 1½ per cent of Britain's present requirements of power can be obtained in this way. The hydro-electric station at the Niagara Falls has a capacity of 1¼ million kW.

By far the most important source of power is the latent power available in fuels, such as coal and oil, in which cases the rate of combustion controls the amount of power developed, and where specially designed machines known as **engines** are required to convert the power to a usable form. Even in the best engines, the thermal efficiency is comparatively low and generally, as in the ordinary domestic grate, there is much waste of fuel and important by-products. It has been proposed that large electric power-stations, gas works and coke ovens be situated in close proximity to coal mines, so that even low-grade and waste coal can be used up and valuable by-products retrieved. The waste low-pressure process steam, coke oven gas, etc., can also be economically employed in local industry around the mines. It is hoped that by some such means efforts will be made to conserve the coal supplies of Britain.

As new coal fields have to be worked farther away from existing industrial centres, new problems involving transport of fuel and redistribution of industries will arise. Other sources of power may then become more economical. These sources, set out below, require expensive equipment, and are not likely yet to supersede, except to a small extent, the obtaining of power from fuel :

(1) Tidal power, using both ebb and flow of the tides.

(2) The sun's rays to evaporate sea-water allowed to flow into open shallow basins.

(3) The heat of the earth, by boring to a depth exceeding five miles or by using water and steam from hot natural springs.

(4) The utilisation of the sun's radiant heat in tropical belts to grow starch and sugar-producing vegetation, so that alcohol can be manufactured in a form suitable for fuel.

(5) The electricity of the atmosphere.

(6) Subatomic energy, or the releasing of the tremendous forces which hold the atoms together and the harnessing of those forces to do useful work. The latent energy in uranium is one million times that available in the same weight of coal.

Transmission of power. Generally, the engineer requires motion allied with force, so that the combination permits of power being transmitted.

A **machine** may be regarded as a mechanism which transmits force, and also as a contrivance by means of which a load may be raised or a resistance overcome to the advantage of the operator. If the resistance is overcome through a distance in a given time, then the machine is transmitting power. The various links of a machine must be strong enough to transmit force, and consequently the size and substance of these links will be appreciable ; force will also be required to move them.

Usually, the force applied to a machine is called the **effort.** For the ideal machine, the following principles apply.

By the **Principle of Work,**

$$\text{Work input} = \text{Work output (Vol. I., p. 124)} \quad \ldots\ldots\ldots\ldots (a)$$

Now, since the time factor is the same for input and output of work, then :

By the **Principle of Power,**

$$\text{Power input} = \text{Power output}, \quad \ldots\ldots\ldots\ldots (b)$$

that is,

Effort × velocity, or speed, of effort = Load × velocity of load. ...(*c*)

Statement (*c*) should be studied carefully, because of its immense importance. It shows why a big force, such as a load, acting at a slowly moving point, can be balanced by a smaller force, such as the effort, at a quickly moving point of application. It should be emphasised that the quickly moving portion can be of much lighter construction than the slowly moving portion of the machine.

$$\text{Rearranging } (c), \quad \frac{\text{Velocity of effort}}{\text{Velocity of load}} = \frac{\text{Load}}{\text{Effort}}, \quad \ldots\ldots\ldots\ldots (d)$$

or **Velocity Ratio** (V.R.) = **Mechanical Advantage** (M.A.). ......(*e*)

In practice, the mechanical advantage, or ratio of load to effort, is always less than the velocity ratio, due to the fact that the actual load raised, or resistance overcome, is less than the theoretical or ideal. This is due to frictional losses, some of the effort being used in overcoming friction in the machine ; but the relative velocities will remain the same whatever the magnitudes of the efforts and loads. The ideal load, or greatest resistance which can be overcome by a given effort, can be calculated from the product

$$\text{Effort} \times \text{velocity ratio}. \quad \ldots\ldots\ldots\ldots\ldots\ldots(f)$$

The statement (*a*) above should be modified in actual machines into the form :

$$\text{Work input} = \text{Work output} + \text{Work done in overcoming friction}. \quad \ldots\ldots(g)$$

From statement (*g*), it follows that the efficiency of any transmission of power by a machine is given by the ratio

$$\frac{\text{useful power output}}{\text{power input}}.$$

As mentioned above, the time factor is the same for both output and input, and therefore this fraction for efficiency may be expressed in other ways (see Vol. I, p. 125), as the following will show.

$$\text{Efficiency} = \frac{\text{Actual load} \times \text{velocity of load}}{\text{Effort} \times \text{velocity of effort}} = \frac{\text{Work output}}{\text{Work input}} = \frac{\textbf{M.A.}}{\textbf{V.R.}}$$

$$= \frac{\text{Actual load}}{\text{Effort} \times \text{velocity ratio}} = \frac{\text{Actual load}}{\text{Ideal load}}. \quad \ldots\ldots\ldots\ldots\ldots(h)$$

When calculating the efficiency of a machine, any of the above fractions may be used and the fraction converted to a percentage if desired.

Velocity ratio of machines. Statement (*f*) above is very important, because as power is transmitted through a machine a knowledge of the relative velocities of the various parts enables the designer to decide on the forces being brought into play and to design accordingly. A different and more experimental treatment of typical machines will be found in Vol. I, pp. 123-150.

The Lever. The ordinary see-saw is one of the simplest machines of the lever type. It is common knowledge that a light boy *A* of weight *E* (Fig. 1) acting as the effort will be much farther from the pivot than a heavier boy *B* of weight *L* acting as the load. Also, when motion starts, *A* will move much faster than *B*. If θ is measured in radians, the distances moved by the forces *E* and *L* will be $r_1\theta$ and $r_2\theta$ respectively. Therefore by the Principle of Power,

$$Er_1\theta = Lr_2\theta \text{ or } Er_1 = Lr_2. \quad \ldots\ldots\ldots\ldots\ldots\ldots(i)$$

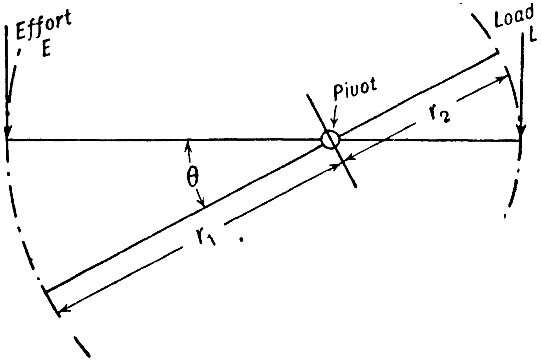


FIG. 1.

It is assumed that the forces act at right angles to the lever. The results can be verified by experiment ; thus

(*a*) the velocity ratio of A to B will equal r_1/r_2.

(*b*) the velocities of A and B and the distances moved through by A and B will be inversely proportional to the weights E and L.

Er_1 and Lr_2 are called the **moments of the forces** E and L about the pivot or fulcrum, since each quantity is the product of the force and the perpendicular distance between the pivot and the line of action of the force. The moment of a force measures its turning effect, and in conjunction with the angular velocity is also a measure of the power available.

Equation (*i*) on p. 9 is a particular case of the **Principle of Moments,** which states that for equilibrium, or balance, the algebraic sum of the moments of the forces about any one point is zero. Clockwise moments are of opposite sign to anticlockwise moments.

Since no machine is perfect, for E to balance L, either r_1 would have to be increased, or r_2 reduced from the theoretical figures obtained from equation (*i*). The efficiency can be obtained by using one of the expressions (*h*).

Weston differential pulley block. This consists of a differential block A and a snatch block B connected by an endless chain. If A is given one complete clockwise revolution in 1 second by the effort

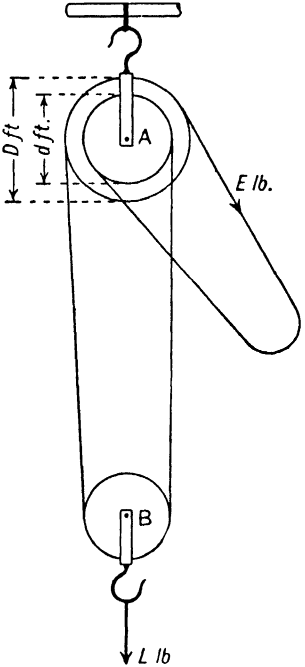


FIG. 2.

E, the large pulley on A will take in πD ft. of chain, while the smaller will pay out πd ft. The difference between πD and πd is shared by the two parts of the chain carrying the snatch block. Hence the snatch block and the load will rise $\frac{1}{2}(\pi D - \pi d)$ ft., while the effort will move πD ft. E will thus move at πD ft. per sec., and L at $\frac{1}{2}(\pi D - \pi d)$ ft. per sec.

Therefore, the velocity ratio will be $\pi D \div \frac{1}{2}(\pi D - \pi d) = 2D/(D-d)$. The theoretical or ideal load raised = Effort × velocity ratio

$$= E \times \frac{2D}{D-d}.$$

Belt, rope and chain drives. Assuming that the belt, rope or chain is inextensible, its speed will be constant at all points. Hence

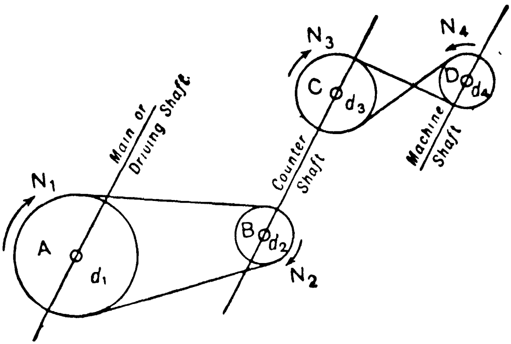


FIG. 3.

for two pulleys A and B, connected by open belt, $\pi d_1 N_1 = \pi d_2 N_2$ (Fig. 3), or,

$$\text{the velocity ratio} = \frac{\text{Speed of A}}{\text{Speed of B}} = \frac{N_1}{N_2} = \frac{d_2}{d_1} = \frac{\text{Dia. of follower}}{\text{Dia. of driver}}\,.$$

For multiple countershaft drives, the velocity ratio is

$$\frac{\text{Speed of A}}{\text{Speed of D}} = \frac{N_1}{N_4} = \frac{d_2 \times d_4}{d_1 \times d_3} = \frac{\text{Product of follower diameters}}{\text{Product of driver diameters}}\,.$$

This result can be readily understood if simple values are given such as $d_1 = d_3 = 2$ ft. and $d_2 = d_4 = 1$ ft. Then the main shaft will rotate at $\frac{1 \times 1}{2 \times 2}$ or $\frac{1}{4}$ the speed of the machine shaft, as will appear obvious by mere inspection of the original problem.

In all problems involving the determination of velocity ratio, it is a good policy in the initial stages to employ simple numerical values for the various parts, as it assists the understanding of problems and the derivation of formulae, which prove useful when more complicated numbers have to be used and when results become more difficult to visualise.

The use of multiple shafts enables a high velocity ratio to be obtained without very large and very small pulleys being in close proximity. It will be noticed that the crossed belt between pulleys

C and D (Fig. 3) produces motion in the *opposite* sense, but the velocity ratio is not altered in magnitude.

Actually, the diameters of the pulleys should be taken to the neutral section of the belting for accurate evaluation of the velocity ratio. The neutral section would be that longitudinal section of the belt which is not stretched or compressed as it is bent round the pulley. The position of this neutral section depends on the type of belting.

Belt of thickness, t	Effective Diameter of Pulley
Single leather (Grain of leather to pulley)	$D+\frac{5}{4}t$
Double ,,	$D+t$
Single ,, (Flesh side of leather to pulley)	$D+\frac{3}{4}t$
Woven or Fabric belting	$D+t$

The process of compounding can be continued to any extent to give the required velocity ratio. Guide or jockey pulleys are used to guide the belts which transmit motion between two pulleys which are not in the same plane. In using jockey pulleys the guiding principle for transference of motion is that the point at which the belt leaves one pulley must lie in the plane of the other pulley on to which it is advancing.

Friction and toothed gearing. In this type of gearing for transmitting power, the wheels are in contact and intermeshing teeth prevent slipping. It should be remembered that a pair of ordinary toothed wheels are equivalent to two cylinders running in contact. The diameter of these cylinders is called the diameter of the pitch circles.

Circular pitch is the name given to the distance measured on the arc of the pitch circle between corresponding points on two consecutive teeth. The **diametral pitch** is obtained by dividing the number of teeth by the pitch circle diameter in inches. With two wheels meshing externally as A and B (Fig. 4), the motion of the follower B is in the opposite direc-

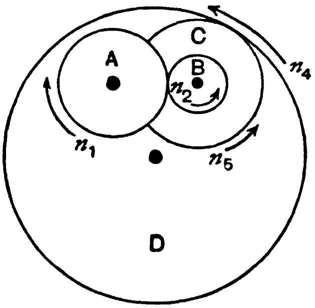


FIG. 4.

tion, similar to a crossed belt drive between two pulleys. The insertion of an idle wheel in between will cause the follower to rotate in the same sense as the driver, producing the same relative motion as with the open belt drive. The idle wheel will not affect the velocity ratio. For internal meshing, as in Fig. 4, the motion of the two wheels C and D is in the same sense. Wheels A and C are drivers and B and D followers. Since the number of teeth on a wheel is proportional to its diameter, the velocity ratio is similar to that of belt drives.

$$\frac{\text{Revs. of A}}{\text{Revs. of D}} = \frac{\text{Product of number of teeth on followers}}{\text{Product of number of teeth on drivers}}.$$

Internal gears can be usefully employed where space is limited, because the pinion is inside the wheel (see, for example, C and D in Fig. 4). With chain drives, slipping is prevented and the velocity ratio may be calculated on the basis of wheel diameters or on the basis of number of teeth per wheel.

Bevel or mitre wheels are employed if the axes of the shafts to be connected intersect at an angle. In this case, the teeth must be cut on wheels in the form of truncated cones instead of on cylinders as in ordinary meshing. The velocity ratios can be obtained as with ordinary toothed and helical gearing.

The screw. The pitch of a screw is the distance advanced or retracted by the nut during one complete revolution of the screw. With single start threads, the distance between any two consecutive threads is equal to the pitch, while with multiple start threads the distance between any two consecutive threads will equal

$$\frac{\text{pitch}}{\text{no. of starts}} \text{ (Vol. 1, p. 145).}$$

If, to turn the screw, the effort is applied at right angles to an arm of radius R and the load is taken by the nut, then the velocity ratio will be $\frac{2\pi R}{\text{pitch}}$.

With the Hunter Differential Screw the nut to the larger screw is fixed, while the larger screw is turned by a power handle of effective radius R. A smaller screw threaded inside the larger screw is fixed to a block which cannot rotate, so that the smaller screw can only move in the direction of its length. The block on the smaller screw supports the load or overcomes the resistance. By making the difference in the screw pitches very small a large velocity ratio can be obtained.

$$\text{Velocity ratio} = \frac{\text{Circumference of power arm}}{\text{Difference in pitches of screws}}.$$

The worm and worm wheel. A very high velocity ratio is obtainable with this machine. The angular velocity ratio between the worm wheel shaft and the worm shaft is (Vol. I, p. 133)

$$\frac{\text{No. of teeth on worm wheel}}{\text{No. of starts on worm}}.$$

N.B. It is characteristic of all screw machines that the efficiency is low and the losses high.

Example 1. *A simple crane or derrick is shown diagrammatically in Fig. 5. E is the force in the tie AC and L the load carried by the jib BC. Imagine a stiff spring fitted in the tie to allow of a small displacement. Find the velocity ratio between E and L.*

The velocities of E and L during this displacement will depend on the lengths of BX and BZ respectively.

$$\text{Velocity ratio} = \frac{\text{BX}}{\text{BZ}}.$$

$$L = E \times \text{velocity ratio} = E \times \frac{\text{BX}}{\text{BZ}} \text{ or } L \times \text{BZ} = E \times \text{BX}.$$

This is the same result as would be obtained by using the Principle of Moments and taking moments about B. The problem is identical with that of finding the velocity ratio for the bell crank lever XBZ pivoted at B under the action of the forces E and L.

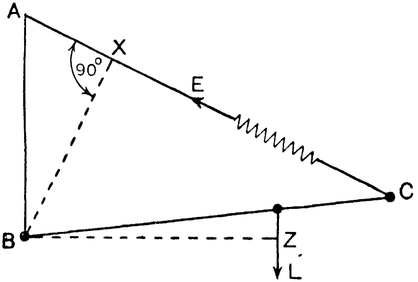


Fig. 5.

Actually, this is a structural problem, but it has been solved by imagining a slight displacement so that the method applied to machines can be employed. The imaginary displacement is termed a **virtual displacement.**

Example 2. *The sluice gate of a lock is operated by an electric motor through a single-worm reduction gear. The wheel of the worm gear is keyed to a long screw which raises or lowers a crosshead, which, in turn, operates the paddles of the sluice gate. The crosshead acts as a nut to the screw. The following particulars are given :*

No. of starts on worm $= 2$; *no. of teeth on worm wheel* $= 36$; *pitch of screw* $= 1$ *in.* ; *motor speed* $= 720$ *r.p.m.* ; *maximum pull required at screw* $= 24$ *tons* ; *travel of crosshead* $= 6$ *ft.* 8 *in.*, *efficiency of worm gear* $= 87{\cdot}5\%$; *efficiency of screw and nut* $= 33\frac{1}{3}\%$. *Calculate,* (1) *the worm reduction ratio,* (2) *the crosshead velocity,* (3) *the time to operate the sluice paddles, and* (4) *the brake horse power of the motor.*

(1) $$\text{Worm reduction ratio} = \frac{\text{No. of teeth on worm wheel}}{\text{No. of starts}} = \frac{36}{2} = \mathbf{18.}$$

(2) $$\text{Speed of screw} = \frac{\text{Motor speed}}{\text{Reduction ratio}} = \frac{720}{18} = 40 \text{ r.p.m.}$$

Crosshead velocity

$$= \text{Speed of screw} \times \text{pitch} = 40 \times 1 \text{ in.} = \mathbf{3\tfrac{1}{3}\ ft.\ per\ min.}$$

(3) $$\text{Time to operate paddles} = \frac{\text{Travel}}{\text{Crosshead velocity}}$$

$$= 6\tfrac{2}{3} \text{ ft.} \div 3\tfrac{1}{3} \text{ ft. per min.} = \mathbf{2\ minutes.}$$

(4) $$\text{Power applied to screw} = \text{Load} \times \text{velocity}$$

$$= 24 \times 2240 \times 3\tfrac{1}{3} \text{ ft. lb. per min.}$$

$$= 179{,}200 \text{ ft. lb. per min.}$$

Overall efficiency of worm gear and screw

$$= 87{\cdot}5\% \times 33\tfrac{1}{3}\% = 29{\cdot}17\%.$$

$$\text{B.H.P. of motor} = \frac{179{,}200}{0{\cdot}2917 \times 33{,}000} = \mathbf{18{\cdot}6.}$$

Example 3. *An electric motor is used to operate a sliding table through a double-reduction chain drive and a rack and pinion. The chain wheel drivers have* 20 *teeth each while the followers have* 120 *teeth. The first driver is keyed to the motor shaft and the last follower to the pinion shaft. There are* 25 *teeth on the pinion of diametral pitch* 5. *The motor speed is* 720 *r.p.m. and its output is* $\frac{3}{4}$ *H.P. If the chain pitch is* $\frac{1}{2}$ *in., calculate* *(a) the speed of each chain and their velocity ratio, (b) the table or rack speed, (c) the ideal turning moment on the pinion, (d) the maximum pressure between the teeth on the rack and pinion.*

(*a*) 1st chain speed $=$ Motor speed $\times$ effective periphery of chain wheel

$$= \frac{720}{60} \times \frac{20}{12} \times \frac{1}{2} = \textbf{10 ft. per sec.}$$

2nd chain speed $=$ Countershaft speed $\times$ chain wheel periphery

$$= \frac{720}{60} \times \frac{20}{120} \times \frac{20}{12} \times \frac{1}{2} = \textbf{1}\tfrac{2}{3}\textbf{ ft. per sec.}$$

$$\text{Velocity ratio} = \frac{\text{1st chain speed}}{\text{2nd chain speed}} = \frac{10}{1\frac{2}{3}} = \mathbf{6} = \frac{\text{Speed of 1st driver}}{\text{Speed of 1st follower}}.$$

(*b*) Velocity ratio between motor and pinion shafts $= \frac{120}{20} \times \frac{120}{20} = 36.$

$$\text{Pitch circle diameter of pinion} = \frac{\text{No. of teeth}}{\text{Diametral pitch}} = \frac{25}{5} = 5 \text{ in.}$$

$$\text{Speed of pinion} = \frac{720}{36} = 20 \text{ r.p.m.}$$

$$\therefore \text{ Speed of rack} = \frac{20 \times 5\pi}{12} \text{ or } \textbf{26·18 ft. per min.}$$

(*c*) Neglecting losses, work done by pinion per min.

$=$ Output of motor per min.

Force $\times$ radius of pitch circle $\times$ 2π $\times$ r.p.m. $= \frac{3}{4} \times 33{,}000$ ft. lb.

or Turning moment $\times\ 2\pi \times 20 = 24{,}750$ ft. lb.

$$\therefore \text{ Turning moment} = \frac{24{,}750}{20 \times 2\pi} \text{ or } \textbf{197 lb. ft.}$$

(*d*) Turning moment on pinion $=$ Pressure between teeth $\times$ Pitch circle radius.

12×197 in. lb. $=$ Pressure in lb. $\times\ 2\frac{1}{2}$ in.

$$\text{Pressure in lb.} = \frac{12 \times 197}{2\frac{1}{2}} \text{ or } \textbf{945·6 lb.}$$

Example 4. *The diagram, Fig. 6, illustrates in one view the power transmission system for a vertical boring and turning machine. The machine shaft is driven by a countershaft which is running at 250 r.p.m. and which has stepped cone pulleys of equal diameters to those shown on the machine shaft. Pinion J is attached to the stepped pulley and both revolve freely on shaft A, while wheel F is keyed to shaft A. Wheel F is engaged with the stepped cone for ordinary drive but is disengaged for the back gear drive, when wheels H and G are also swung into gear with J and*

F respectively. Find the speed of the turntable in revolutions per minute in each of the available speeds (*a*) *without back gear,* (*b*) *with back gear.*

Velocity ratio between the turntable and shaft $\mathsf{A} = \frac{18}{45} \times \frac{25}{125} = \frac{2}{25}$.

(*a*) Without back gear. For top speed of turntable the 19 in. pulley on the countershaft drives the 13 in. pulley on the stepped cone pulley.

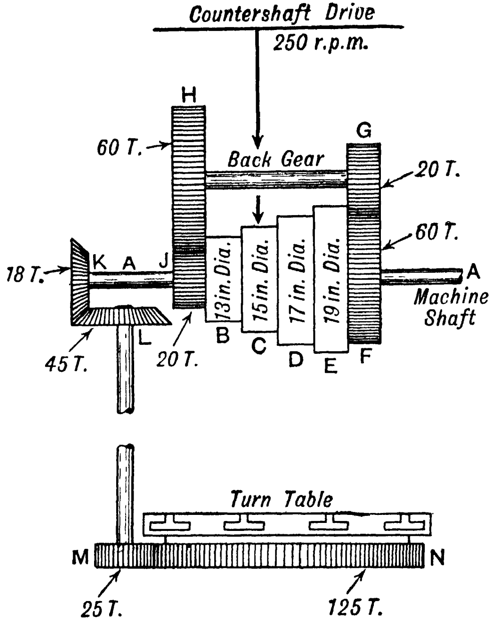


FIG. 6.

Top speed $= 250 \times \frac{19}{13} \times \frac{2}{25} = 29\frac{3}{13}$ r.p.m.

Second speed $= 250 \times \frac{17}{15} \times \frac{2}{25} = 22\frac{2}{3}$ r.p.m.

Third speed $= 250 \times \frac{15}{17} \times \frac{2}{25} = 17\frac{11}{17}$ r.p.m.

Fourth speed $= 250 \times \frac{13}{19} \times \frac{2}{25} = 13\frac{13}{19}$ r.p.m.

(*b*) With back gear. In this case the velocity ratio between the stepped pulley BCDE and shaft $\mathsf{A} = \frac{20}{60} \times \frac{20}{60} = \frac{1}{9}$.

The four speeds now available will each be $\frac{1}{9}$th of those above.

Fifth speed $= 29\frac{3}{13} \times \frac{1}{9} = 3\frac{29}{117}$ r.p.m.

Sixth speed $= 22\frac{2}{3} \times \frac{1}{9} = 2\frac{14}{27}$ r.p.m.

Seventh speed $= 17\frac{11}{17} \times \frac{1}{9} = 1\frac{147}{153}$ r.p.m.

Eighth speed $= 13\frac{13}{19} \times \frac{1}{9} = 1\frac{89}{171}$ r.p.m.

Momentum. The momentum of a body may be regarded as the quantity of motion it possesses and is measured by the product of its mass and velocity. Thus

$$\text{Momentum} = \frac{WV}{g} \text{ in engineer's units of momentum,}$$

where W is the weight and W/g is the mass of the body in engineer's units and V the velocity in ft. per sec.

If a body suffers a change of momentum and the velocity changes from V_1 to V_2, the change of velocity is $V_2 - V_1$, which is related to the acceleration $(V_2 - V_1)/t$, if t is the time of the change in seconds. Since force equals mass times acceleration, it follows that the rate of change of momentum equals

$$\frac{W}{g}\frac{(V_2 - V_1)}{t} = \frac{W}{g} \times \text{acceleration,}$$

and the rate of change of momentum in engineer's units equals force producing that rate of change. (See Vol. I, p. 262, and also p. 261.)

Example. *The speed of a sliding part of a mechanism of mass 7 lb. is reduced by constraint from 5 ft. per sec. to 2 ft. per sec. in 0·04 sec. Calculate the average force applied through the constraint.*

Force = Rate of change of momentum.

$$F = \frac{7}{g}\left\{\frac{5-2}{0{\cdot}04}\right\} = \frac{2100}{4g} = 16{\cdot}3 \text{ lb.}$$

Energy. In Engineering Science the term energy means capacity or ability to do work. If energy is due to relative position, or the static storage of energy as in a spring, it is called **potential** energy. The energy of a compressed spring or of water in a lake high up amid mountains is potential. If the compression on the spring or the water be released, both are capable of doing work. If the energy is due to motion, either translatory or rotary or both, then the energy is termed **kinetic.** A revolving flywheel or the tup of a hammer about to strike a forging possess kinetic energy. Apart from these mechanical forms, energy may also take the form of

chemical, electrical, heat, light and sound energy. The various types of energy can be changed in form, but the actual total quantity can never be destroyed. The latter statement is known as the **Principle of Conservation of Energy.** The change of energy from one form to another is usually extremely wasteful, inasmuch as it is almost impossible to convert one form of energy completely to another form, without a dissipation of some of the energy in another and unusable form. This dissipation may be regarded as a loss from the conversion point of view, although the lost energy is not destroyed. For example, in the ordinary filament lamp used to change electrical to light energy, most of the electrical energy is lost so far as lighting is concerned by conversion to heat energy.

The following constants may be employed in calculations on the change of energy, and represent a rate of exchange for efficient conversion of mechanical and heat or electrical energy.

Mechanical and heat energy.

778 ft. lb. of work are equivalent to 1 **British Thermal Unit** (B.Th.U.); that is, the average heat required to raise 1 lb. of water through 1 Fahr. degree.

1400 ft. lb. of work are equivalent to 1 **Centigrade Heat Unit** (C.H.U.); that is, the average heat required to raise 1 lb. of water through 1 Cen. degree.

The above constants are known as the **Mechanical Equivalent of Heat** or **Joule's equivalent** in different units.

For larger changes,

1 horse power hour (h.p. hour), that is, 1 horse power maintained for 1 hour or $33{,}000 \times 60$ ft. lb. of energy is equivalent to 1414 C.H.U. or 2545 B.Th.U.

Mechanical and electrical energy.

1 ft. lb. is equivalent to 1·356 **joules or watt-seconds** (1 watt-second = 1 watt maintained for 1 second).

1 h.p. hour is equivalent to 0·746 B.T.U. (Board of Trade Unit), where 1 B.T.U. = 1 kW. maintained for 1 hour (1 kWh.). This must not be confused with B.Th.U. (see below).

1 kWh. is equal to 3,600,000 joules or watt-seconds.

Heat to electrical energy.

1 gram calorie = 4·187 joules and 1 B.Th.U. = 1055 joules.
1 kWh. = 1 B.T.U. = 1896 C.H.U. = 3412 B.Th.U.

Example. *An engine develops* 100 *Brake Horse Power at* 360 *r.p.m. and is used to drive a generator at* 1400 *r.p.m. through a belt drive having a velocity ratio of* 4. *Calculate* (*a*) *the percentage belt slip,* (*b*) *the electrical input and output of the generator on a power basis if its efficiency is* 90%, (*c*) *the energy available in C.H.U. if the output is maintained for one hour.*

(*a*) $\text{Slip} = \left(\frac{4 \times 360 - 1400}{1400}\right) 100 = 2{\cdot}86\ \%.$

(*b*) $\text{Input to generator} = \left(\frac{100 - 2{\cdot}86}{100}\right) \times 100$ or **97·1 H.P.**

$\text{Output of generator} = \frac{90}{100} \times 97{\cdot}1 = 87{\cdot}4\ \text{H.P.}$

$= 0{\cdot}746 \times 87{\cdot}4 = \mathbf{65{\cdot}2\ kW.}$

(*c*) $\text{Energy available} = 65{\cdot}2\ \text{kWh.} = 65{\cdot}2 \times 1896\ \text{C.H.U.}$

$= \mathbf{123{,}640\ C.H.U.}$

EXERCISES ON CHAPTER I

1. Explain why rest and motion, as used in Engineering Science, are actually relative terms. Distinguish between speed, velocity and acceleration.

2. What is a radian? Make the following conversions :

   (*a*) 20 ft. per sec. to miles per hour,
   (*b*) 40 knots to ft. per sec.,
   (*c*) 1·5 ft. per sec. per sec. to miles per hour per hour,
   (*d*) 2400 revs. per min. to radians per second,
   (*e*) 2·5 radians per sec. per sec. to revs. per min. per min.,
   (*f*) 42 revs. per min. per min. to radians per sec. per sec.

3. What is the linear speed in ft. per sec. of a train rounding a curve of ·25 chains radius at 45 m.p.h.? What is the angular velocity in rad. per sec.? If the speed of the train gradually decreases to 36 m.p.h. in 4 sec., what are the linear and angular retardations?

4. The angular velocity of a flywheel is increased from 90 to 120 r.p.m. in 15 sec. Calculate the angular acceleration in radians per sec. per sec. and the tangential acceleration of a point on the rim of the flywheel 2 ft. from its axis.

5. What is a mechanism? Differentiate between a mechanism and a machine. Why is a fast-moving part of a machine generally not so heavily constructed as a slow-moving part?

6. Distinguish clearly between the terms mass and weight. A machine is placed on rollers and raised up an inclined plane. Explain how you would calculate the work done against gravity.

7. Define force and velocity ratio. In the ideal machine, what is the connection existing between the velocity ratios of any two moving parts and the forces employed there?

8. A force of 400 lb. acting along a rope inclined at 30° to the horizontal is used to drag a machine weighing 1500 lb. along level ground. How many ft. lb. of work are done if the machine is dragged 20 ft.?

9. A cable of diameter 2 inches weighing 6·456 lb. per ft. raises 17,000 lb. of ore in a skip weighing 9470 lb. from a depth of 3500 ft. in 1 minute. Calculate the amount of work done in ft. tons.

10. Define work and explain clearly how it is measured. A truck weighing ½ ton rests on horizontal rails with its centre at a point where the radius of the circular track is 500 ft. A horizontal force of 200 lb. is applied continuously to the truck and kept at an angle of 45° to this initial radius, and the truck moves 40 ft. along the rails. Calculate the work done by this force. What is the work done against gravity?

11. What meaning does the engineer attach to the word "power"? What are the sources of natural power? Indicate briefly how this natural power is converted to use.

12. A dredger can lift 1500 tons of mud and sand in 41 minutes from an average depth of 50 ft. Neglecting losses, calculate the work done and the average useful horse power developed.

13. An escalator has 100 steps each 8 in. high and the journey from top to bottom takes 96 sec. If when fully loaded it carries 32 stones on each step, calculate the horse power of the motors, assuming an overall efficiency of 60%.

14. What do you understand by the Principles of Work and Power as applied to devices for transmitting work and power? What is the general name given to such devices?

15. What is the moment of a force? State the Principle of Moments. How can this principle be verified experimentally?

16. A grinding machine spindle is driven from a main shaft revolving at 120 r.p.m. The drivers on the two countershafts employed are 18 in. and 12 in. in diameter and the followers each 8 in. in diameter. If the pulley on the main shaft is 24 in. in diameter and on the machine spindle is 4 in. in diameter, calculate the velocity ratio between the machine and main shafts. Taking the efficiency of the transmission of power as 60%, find the tangential force available at the periphery of a grinding wheel 12 in. in diameter per horse power available at the main shaft.

17. Part of the drive used in a sand-blasting machine consists of a double-reduction chain drive in which the drivers on the motor shaft and first countershaft have 30 teeth each, while the followers have respectively 108 and 120 teeth. Calculate the velocity ratio of the drive. What is the velocity ratio between the two chain speeds if the chain pitch is $\frac{1}{2}$ inch in each case?

18. Fig. 7 shows a chain drive used to operate the horizontal spindles P, Q, R to which are keyed chain wheels F, G and H, each having 24 teeth. Motor-chain wheel A with 20 teeth rotates at 720 r.p.m. and drives chain wheels C and E which have 100 teeth each. Chain wheels B and D have 16 teeth each and are keyed to the same shafts as C and E respectively. Calculate the velocity ratio between the motor shaft and the spindles and also their actual velocity. What is the velocity ratio between the chains at X and V if the chain pitch is the same for each chain? In theory, what should be the relative strengths of the chains at X and V?

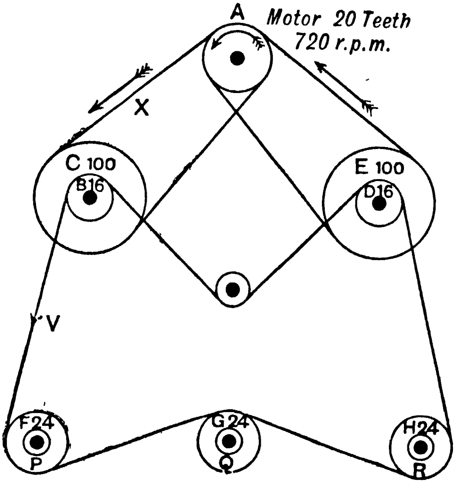


FIG. 7.

19. In the double-reduction helical gear for a turbine, a wheel with 50 teeth on the turbine shaft drives a follower with 330 teeth. Keyed to the same shaft as the latter is another pinion with 90 teeth, which drives a follower with 630 teeth on the propellor shaft. Calculate the propellor speed if the turbine rotates at 3250 r.p.m. Also calculate the ideal turning moment available at the propellor shaft per H.P. delivered by the turbine.

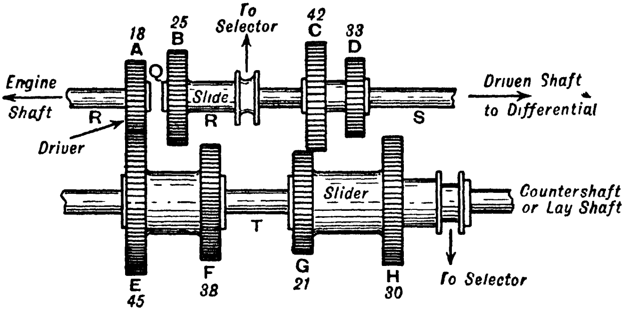


FIG. 8.

20. Fig. 8 shows a four-speed gear in which the driving shaft is R, S the driven shaft and T the lay or countershaft. All the wheels are keyed, but B, G and H can be made to slide along their shafts. Calculate the speeds of revolution of shafts for each gear, if R revolves at 1000 r.p.m. For the first speed C and G are in gear; for the second, D and H; for the third, B and F; while for the fourth speed the shaft R drives S directly through a clutch at Q. The number of teeth on each wheel is marked alongside the wheel in the figure.

21. A variable-speed belt drive is shown in Fig. 9 in which the belt may be moved along the pulleys B and C by a central striking fork and screw. The pulley A is 14 in. in diameter and revolves at 100 r.p.m., and the taper pulleys B and C taper from 6 in. to 11 in. diameter,

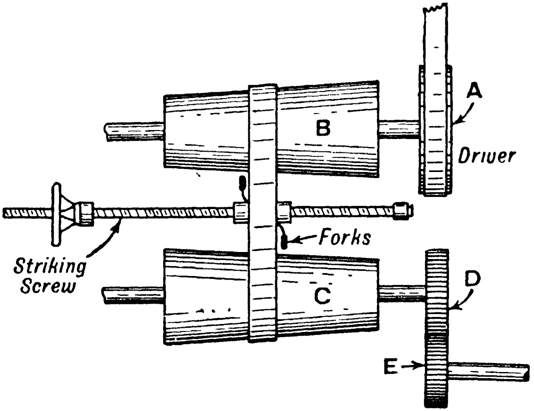


FIG. 9.

measured at the centre of the belt. The drive is completed by two toothed wheels D and E with 80 and 32 teeth respectively. Calculate, (*a*) the maximum and minimum velocity ratios between A and E; (*b*) the effective belt pull at A, if the H.P. received is 2; (*c*) the maximum pressure between the teeth at D and E when 2 H.P. is being transmitted. Neglect all losses. The diametral pitch is 5.

22. Calculate the speed of the driven shaft in the double reduction and reversal arrangement shown in Fig. 10 when the motor shaft speed is 1200 r.p.m. If the motor efficiency is 85% and the gear efficiency is 60%, find the normal force in a plane at right angles to the shaft and at a radius of 6 in. from the centre of the driven shaft, if the input of the motor is 6 kW.

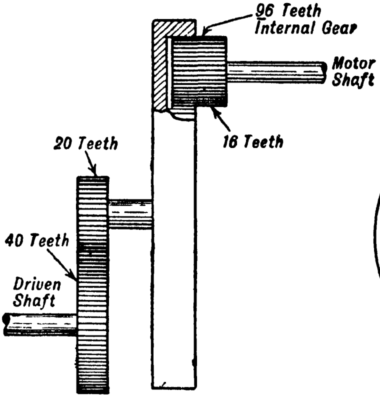


FIG. 10.

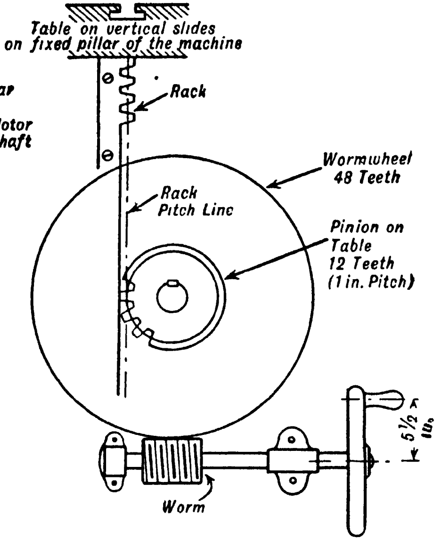


Table on vertical slides on fixed pillar of the machine
Rack
Wormwheel 48 Teeth
Rack Pitch Line
Pinion on Table 12 Teeth (1 in. Pitch)
5½ ins
Worm

FIG. 11.

23. The table of a drilling machine Fig. 11 is raised or lowered along its vertical slides by a pinion working against a rack on the vertical face of the frame of the machine. The pinion is rotated by a handwheel through a worm and worm-wheel. If the weight of the table is 400 lb. and the work on it weighs 500 lb., find the least force at the handle to raise and lower the table if the overall efficiency is 33⅓%.

24. The mechanism of a Bourdon Pressure Gauge is shown in Fig. 12. The sector and pinion are cut to a metric module pitch (Vol. I, p. 165) of $\frac{1}{4}$. Find (*a*) the movement of the end of the indicator needle, which is of radius 75 mm., for a movement of 0·3 mm. on the pressure tube, (*b*) the velocity ratio existing between the needle movement and that of the pressure tube, (*c*) the movement of the pressure tube to register 100 lb. per sq. in. on the dial if the dial scale is to show 1 in. of needle movement for 100 lb. per sq. in. pressure in the pressure tube.

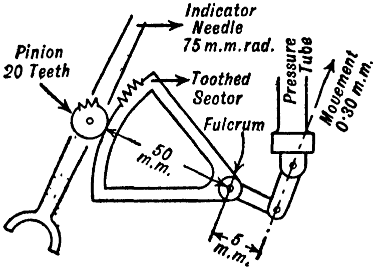


FIG. 12. Mechanism of a Bourdon Pressure Gauge.

25. Fig. 13 shows an arrangement for taper turning on a lathe in which the cross feed of the tool box is geared through the leading screw, which operates the saddle, to the lathe mandril. Find the taper, that is, twice the transverse movement of the tool, for each one-foot traverse of the saddle along the leading screw.

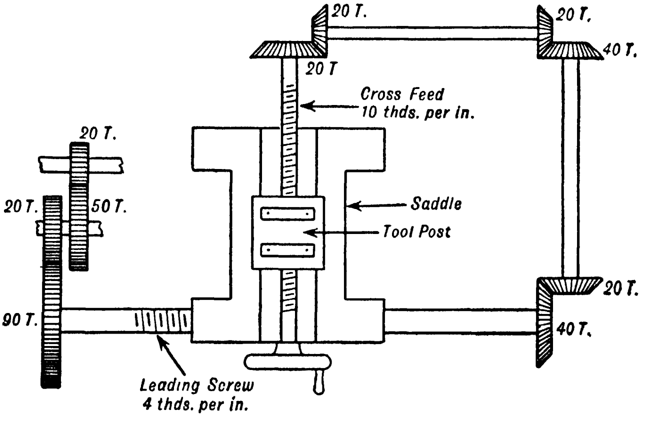


FIG. 13.

26. Fig. 14 shows the train of wheels, pinion and rack for moving the table of a planing machine along horizontal slides. The number of teeth on each wheel is marked alongside the wheel in the figure, while the rack has teeth of 1¼ in. pitch. If the driver revolves at 30 r.p.m. and is receiving 1 H.P., find the speed of the table in ft. per min. and the force urging it forward if the efficiency of transmission is 60%.

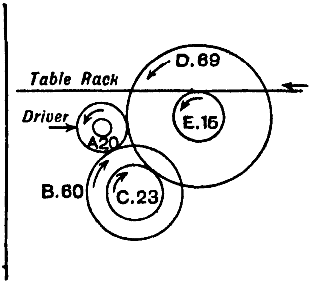


FIG. 14.

27. In an automobile factory a travelling assembly platform driven by an electric motor uses a combined spur, worm and double-reduction gearing shown diagrammatically in Fig. 15, where the numbers refer to the number of teeth on the gear wheels. The worm is two-start and operates on a worm-wheel of 40 teeth. The wheels are keyed to shafts A, B, C, D, E. If the shaft E drives the platform, calculate the velocity ratio between the motor shaft A and the shaft E, and the ideal turning moment which can be exerted on shaft E for each horse power delivered by the motor, when the speed of the latter is 1200 r.p.m.

28. A fuel distributor in the form of a large screw of 1½ ft. pitch is used to supply a boiler. This screw, as it turns, propels the fuel forward to the required feeding position, and is driven by a motor through a

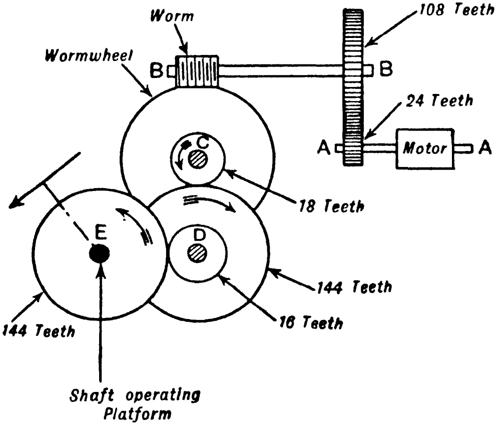


FIG. 15.

double-reduction worm and worm-wheel gear. Each worm is double-start with worm-wheels having 60 and 64 teeth. Calculate (*a*) the velocity ratio between the revolutions of the motor and screw, (*b*) the H.P. of the motor when its speed is 1200 r.p.m. and the axial thrust on the screw is 1 ton. Assume an overall efficiency of 20%.

29. With a Hunter differential screw the power handle has an effective radius of 2 ft. and the screw pitches are $\frac{1}{4}$ in. and $\frac{7}{32}$ in. respectively. Calculate the maximum axial resistance which the nut on the screw with the smaller pitch can overcome if a force of 30 lb. is applied at right angles to the end of the power arm and in a plane at right angles to the axis of the screw. Take the efficiency as 15%.

30. What do you understand by the term momentum?

A projectile weighing 620 lb. and having a velocity of 1000 ft. per sec. is brought to rest by armour plating in a distance of 12 in. Calculate (*a*) the average retardation, (*b*) the average resisting force, (*c*) the work done in the penetration.

31. What do you understand by the term, " energy of a body "? How would you distinguish between power and energy? State the Principle of Conservation of Energy and mention the various types of energy.

32. Man, the ass, the ox, the average horse and the camel can develop respectively 70, 110, 300, 450, and 900 ft. lb. per sec. Convert each of these powers to horse power, kilowatts, and also to B.Th.U.'s per minute.

33. Distinguish between a horse power and a horse power hour. Show how to convert these mechanical units to the corresponding electrical and heat units. Calculate the equivalent of 1 H.P. in ft. tons per min. and ft. tons per hour.

34. Define work and energy. A shaft of a coal mine is 660 yd. deep. The cage and load weigh 5 tons and the cable weighs 24 lb. per fathom. Calculate the energy expended in raising the load to the surface. If the operation takes 55 sec., calculate the average useful H.P. required. (*N.B.*—The actual maximum horse power will exceed this average figure because extra power is required to set the cage in motion or accelerate it, there are considerable losses in friction in the winding gear, etc., and the full length of the cable has to be lifted at the beginning.)

35. How many gallons of water can be raised from a depth of 165 ft. by an expenditure of 150 H.P. hours? If the efficiency of the pump and motor combined is 60%, calculate the mechanical horse power and electrical output in kW. of the motor capable of raising this quantity of water in 1 hour.

CHAPTER II

VECTOR QUANTITIES. THE POLYGON LAW. GENERAL CONDITIONS OF EQUILIBRIUM OF FORCES IN ONE PLANE. CENTRE OF GRAVITY. STABILITY

Scalar and vector quantities. Quantities such as the length of a body, the speed of a body, or the energy stored in a moving body, all of which can be represented completely by a magnitude expressed in appropriate units and do not involve direction, are *scalar* quantities. If, in addition, the quantities involve the idea of direction as well as magnitude, as for example, velocity, acceleration and force, they are called *vector* quantities.

A **vector** is a straight line of definite length which is drawn to indicate a definite direction and marked to show the sense of that direction. In other words, it represents to scale a magnitude and indicates a direction and sense of direction. Displacements, velocities, accelerations are called vector quantities because they can be completely represented by vectors. If an arrow is placed on a vector it can be referred to by a single letter such as R in Fig. 16 (a). When no arrow is provided, two letters are necessary such as PQ in Fig. 16 (a), which indicates that the direction is PQ and the sense of direction from P to Q. Should the direction be from Q to P, the vector is represented by the letters QP.

Some difference of opinion exists as to whether the sense of a vector be indicated by an arrow or by the sequence of lettering or by both. The authors prefer to indicate where possible by both sequence of lettering and an arrow on the line of action, and this practice will be followed in the work.

Mathematically, the vector may be represented, Fig. 16 (a), in the forms R_θ or PQ_θ, where θ is the angle which R (or PQ) makes with the standard direction of reference, which is generally horizontal.

Components. If any triangle afe, Fig. 16 (b), be drawn about a vector ae, then af and fe are called the component vectors of the vector ae. af and fe will together be equivalent to ae and may take its place. If af and fe are at right angles, Fig. 16 (c), they are referred

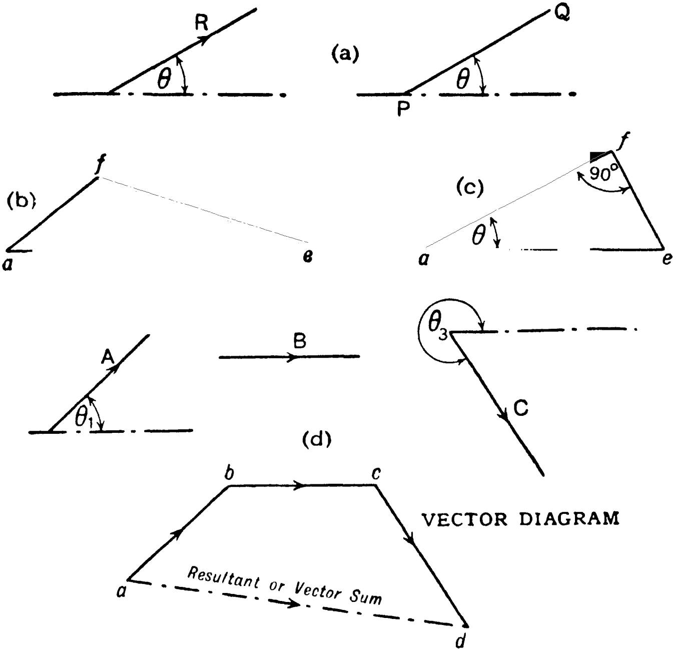


FIG. 16.

to as the **rectangular components** of *ae*. In the case of rectangular components,

$$af = ae \cos \theta \quad \text{and} \quad fe = ae \sin \theta.$$

This reasoning is in accordance with the triangle of forces, Vol. I, p. 73.

Scalar quantities, being magnitudes, are added and subtracted by the ordinary rules of arithmetic, but vectors need a graphical or trigonometrical process for their complete addition and subtraction.

Addition of vectors. Let A, B, C, Fig. 16 (*d*), be three vectors which are to be added. Draw *ab*, *bc*, *cd* parallel to and equal or proportional to A, B and C respectively. Join *da*. Then the vector *ad* is the resultant and equivalent to the vector sum of A, B and C. It should be noted that the order of adding the vectors is immaterial. The polygon *abcd* is called the **vector polygon,** and thus vectors may be added by what is known as the **polygon law** (Vol. I, p. 74).

The summation of these vectors may be expressed mathematically in the form

$$ab_{\theta_1} + bc_{\theta_2} + cd_{\theta_3} = ad_{\theta_4},$$

where the plus sign and the equal sign represent vectorial addition and equality respectively. The student will already have had experience of the triangle, parallelogram and polygon laws when applied to forces and velocities (see experimental work, Vol. I, pp. 73, 74, 265), and will have little difficulty in understanding that the above is a general law applying to the addition of all vector quantities, whether forces, accelerations, impulses, momenta, etc.

Mathematical addition of vectors. The vector sum may be obtained mathematically as follows :

(*a*) Find the sum (H) of the horizontal components of all the vectors, and the sum (V) of all the vertical components.

(*b*) Find the resultant (R) of the sum of the vertical components and the sum of the horizontal components, using the relation

$$R = \sqrt{V^2 + H^2}.$$

(*c*) Find the direction θ that this resultant makes with the horizontal, using the relation $\tan\theta = V/H$ or $\theta = \tan^{-1} V/H$.

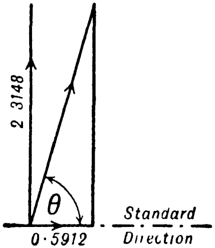


FIG. 17.

Should $H=0$ and $V=0$ and therefore $R=0$, this represents a special case of zero resultant, and corresponds to the condition when the vector polygon closes.

Example. *Find the value of* $6_{70^\circ} + 3_{320^\circ} + 4_{200^\circ}$.

(*a*) Sum of horizontal components $= 6\cos 70^\circ + 3\cos 320^\circ + 4\cos 200^\circ$.

$$\left[\begin{array}{l}\cos 320^\circ = +\cos 40^\circ \\ \cos 200^\circ = -\cos 20^\circ\end{array}\right] \qquad \begin{aligned} &= 6\cos 70^\circ + 3\cos 40^\circ - 4\cos 20^\circ \\ &= 6\times 0{\cdot}342 + 3\times 0{\cdot}766 - 4\times 0{\cdot}9397 \\ &= 0{\cdot}5912.\end{aligned}$$

Sum of vertical components $= 6\sin 70^\circ + 3\sin 320^\circ + 4\sin 200^\circ$

$$\left[\begin{array}{l}\sin 320^\circ = -\sin 40^\circ \\ \sin 200^\circ = -\sin 20^\circ\end{array}\right] \qquad \begin{aligned} &= 6\sin 70^\circ - 3\sin 40^\circ - 4\sin 20^\circ \\ &= 6\times 0{\cdot}9397 - 3\times 0{\cdot}6428 - 4\times 0{\cdot}342 \\ &= 2{\cdot}3418.\end{aligned}$$

(*b*) $\text{Resultant} = \sqrt{0{\cdot}5912^2 + 2{\cdot}3418^2} = 2{\cdot}415$.

(*c*) To find the direction and sense of the resultant:

$$\tan\theta = \frac{2{\cdot}3418}{0{\cdot}5912} = 3{\cdot}962;$$

$$\therefore\ \theta = 75^\circ\ 50';$$

and resultant vector $= 2{\cdot}415_{75^\circ\ 50'}$.

Subtraction of vectors. Suppose it is required to find the value of $P_{\theta_1} - Q_{\theta_2}$. In the graphical representation of this case, Q must be drawn in the opposite sense to that in which it would be drawn if preceded by the positive sign.

Thus, $$-Q_{\theta_2} = +Q_{\theta_2 \pm 180^\circ}.$$

The change of sign is equivalent to the addition to or subtraction of 180° from the angle of the vector.

Example. *Evaluate* $5_{50^\circ} - 6_{115^\circ}$.

A graphical solution is shown in Fig. 18 where the answer is R_θ or, alternatively, the value of R and θ may be found mathematically by solving the triangle *abc*. In this case,

$$\angle abc = 115^\circ - 50^\circ = 65^\circ.$$

Using the cosine rule to find R,

$$R^2 = P^2 + Q^2 - 2PQ\cos\angle abc,$$

$$R^2 = 5^2 + 6^2 - 2 \times 6 \times 5 \times \cos 65^\circ$$

$$= 25 + 36 - 60 \times 0{\cdot}4226 = 35{\cdot}64;$$

$$\therefore\ R = 5{\cdot}97.$$

Using the sine rule to find $\angle bac$

$$\frac{\sin\angle bac}{6} = \frac{\sin\angle abc}{R}$$

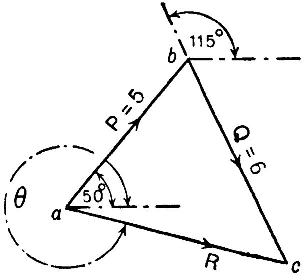


FIG. 18.

or $$\sin\angle bac = \frac{6\sin 65^\circ}{5{\cdot}97};$$

$$\therefore\ \angle bac = 65^\circ\ 37'.$$

Thus, $$\theta = 360^\circ + 50^\circ - 65^\circ\ 37' = 344^\circ\ 23',$$

or $$R = 5{\cdot}97_{344^\circ\ 23'}.$$

When a number of vectors have to be added or subtracted, a graphical solution will involve the drawing of a polygon in which each side of the polygon will represent a vector and the closing line the resultant vector.

Example. *Evaluate* $6_{70°} - 3_{140°} - 4_{20°}$.

The resultant vector may be found graphically by using the polygon law as on p. 30, or by the method of components as in the previous example on p. 31.

Since a change of sign means turning the vector through 180°, the expression $6_{70°} - 3_{140°} - 4_{20°}$ is equivalent to

$$6_{70°} + 3_{140°+180°} + 4_{20°+180°} \text{ or } 6_{70°} + 3_{320°} + 4_{200°}.$$

The latter is the same expression as in the example on p. 31, and the resultant vector or vector sum is $2{\cdot}415_{75°\,50'}$.

Conditions of equilibrium of a body under the action of coplanar forces, that is, forces all in one plane. If a body is at rest or is moving with uniform motion in a straight line, that is, without acceleration or change of direction, it is said to be in equilibrium. This means there must be neither a resultant force nor a resultant turning moment to alter or deflect that state of rest or of uniform motion in a straight line. Since forces are vector quantities, the methods adopted earlier in this chapter for adding and subtracting vectors, and finding component and resultant vectors, apply equally well to problems involving forces. Some special cases will now be considered.

Two forces acting. When only two forces act on a body in equilibrium, then the two forces must be equal and opposite in sense, and they must act along the same straight line otherwise equilibrium will not be maintained. Expressed as vectors, the forces could be represented by

$$P_\theta \text{ and } P_{\theta \pm 180°}.$$

In such cases as where the weight of one body is supported by another body, the supporting force exerted by the latter body is usually termed the **reaction.** The forces in a tug-of-war rope serve as an example.

Three forces acting. If three forces produce equilibrium, the resultant of two of the forces must be equal but opposite in sense to the third force. Consequently, the lines of action of the three forces must either meet in one point or be parallel. This fact is of use in solving problems (see example 6, p. 50). If three forces are in equilibrium, each force is proportional to the sine of the angle between the other two. This follows from the Law of the Triangle of Forces and from the Sine Rule as applied to triangles, and is known as Lamé's Theorem.

Four or more forces acting. In order that there shall be no resultant force to alter the state of rest or uniform motion of the body in a straight line, the following two conditions must hold :

To prevent translational motion.

(*a*) The sum of the vector components of forces resolved into *each of two directions* must be zero, since there must be no unbalanced force tending to move the body in either of those directions.

To prevent rotational motion.

(*b*) In order that there may be no resultant turning moment to rotate the body, the algebraic sum of the moments (p. 10) of all the forces acting on the body about any *one* point in the plane of the forces, must be zero, that is, the Principle of Moments must be satisfied.

Thus, the three conditions of equilibrium in (*a*) and in (*b*) are all necessary and sufficient. They may be varied to suit the solution of particular problems, as will be shown later in the chapter. For example, conditions involving graphical methods can be employed. Condition (*a*) above can be replaced as follows :

For graphical solution.

(*c*) The vector polygon of forces, in which each side taken in order represents in magnitude, direction and sense one of the forces acting on the body, *must close* for equilibrium ; that is, there is to be no unbalanced force.

The corresponding condition to take the place of (*b*) above is :

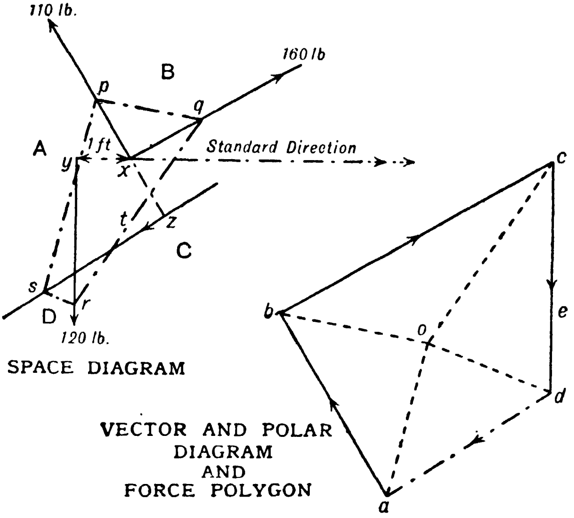
(*d*) The funicular or link polygon (see p. 91, Vol. I) for all the forces acting on the body *must close*, that is, there is to be no unbalanced moment.

It should be noticed that in the special case of four or more forces *acting at one point* and conforming with conditions (*a*) or (*c*), there can be no resultant turning moment.

Bow's notation. In Fig. 19 capital letters have been placed in the spaces between the forces. In Bow's notation a force is denoted by the two letters in the spaces on each side of it, that is, every force is between two letters. The same letters in small type denote the force in the force polygon.

In the following example the conditions (a) and (b) are applied to obtain a numerical solution, and then conditions (c) and (d) applied to obtain a graphical solution.

Example 1. *The three forces* $AB = 110$ *lb.,* $BC = 160$ *lb.,* $CD = 120$ *lb., shown in Fig.* 19, *act in one plane on the table of a machine. The angles which these forces make with the standard direction are respectively* 120°, 30° *and* 270°. *Find the magnitude, direction and sense of the force, and a point in the line of action necessary to produce equilibrium.* AB *and* BC *intersect at* X *and* $xy = 1$ *ft.*



SPACE DIAGRAM

VECTOR AND POLAR DIAGRAM AND FORCE POLYGON

FIG. 19.

Method (1). Condition (a).

$$\text{Sum of horizontal components} = 160 \cos 30° + 110 \cos 120° + 120 \cos 270°$$
$$= 138{\cdot}56 - 55 + 0$$
$$= 83{\cdot}56 \text{ lb.}$$

$$\text{Sum of vertical components} = 160 \sin 30° + 110 \sin 120° + 120 \sin 270°$$
$$= 80 + 95{\cdot}26 - 120$$
$$= 55{\cdot}26 \text{ lb.}$$

Resultant $ad = \sqrt{83{\cdot}56^2 + 55{\cdot}26^2} = 100{\cdot}2$ lb.

Direction of resultant $= \tan^{-1}\dfrac{55{\cdot}26}{83{\cdot}56} = 33^\circ\ 29'$.

Resultant vector representing the resultant force $= 100{\cdot}2_{33^\circ\ 29'}$.

$\therefore$ the equilibrant da is represented by the vector $100{\cdot}2_{213^\circ\ 29'}$.

Condition (*b*). This condition enables a point in the line of action of the equilibrant to be determined.

For equilibrium, the sum of the clockwise moments taken about any one point in the plane of the forces equals the sum of the anticlockwise moments about the same point. Two of the forces have no moment about x, which is their point of intersection, so that x is a convenient point about which to take moments.

$$160 \times 0 + 110 \times 0 + \mathsf{AD} \times xz = 120 \times xy.$$

$$\therefore\ xz = \frac{120 \times xy}{\mathsf{AD}} = \frac{120 \times 1}{100{\cdot}2} = 1{\cdot}198 \text{ ft.}$$

These conditions permit of the equilibrant being completely determined and thus,

Magnitude of equilibrant = 100·2 lb. Direction and sense = 33° 29′ with standard direction. Point in line of action is z, which is at the foot of the perpendicular 1·198 ft. long drawn from x on to the line of action.

Method (2). **Condition** (*c*). In this case the vector force polygon *abcd* (Fig. 19) would be drawn first, the closing line *da* representing the magnitude, direction and sense of the equilibrant.

Condition (*d*). **The link polygon** enables a point *s* in the line of action of the equilibrant to be determined. Choose any point *o*, and join *o* to each of the vertices *a*, *b*, *c*, *d*. Now, choose any point *p* in the line of action of *AB* and in space B draw *pq* parallel to *ob*. From *q* on the line of action of *BC* draw in space C a line *qr* parallel to *oc*, and from *p* and *r* draw lines *ps* and *rs* respectively parallel to *oa* and *od* meeting at *s*. Through *s* draw AD parallel to *ad* in the vector diagram. The equilibrant AD passing through the point *s* has been completely determined.

N.B.—The choice of a different pole for the link polygon would give a differently-shaped link polygon, but the choice of a suitable pole will lead to a well-shaped polygon in which the intersections of adjacent sides are not at very acute angles. In every case the points in the line of action of the equilibrant, obtained by using different poles, will all lie in a straight line coincident with the position of the equilibrant.

Proof of construction of link polygon. Let each of the forces *AB*, *BC* and *CD* (Fig. 19) be resolved along the two sides of the link polygon which meet on its line of action; for example, *AB* along *sp* and *qp*, *BC* along *pq* and *rq*, and so on. The corresponding sides of the triangle of forces in the vector and polar diagram will give the magnitude of two components; that is, *AB* may be replaced by components in the directions *sp* and *qp*, of which the magnitudes are represented respectively by the vectors *ao* and *ob*. In the same way, *BC* is replaced by components of which the magnitudes are represented respectively by *bo* and *oc*. On completing this process, it will be found that the original three forces are replaced by components the lines of action of which are the four sides of the link polygon. For example, the component of *AB* along *qp* is represented by *ob*, while the component of *BC* along *qp* is represented by *bo*. Obviously, *ob* and *bo* balance, and so do the components of *BC* and *CD* along *qr*. The components in the lines *ps* and *rs* represented by the vectors *oa* and *do* do not balance, but have a resultant *da* representing *DA*, acting through *s*, a point on the intersection of their lines of action. The placing of *DA* in the space diagram parallel to *da* and passing through *s* enables all the pairs of components resolved along the sides of the link polygon to balance, and hence *DA* represents the equilibrant of the three given forces in magnitude, direction and position. Thus, for equilibrium, it is essential for the link polygon to be a closed figure.

In the above example, the equilibrant was determined by finding its magnitude, direction and position. In the following example two equilibrants are required, the direction and position of one, and the position of the other, being given.

Example 2. *Suppose in Example 1, Fig. 19, the known quantities were changed to AB = 110 lb., BC = 160 lb., the force CD to pass through y in the direction yr and a fourth force DA to produce equilibrium is to pass through the arbitrary point s. It is required to find*

(*a*) *the magnitudes of the forces CD and DA,*
(*b*) *the direction of the force AD.*

In the vector diagram draw *ab*, *bc*, to represent the forces *AB* and *BC* and also *cd* of indefinite length parallel to *CD*. Choose a pole *o* and join it to the known vertices, namely *a*, *b* and *c*. Starting at point *s* in the space diagram in space A, draw *sp* parallel to *oa*, *pq* parallel to *ob* in space B, and *qt* parallel to *oc* meeting the line of action of force *CD* in *r*. Join *sr*. In the vector diagram draw *od* parallel to *sr* and meeting *ce* produced in *d*. Join *ad*.

Then *cd* to scale gives the required magnitude of the force *CD*, and *da* gives the magnitude and direction of the fourth force *DA*.

EXPT. 1. THE LINK POLYGON.

OBJECT. *To find the pole of a given link polygon.*

APPARATUS. Drawing boards and supports, pulleys and supporting stands, weights, small rings. See Fig. 20.

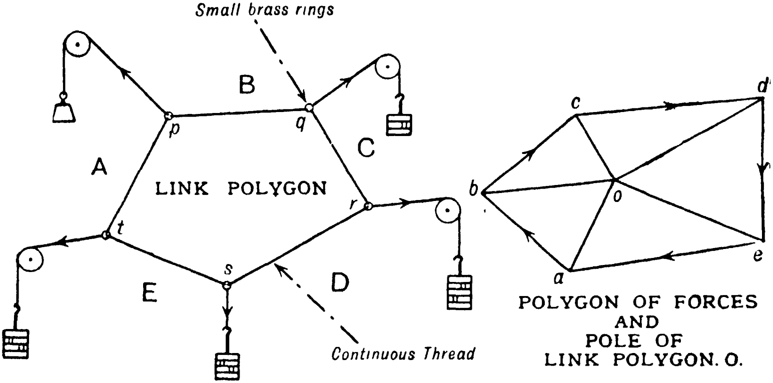


FIG. 20.

METHOD OF PROCEDURE. Pin a sheet of paper to the drawing board and fix it in a vertical plane. Thread five small brass rings to a piece of thread and tie the free ends of the thread. To each of the rings tie another piece of thread so that forces in a parallel plane and close to the board can be applied by means of the pulleys and weights as shown in the diagram. When the arrangement is in equilibrium, mark off carefully on a piece of paper on the drawing board, the shape of the link polygon and the direction of the forces *AB*, *BC*, *CD*, *DE*, *EA* acting parallel to the plane of the paper. Remove the weights and threads and draw on the sheet of paper a polygon of forces *abcde* for the forces acting. Then draw a line from *b* parallel to *pq*, from *c* parallel to *qr*, from *d* parallel to *rs*, and so on.

OBSERVATIONS AND DERIVED RESULTS. It will be found that all these lines drawn from the vertices of the force polygon meet in one point called the pole of the link polygon.

Hints on the solution of problems. From Examples 1 and 2, it will be seen that the necessary conditions of equilibrium serve to determine three unknown quantities in any problem dealing with forces in one plane. For example, it is possible to determine (*a*) three unknown magnitudes, or alternatively (*b*) one unknown magnitude, an unknown direction and an unknown position, or other similar combinations. In solving problems by the preceding principles, it is desirable to select carefully the points about which to take moments. Thus, moments about a point in the line of action of an unknown force, about which it will have zero moment, will enable another unknown quantity to be determined. Also the careful choice of direction for the resolution of forces into components may lead to a more ready solution ; for example, if the forces are resolved in a direction perpendicular to an unknown force, then no component of the unknown force can enter into the equation derived from this resolution.

Example 3. *The four parallel forces AB, BC, CD, DE act on a rod yz and make an acute angle θ with it. Find the resultant force in magnitude, direction, sense, and point of application.*

Method 1. Graphical solution. In Fig. 21 the vectors *ab*, *bc*, *cd*, *de* are drawn in line respectively parallel, and to represent the given forces. Then *ae* is the vector force polygon or **load line,** so that *ae* represents the required resultant in magnitude and *ea* the equilibrant of the

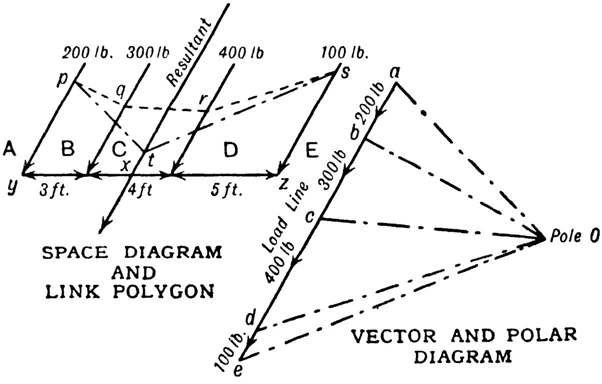


FIG. 21.

forces. Choose any pole *o* and join to the points *a*, *b*, *c*, *d* and *e*. In spaces B, C, D, A and E draw lines parallel to the lines *ob*, *oc*, *od*, *oa* and *oe* respectively and produce the last two links to meet at *t* and so form the link polygon *pqrst*. Through *t* draw a line parallel to *ae*. This line represents the resultant force and x and *t* are points in its line of action. It is instructive to note that x is a fixed point in the rod *yz* for the forces shown irrespective of the value of the angle θ. x is called the **centre** of the parallel forces and is a fixed point in relation to the forces.

Method 2. Resultant force $= ae = 200 + 300 + 400 + 100$, or 1000 lb.

To find the point of application x, take moments about y.

Moment of resultant = Sum of moments of parallel forces.

$$1000 \, . \, yx \, . \sin\theta = 300 \times 3 \sin\theta + 400 \times 7 \sin\theta + 100 \times 12 \sin\theta.$$

$$1000 \, . \, yx = 900 + 2800 + 1200 = 4900.$$

$$yx = 4{\cdot}9 \text{ ft.}$$

Hence both methods show that the resultant is a force of **1000 lb. acting at a point 4·9 ft. from *y* and making an angle θ degrees with the bar.** The forces shown would be called a system of parallel forces and they have been compounded into a single force of 1000 lb. acting through points x and *t*.

Conversely, it is possible to replace this single resultant by two parallel forces through given points. Thus two equilibrants can be found to pass through given points for a system of parallel forces as in the case of the two reactions of a loaded beam (Chap. VIII).

Couples. Two equal parallel forces form a couple when their lines of action are not the same and their senses are opposite. The **arm of a couple** is the perpendicular distance between the lines of action of the two forces forming the couple. The **moment of a couple** is the product of the magnitude of one of the forces and the arm of the couple. This fact can easily be verified by taking moments about any point in the plane of the couple. The moment is always the same, and since a couple can only produce or tend to produce rotation, and not translation, a couple of equal moment and opposite sign is necessary to effect a balance. Any number of couples in the same plane acting on a body are equal to a resultant couple the moment of which is equal to the algebraic sum of the moments of the couples. Also the effect of a couple on a body is unaltered if it is transferred to any plane parallel to its own plane, so long as the arm of the couple remains parallel to its original position.

Example. *Consider the forces acting on an ordinary door* (*Fig.* 22) *which is supported by two hinges.* The weight W of the door is the resultant of the system of parallel forces caused by the force of gravity acting on the particles forming its mass. The hinges must supply a vertically upward force S of magnitude equal to W, and thus the two forces S and W form a couple tending to produce clockwise rotation. The necessary balancing couple is formed by the equal and opposite horizontal forces P and Q acting at the hinges. For equilibrium, $Pa = Wb$.

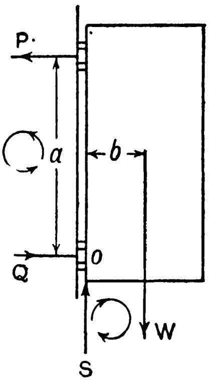


FIG. 22.

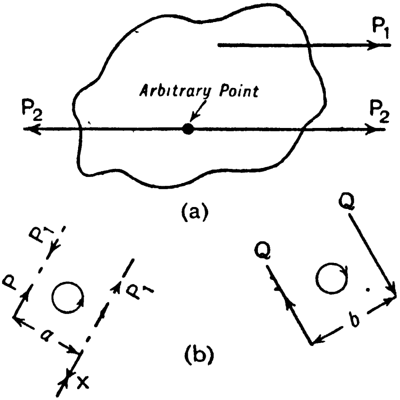


FIG. 23.

Any force acting on a body can be replaced by a couple together with another force acting at any arbitrary point. This is because the introduction of two equal and opposite forces, parallel to the original force, at any point on the body will not affect the equilibrium. In Fig. 23 (a) the introduction of the two equal and opposite forces P_2, where $P_2 = P_1$, does not affect the equilibrium.

Also a force can be substituted for a given force and a given couple. For example, suppose P [Fig. 23 (b)] be the given force and QQ the given couple. By adding two equal and opposite forces P_1 such that $P = P_1$ and $P_1 a = Qb$, the couples balance, the original force P is balanced and the substituted force P_1 is left acting through X.

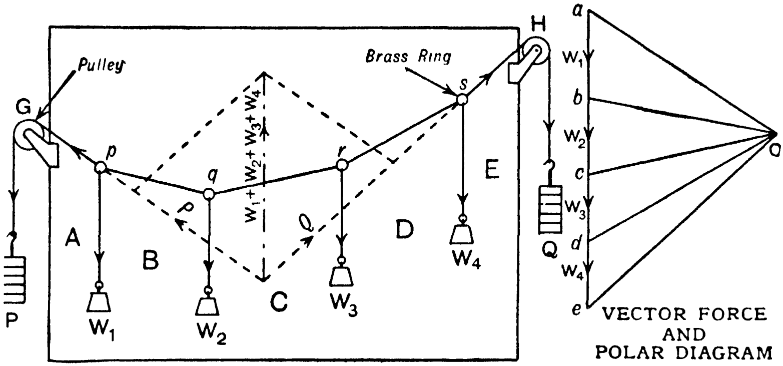


Fig. 24.

Expt. 2. Objects. *To study the shape of a hanging cord in equilibrium and to find the resultant of a number of parallel forces.*

Apparatus. Drawing board and paper. Thread, small brass rings, weights. See Fig. 24.

Method of procedure. A light cord joins small rings *p*, *q*, *r*, *s*, and is passed over pulleys G and H attached to a drawing board placed in a vertical plane. Weights W_1, W_2, W_3 and W_4 are attached to the rings and P and Q to the ends of the cord, which pass over pulleys clamped to the side of the board. Draw the force polygon *abcde* on the paper on the board, and after choosing a pole O, join *Oa, Ob, Oc, Od and Oe.* Now fix the ring *p* by a pin and raise or lower pulley G until the cord *p*G is parallel to *a*O ; then fix the ring *q* so that *pq* is parallel to *b*O. Also fix the other rings in turn so that the cords *qr*, *rs*, *s*H are parallel to *c*O, *d*O, *e*O respectively. Now adjust the magnitudes of the forces P and Q to represent the magnitudes of O*a* and O*e* respectively to the scale used in the vector diagram.

Observations and derived results. It will be noticed that on removing the pins securing each of the rings, the cords remain in equilibrium without change of position. Also by producing the lines G*p* and H*s* until they meet and completing the parallelogram of forces for P and Q, the single resultant of W_1, W_2, W_3 and W_4 is determined.

Experiment 2 illustrates the principle of the suspension bridge, in which the main cables support the road or railway underneath by means of vertical hangers attached to the cables.

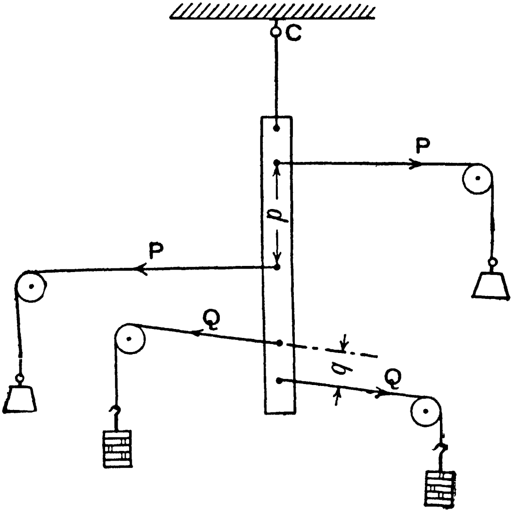


FIG. 25.

EXPT. 3. OBJECT. *To study the balancing effect of two equal and opposing couples.*

APPARATUS. Scale, four pulleys, thread and weights. See Fig. 25.

METHOD OF PROCEDURE. The scale is suspended by a single thread from a fixed support and a couple of moment Pp is applied in a clockwise sense. Apply additional forces by means of the threads, pulleys and weights in various directions.

OBSERVATIONS AND DERIVED RESULTS. It will be found that the scale cannot be restored to its position of equilibrium, which means hanging vertically from C, unless an equal and opposing couple is applied, such as a couple of moment Qq tending to produce motion in an anticlockwise sense with Qq equal to Pp. In this experiment p and q are the arms of the couples PP and QQ respectively.

Alternative methods of stating the conditions of equilibrium. At this stage it is possible to express in different words the principles already dealt with, which apply to forces acting in one plane.

A system of coplanar forces must reduce either to (1) a single resultant force, or (2) to a couple, or (3) be in equilibrium. For on p. 34 the two conditions of (*a*) and condition (*b*) were given as the necessary and sufficient conditions for equilibrium. Suppose the forces reduce to zero resultant to conform with (*a*), then the forces will be in equilibrium unless they form a couple ; but a couple has the same moment about *every* point in its plane, so if the moment about any *one* point is zero, then there will be equilibrium in conformity with condition (*b*).

The necessary conditions of equilibrium may also be expressed in the following way. If a system of forces acts in one plane upon a rigid body, and the algebraic sum of their moments about each of *three* points in the plane but not in the same straight line are each zero, the system of forces is in equilibrium.

In this case, the forces cannot reduce to a couple since the moments are zero ; and they cannot reduce to a single resultant because, although this may have zero moment about two points if it passed through them, it cannot have zero moment about a third point not in the same line as the other two points.

The equilibrium of coplanar forces is also assured if the following three conditions are complied with : the forces must have zero moment about each of two points in the plane, and must have no total component force in the direction of the line joining these two points.

Smooth bodies. A smooth body is defined as one which offers no tangential resistance to another sliding over it. When two smooth bodies are in contact, the reaction or force exerted by each body on the other is normal or perpendicular to the tangent plane between the surfaces in contact. This is because there is no frictional component tangential to the smooth surface.

Although no surface is absolutely smooth or frictionless, many are smooth enough for frictional forces to be ignored. A hinge satisfies these conditions, since the reaction may be assumed to pass through the centre of the pin of the hinge. Line contact of a roller and point contact of a ball bearing afford further examples ; the reactions in these cases will pass along a diameter through the points of contact.

First moments. The product of an $\left\{\begin{matrix}\text{area}\\ \text{volume}\\ \text{mass}\\ \text{force}\end{matrix}\right\}$ by its distance x

from a given point or axis is called the **first moment** of the $\left\{\begin{array}{l}\text{area}\\ \text{volume}\\ \text{mass}\\ \text{force}\end{array}\right\}$ about the given point or axis. Often it is simply referred to as the **moment.**

The **first moment centre** of an area or **centroid** is that point at which the whole area may be deemed to be concentrated in order that its moment about any given line will be equal to the first moment of the area about the same line.

Hence if A is the total area and $\bar{x}$ the distance of the centroid from the given line,

$$A\bar{x} = \Sigma \text{ First moments, where } \Sigma \text{ indicates `` sum of ''}$$

or

$$\bar{x} = \frac{\Sigma \text{ First moments}}{\Sigma \text{ Areas}} = \frac{\Sigma \text{ First moments}}{\text{Total area}}.$$

The summation of first moments and areas may be effected by the ordinary rules of mensuration or by integration, as the following example will show.

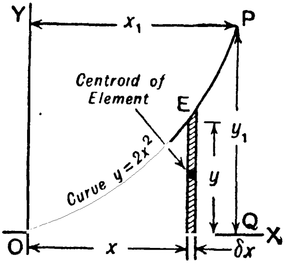


FIG. 26.

Example. *Find the centroid of the area OEPQ, if the equation to the curve OEP is* $y = 2x^2$.

Consider a very thin elemental area of thickness δx and length y and distant x from the origin O.

Area of element $= y\,\delta x$.

Moment of element about OY

$= \text{Area} \times \text{distance}$

$= y\,\delta x\,.\,x$.

$$\text{Whole area} = \int_{x=0}^{x=x_1} y\,dx = \int_{x=0}^{x=x_1} 2x^2\,dx = \tfrac{2}{3}x_1^3.$$

In the limit, the sum of moments of area about OY

$$= \int_{x=0}^{x=x_1} xy\,dx = \int_{x=0}^{x=x_1} 2x^3\,dx = \tfrac{1}{2}x_1^4.$$

$$\text{Distance of centroid from OY} = x = \frac{\text{Sum of moments of area}}{\text{Whole area}}$$

$$= \frac{\tfrac{1}{2}x_1^4}{\tfrac{2}{3}x_1^3} = \tfrac{3}{4}\mathbf{x}_1.$$

Similarly, moment of elemental strip about $\mathrm{OX} = y\,\delta x \,.\, \dfrac{y}{2}\cdot$

Sum of moments about $\mathrm{OX} = \displaystyle\int_{x=0}^{x=x_1} \tfrac{1}{2}y^2\,dx = \int_0^{x_1} 2x^4\,dx = \tfrac{2}{5}x_1{}^5.$

Distance of centroid $\bar{y}$ from $\mathrm{OX} = \dfrac{\text{Sum of moments of area}}{\text{Whole area}}$

$$= \frac{\frac{2}{5}x_1{}^5}{\frac{2}{3}x_1{}^3} = \tfrac{3}{5}x_1{}^2.$$

but $y_1 = 2x_1{}^2$, so that $x_1{}^2 = \tfrac{1}{2}y_1$;

$$\therefore\ \bar{y} = \tfrac{3}{5}x_1{}^2 = \tfrac{3}{10}y_1.$$

Hence the centroid is situated at a distance $\tfrac{3}{4}x_1$ from axis OY and $\tfrac{3}{10}y_1$ from OX.

Thus in general the distance of the centroid from OY is given by $\dfrac{\int y\,x\,dx}{\int y\,dx}$ taken between appropriate limits ; and similarly the distance from OX by $\dfrac{\int \frac{1}{2}y^2\,dx}{\int y\,dx}\cdot$

In addition to the algebraic method given above, the centroid may be found by graphical or by combined graphical and algebraic methods. (See p. 588, Vol. I.)

Centre of gravity. The centre of $\left\{\begin{matrix}\text{volume}\\ \text{mass}\\ \text{gravity}\end{matrix}\right\}$ is the point at which the whole $\left\{\begin{matrix}\text{volume of the body}\\ \text{mass of the body}\\ \text{gravitational forces on all particles of the body}\end{matrix}\right\}$ can be considered to be concentrated in such a way that its first moment about any given line will be equal to the first moment of the $\left\{\begin{matrix}\text{volume}\\ \text{mass}\\ \text{gravitational forces}\end{matrix}\right\}$ about the same line.

For any homogeneous solid, of uniform density throughout, the centres of volume, mass and gravity will be one and the same point, which will always lie on an axis of symmetry. If the body possesses two axes of symmetry, then the centre must lie on a line passing through their point of intersection. If there are three axes of symmetry, then the centre must lie at the intersection of these axes.

In other words, for many regular geometrical solids the centre of gravity and the other centres will lie at the geometrical centre. Examples of such solids are a uniform bar, regular prism, cylinder, cube, ellipsoid, etc.

The centre of gravity of a body may also be defined as the point through which the line of action of the weight of the body always passes.

The **centre of mass** is sometimes called the **centre of inertia.**

Previous reference has been made to the resultant of parallel forces due to gravitation acting on particles comprising a body. This resultant force, irrespective of the position of the body, always acts through the centre of gravity, whether it is within or outside the body, so that the centre of gravity is always a fixed point in relation to the body.

In the simplest case, consider two particles *A* and *B* of which the weights w_1 and w_2 respectively comprise a pair of parallel forces. The resultant will act at the point C in the line AB such that

$$w_1 \,.\, \mathrm{AC} = w_2 \,.\, \mathrm{BC},$$

or

$$\frac{w_1 + w_2}{\mathrm{AB}} = \frac{w_2}{\mathrm{AC}} = \frac{w_1}{\mathrm{BC}}.$$

In other words, AC and BC are inversely proportional to the adjacent weights. Thus the system may be replaced by one imaginary particle placed at C and weighing $w_1 + w_2$; and C may be regarded as the centre of gravity of A and B. This imaginary particle may be combined with a third particle of weight w_3 and the process continued until the position of a single particle situated at the centre of gravity of the system and with a weight equal to the total weight is found.

In many cases the centre of gravity of solids may be found mathematically, because the distance $\bar{x}$ of the centre from a point O is by definition

$$\bar{x} = \frac{\Sigma \text{ First moments of weights about the axis through O}}{\Sigma \text{ Weights}};$$

in which expression due allowance must be made for variations in density and cross-sectional area.

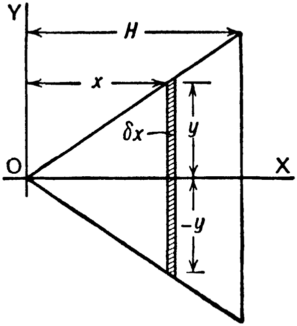


FIG. 27.

Example 1. *Find the centre of gravity of the solid traced out or enveloped when the line $y=2x$ revolves about the axis of x between the limits of $x=H$ and $x=O$. Assume the solid contained by the revolving line is of uniform density.*

Imagine the solid to be divided into a large number of thin discs of circular section, of thickness δx, cut perpendicularly to the x-axis. Consider one disc at a distance x from O which has a radius y. Then, weight of disc $=\rho\pi y^2 . \delta x$, where ρ is the density.

$$\bar{x}=\frac{\Sigma \text{ First moments of weights}}{\Sigma \text{ Weights}}=\pi\rho\int_{x=0}^{x=H} xy^2\,dx \Big/ \pi\rho\int_{x-0}^{x=H} y^2\,dx,$$

but

$$y=2x, \therefore \bar{x}=\int_0^H 4x^3\,dx \Big/ \int_0^H 4x^2\,dx=\Big[x^4\Big]_0^H \Big/ \Big[\frac{4}{3}x^3\Big]_0^H=\tfrac{3}{4}\text{H}.$$

Hence the C.G. is **at a distance equal to $\frac{3}{4}$ of the axial length from the vertex of the cone.**

Example 2. *A uniform cylindrical, disc-type flywheel weighs* 30 *lb., but it is found that its centre of gravity is* 0·05 *in. from its geometrical centre. A small hole is drilled* 4 *in. from the geometrical centre so as to fix the centre of gravity at this centre. Calculate the weight of metal which must be removed by the drill.*

Let w lb. be the weight of metal to be removed.

Before drilling, if the disc is horizontal, a downward force of 30 lb. acts 0·05 in. from the geometrical centre and must be balanced. The removal of w lb. of metal is equivalent to an upward force acting 4 in. from the centre. Taking moments about the centre, a moment of $4w$ lb. in. must correct the moment of 30 × 0·05lb. in.

$$\therefore\ w=\frac{30\times 0{\cdot}05}{4}=\tfrac{3}{8}\text{ lb.}$$

Example 3. *Find the position of the centre of gravity of a plate* 6″ *by* 3″ *which has a* 2″ *diameter hole cut in it as shown in Fig.* 28.

The weight of the plate will be proportional to the area, which is 18 sq. in. The weight of the metal to be removed at the hole will be proportional to π sq. in. C is the position of the new centre of gravity on the line OG produced, and its position can be found by finding the resultant of two imaginary opposing parallel forces of magnitude π and 18 units and $1\frac{1}{2}$ in. apart.

Resultant $= (18 - \pi)$;

and taking moments about G,

$$(18 - \pi)\bar{x} = 1\tfrac{1}{2}\pi$$

or

$$\bar{x} = \frac{1\frac{1}{2}\pi}{(18 - \pi)} = 0{\cdot}317 \text{ in.}$$

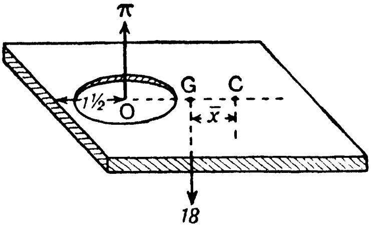


FIG. 28.

Example 4. *A uniform bar* 10 *ft. long is pivoted at its middle point and a* 200 *lb. weight is fastened to the ends of the bar by two cords* 9 *ft. and* 6 *ft. long respectively. Find the inclination of the bar to the horizontal and the tension in each of the cords.*

The 200 lb. wt. will come to rest with its centre of gravity vertically below the pivot C, which is the point of support. Thus CD (Fig. 29) will be vertical.

Draw the triangle ABD and join CD.

Now that the directions AD, BD and CD are determined, the triangle of forces *pqr* can be drawn in which *qr* represents 200 lb.

Inclination of AB $= 90° - \angle$ BCD $= 23°$ by measurement.

Tension in AD $= pr = 161$ lb.

" " BD $= pq = 105$ lb.

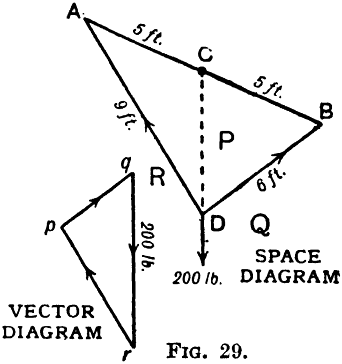


FIG. 29.

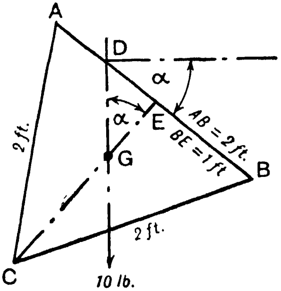


Fig. 30.

Example 5. *A rigid frame ABC of triangular shape, in which each bar is 2 ft. long, is suspended from a point D on AB, 6 inches from one vertex A. If the frame weighs 10 lb., find the inclination of the top bar AB to the horizontal. What weight W hung from the vertex A will restore the top bar to the horizontal position?*

The triangle ABC will come to rest with its centre of gravity G vertically below the point of support D.

$$GE = \tfrac{1}{3} \text{ of } CE = \tfrac{1}{3}\sqrt{3} \text{ ft.} \quad DE = \tfrac{1}{2} \text{ ft.}$$

$$\alpha = \tan^{-1} DE/GE = \tan^{-1} \sqrt{3}/2 = 40^\circ\ 55'.$$

Taking moments about D, when AB is horizontal,

$$W \times \tfrac{1}{2} = 10 \times \tfrac{1}{2}, \text{ or } W = 10 \text{ lb.}$$

Example 6. *A uniform boom* 30 *ft. long weighing* 1000 *lb. is hinged at the end p (Fig.* 31) *and is supported in a horizontal position by the tie qs inclined at* 30 *degrees to pr and attached at q,* 10 *ft. from r. s is vertically above p and a vertical load of* 2000 *lb. is placed at r. It is required to find the magnitude of the force in the tie and the thrust at the hinge p in magnitude and direction.*

The resultant of the 1000 lb. and 2000 lb. forces will act through *t*, which is 5 ft. from *r*, because if moments are taken about *r*

$$tr \times 3000 = 1000 \times 15,$$
$$tr = 5 \text{ ft.}$$

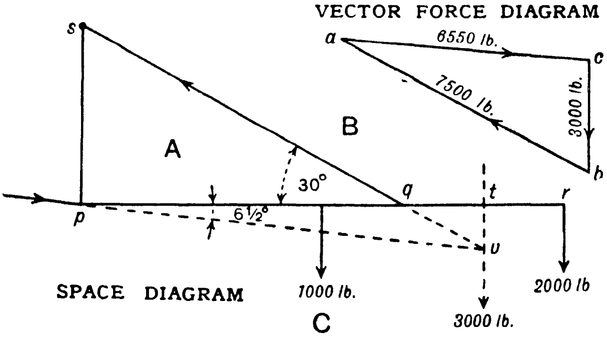


Fig. 31.

This resultant, together with the pull in the tie and the force at the hinge, keep the boom in equilibrium ; and therefore their lines of action must meet in one point.

Produce the line of action of AB to meet that of BC the 3000-lb. force at *u*. Join *pu*. Then *pu* will be the line of action of CA, the balancing force at the hinge.

Draw *cb* in the vector force diagram to represent 3000 lb. and complete the diagram by drawing *ba* and *ac* parallel respectively to the forces BA and AC.

Magnitude of force in tie AB by measurement = 7500 lb.

" " " at hinge " = 6550 lb.

Direction of equilibrant at hinge " = 6·5° with line pr.

Centre of gravity and work done in lifting. Knowledge of the position of the centre of gravity of a body or system of bodies facilitates the calculation of the work done in problems on lifting and tilting. For example, in calculating the work done in winding a cable, or in pumping water, it is necessary to find the position of the centre of gravity of the cable or of the water, both before and after lifting.

By definition, the centre of gravity is a point through which the weight of the body (or system of bodies) may be deemed to act for all positions of the body (or system of bodies). Hence *the work done in lifting will be equal to the weight of the body multiplied by the vertical distance through which the centre of gravity has been raised.* This principle applies even if, during lifting, the relative position of the centre of gravity alters in regard to the constituent parts of the body, or system of bodies.

Experimental methods of determining the centre of gravity. When a body or lamina is suspended from a point or balanced on a knife edge and the body is in equilibrium, the centre of gravity must lie vertically below the point of suspension or directly above the knife edge, as the case may be. This principle is utilised in determining practically the position of the centre of gravity of engineering components and machines when these are comparatively small, by suspending the part from two or three different points or balancing upon a knife edge in two or three different positions. The number

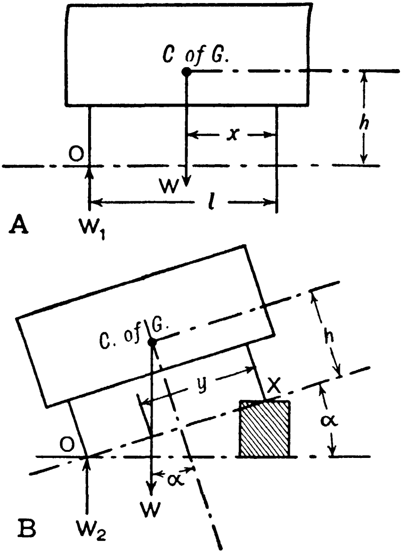

C of G.
h
x
O
W
l
A
W_1
C. of G.
h
y
X
α
O
α
W
B
W_2

FIG. 32.

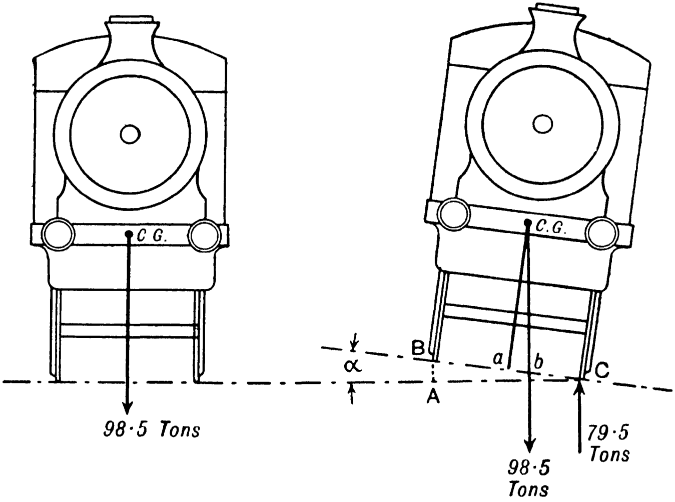

C G.
98·5 Tons
α
B
a
b
C
A
C.G.
98·5 Tons
79·5 Tons

FIG. 33.

of balancing positions will depend upon whether the part possesses any axes of symmetry. (See Vol. I, p. 196.)

Method of inclination. For vehicles and locomotives and large parts, the method of inclination may be used. The procedure may be to run the vehicle on to a weighbridge and obtain the total weight W. The latter may also be obtained by adding together the weights of the various parts of the vehicle. Then a crane or jack is employed to lift one side of the vehicle so that the tyres are only just clear of ground level. The new weight W_1 registered by the other side is then noted. Afterwards the same side of the vehicle is lifted a measured amount, so that the vehicle is tilted through a known angle a and blocked up clear of the weighbridge and the third reading W_2 taken.

Fig. A. Taking moments about X:

$$W_1 l = Wx \quad \text{or} \quad x = \frac{W_1 l}{W}.$$

Fig. B. Taking moments about X:

$$Wy \cos a = W_2 l \cos a \quad \text{or} \quad y = \frac{W_2 l}{W};$$

$$\therefore \; y - x = \frac{W_2 l}{W} - \frac{W_1 l}{W} = \frac{(W_2 - W_1) l}{W}.$$

But

$$h = (y - x) \cot a$$

$$= \frac{(W_2 - W_1) l \cot a}{W},$$

where a is the angle through which object is inclined.

The above method gives the position of the centre of gravity in relation to a cross-section. Similarly, the position of the C.G. on a longitudinal section could be found by tilting the vehicle through a known angle by lifting the front axle through a known distance.

To find the position of the centre of gravity of a locomotive above its track by the method of inclination, assuming the centre of gravity lies on a longitudinal plane of symmetry.

METHOD OF PROCEDURE. The locomotive is run on to a weighbridge and its total weight obtained [Fig. 33 (*a*)]. The wheels on one side of the locomotive are then raised a measured distance AB [Fig. 33 (*b*)] by a crane or other means. The weighbridge then records the pressure exerted downwards by the wheels on the side

C. If c is the centre of gravity of the locomotive, then, after inclination, the resultant gravitational force acts along the line cb. By taking moments about B, the distance Bb and hence ab may be found. Since ac and cb are respectively at right angles to BC and AC, the angles bca and BCA are equal. Then $\frac{ab}{ac} = \tan\alpha = \frac{AB}{AC}$.

Of these quantities, AB, AC and ab are known or can be calculated, so that ac the required distance can be determined.

Example. *A tank locomotive in working order weighs* 98·5 *tons and the weighbridge reading when the offside wheels are raised* 15 *in. is* 79·5 *tons. Find the height of the centre of gravity above rail-level if the gauge width is* 56½ *in.*

Taking moments about B, 98·5 tons × Bb cos α = 79·5 tons × BC cos α.

$$\therefore \quad Bb = \frac{79{\cdot}5 \times BC}{98{\cdot}5} = \frac{79{\cdot}5 \times 56{\cdot}5}{98{\cdot}5} = 45{\cdot}6 \text{ in.}$$

Then $$ab = Bb - Ba = 45{\cdot}6 - 28{\cdot}25 = 17{\cdot}35 \text{ in.}$$

Also $$AC = \sqrt{56{\cdot}5^2 - 15^2} = 54{\cdot}47 \text{ in.};$$

$$\therefore \text{ since } \frac{ac}{ab} = \frac{AC}{AB}, \text{ then } ac = \frac{AC \times ab}{AB} = \frac{54{\cdot}47 \times 17{\cdot}35}{15} = 63 \text{ in.}$$

Ans. The centre of gravity of the tank locomotive is 5 ft. 3 in. above rail-level.

Stability. A body is said to be in **stable equilibrium** when, if it is slightly displaced from its position of equilibrium, the forces acting on the body tend to restore it to its position of equilibrium.

The body is said to be in **unstable equilibrium** when, if slightly displaced, the forces acting tend to move it still further from its position of equilibrium.

The body is said to be in **neutral equilibrium** if, after a slight displacement, the forces acting are in equilibrium and the body remains in the new position.

Stability can also be tested from the energy point of view. Because the potential energy of a body in any given position depends upon the product of the mass of the body and the vertical height of its centre of gravity above the chosen datum-level, the stability of its equilibrium will depend upon the amount of potential energy it possesses after any given slight *tilt*. For example, if a body receives an angular displacement or tilt, the equilibrium will be neutral if the amount of potential energy is constant. Again, if a slight tilt causes the centre of gravity to drop and the potential energy to become less, and this condition is maintained, there will be no energy available to restore the body to its former position and the body will be in unstable equilibrium. On the other hand, if the centre of gravity rises and the potential energy increases, energy will be available to restore the body to its former position and it will be in stable equilibrium.

Generally speaking, if a body has more than one *position* of equilibrium, the one in which its centre of gravity is lowest and the potential energy least will be the position of stable equilibrium, while that in which the centre of gravity is highest and the potential energy greatest will be the unstable one.

The stability of a body is increased by: (1) lowering the centre of gravity; (2) increasing the area of the base; (3) increasing the weight.

A solid resting on an inclined plane will overturn if a vertical line through its centre of gravity just passes outside the base. The base is defined as the area within the polygon formed by joining all adjacent points of contact with the plane with straight lines.

In all cases of a simple rigid body in contact with a supporting surface, the path of the centre of gravity as the body *rolls* over a plane surface gives a definite indication of stability.

Lowest points on this path usually indicate positions of stable equilibrium; highest points positions of unstable equilibrium.

If the path is a straight line parallel to the plane surface, then neutral equilibrium is indicated.

EXERCISES ON CHAPTER II

1. What is a vector? Find the value of $6_0 + 6_{180}$, $5_{240} + 5_{60}$, and $6_{30} - 5_{390}$.

2. Evaluate $4_{60} + 3_{270}$.

3. Simplify $9_{40} + 6_{200} - 7_{110}$.

4. Simplify (*a*) $7_{28} - 4_{120} + 12_{90} + 10_{180}$; (*b*) $11_{135} + 6_{270} - 4_{40}$.

5. At a node or joint of a bridge girder the forces acting are represented by the vector expression

$$20_{270} + 16{\cdot}43_{180} + 17_0 + 1{\cdot}672_{90} + 13_{60} - 10_{315}.$$

Show that the joint is in equilibrium.

6. The following system of forces has a zero vector sum: $4{\cdot}55_{294}$, $3{\cdot}5_{110}$, 2_{70}, ? Find the missing values.

7. Fig. 34 shows the forces acting on a rod AB. Determine the magnitude, direction and point of application of the equilibrant.

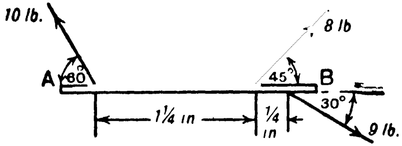


FIG. 34.

8. Four parallel forces of magnitudes 2 lb., 4 lb., 4 lb. and 3 lb. act in the same sense, and the distances between the forces are respectively 2 ft., 5 ft. and 4 ft. Find the resultant of the forces using the link polygon method, and check your answer by using the Principle of Moments.

9. A square plate ABCD of side 2 ft. has forces of values 2 lb., 3 lb., 4 lb. and 5 lb. acting respectively along the edges AB, BC, CD and DA. Find, completely, the force required in order to balance the plate.

10. An isosceles triangular frame ABC in which the span AB is 15 ft. and rise 5 ft. has vertical loads of 400 lb. and 600 lb. acting respectively in the centre of the bar AC and at C, and also a load of 800 lb. acting at the centre of the bar CB and at right angles to it. The reaction at B is vertical. Find the magnitude and direction of the other reaction at A and the magnitude of the reaction at B.

Hint.—First, by taking moments about A, find the magnitude of the reaction at B.

11. Fig. 35 shows the magnitudes and directions of forces acting at the corners of a square. Determine the magnitude, direction and sense of the equilibrant.

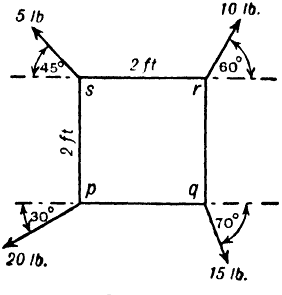


FIG. 35.

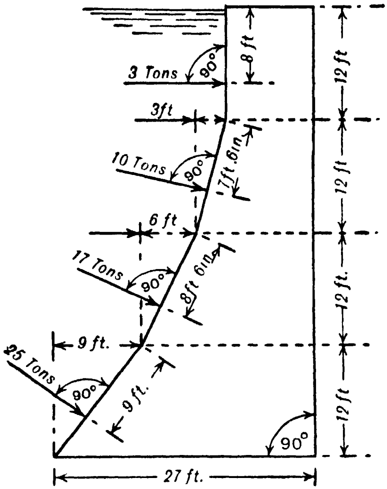


FIG. 36.

12. Fig. 36 shows the forces acting on a dam. Determine the resultant force on the face of the dam, and the turning moment it exerts about the vertex marked 90°.

13. A symmetrical three-hinged arch ABC of span $AC = 12$ ft. and rise 4 ft. has hinges at A, B and C. The arch takes a vertical load of 10 tons at a point on the span AC such that the line of action of the load is 2 ft. from the vertical through C. Find the magnitudes and directions of the forces acting at each of the hinges.

14. A jib 30 ft. long is hinged at its lower end A and is inclined 45° to the horizontal. Its upper end B is loaded with 10 tons, while the weight of the jib is $\frac{3}{4}$ ton and its C.G. is 10 ft. from A. A stay inclined at 90° to the jib is fastened to a point on the jib 15 ft. from A and to a vertical post through A. Find the magnitude and direction of the force at the hinge and the pull in the stay.

15. A uniform rod AB weighs W lb. and is hinged at one end A and is supported in a position making an angle of 30° with the horizontal by a force of 60 lb. at B acting at right angles to AB. Find the value of W and the reaction of the hinge at A both in magnitude and direction.

16. One end of a uniform bar AB 4 ft. long rests with one end A against a smooth wall while the other end B is supported by a cord 6 ft. long with its other end secured to a point vertically above A. Determine the angle which the bar makes with the vertical.

17. A smooth hemispherical bowl of radius 20 in. has a rod 30 in. long placed inside it. Determine the position in which the rod will remain in equilibrium if: (*a*) the rod is uniform, and (*b*) if the centre of gravity of the rod is 8 in. from one end.

18. Prove that it is impossible for a ladder to remain in equilibrium when resting against a smooth vertical wall if the ground upon which the ladder is resting is also smooth.

19. A gate weighing 200 lb. is hung on two hinges 5 ft. apart in a vertical line distant 6 ft. from the centre of gravity of the gate. Assuming that the whole weight of the gate is borne by the lower hinge, find the resultant force acting at each hinge.

20. The weight of a kite is 6 lb. and it is inclined 50° to the horizontal. If the resultant thrust of the air acts at a point 7 in. above its centre of gravity and the string is tied 9 in. above it, find the thrust of the air and the tension of the string.

21. A rigid rectangular framework measuring 16 in. by 20 in. is made of thin uniform bars with a total weight of 4 lb. The framework is suspended in the vertical plane by a string fastened to a point 2 in. from the end of one of the long bars. Find the angle which this bar will make with the vertical.

22. A load of 1 ton is free to slide along a smooth wire 10 ft. long. The ends of the wire are connected to two points A and B, in such a way that B is 5 ft. measured horizontally from A and 1 ft. measured vertically below A. Determine the position of the load when it comes to rest and the tension in the wire.

23. A telescope consists of four tubes each 10 inches long sliding one within the other, and their weights are 8, 7, 6 and 5 oz. respectively. The telescope is to be pivoted at a point under the C.G. when the tubes are fully extended. Find the position of the C.G. of the extended telescope from the smaller end.

24. A uniform square board of mass 2 lb. has masses of 3, 5, 7 and 9 lb. placed at the four corners. Determine the position of the centre of gravity of the system.

25. Where must a circular hole of 3 in. radius be punched from a circular sheet of tinplate of 1 ft. radius in order that the centre of gravity of the remainder may be only $\frac{1}{2}$ in. from the original centre of the sheet?

26. Find the position of the centre of gravity of a piece of uniform angle iron 6 ft. long if the angle section is 6 in. × 6 in. × $\frac{1}{2}$ in.

27. An angle section of rolled steel bar is 6 in. × 5 in. outside dimensions and 0·5 in. thick throughout. Find the position of the centroid of the angle section.

28. A locomotive weighs 400 tons and its centre of gravity is 4 ft. 6 in. above the rails, gauge 4 ft. 8½ in. To what angle can the locomotive be tilted before it upsets, and find the horizontal force through the centre of gravity necessary to do this?

29. The portion of the balance weight of the driving wheel of a locomotive situated between two consecutive spokes is of uniform thickness and shaped as shown in Fig. 37. By cutting out the shape in cardboard and by suspending it from two different points, determine the position of its centre of gravity.

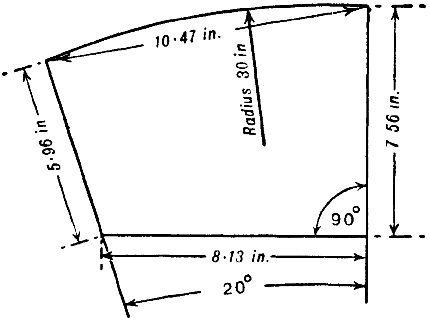


FIG. 37.

30. Describe carefully with the aid of sketches how the centre of gravity of a bus could be determined by the method of inclination, both in relation to a cross-section and the mid-longitudinal section.

31. A locomotive designed for a gauge of 5 ft. weighs 120 tons. The wheels on one side are raised 12 in. and supported on blocks, while the other wheels exert a load of 89·4 tons on the weighbridge. Find the height of the centre of gravity above rail-level.

32. To support a motor cycle with wheel centres 60 in. apart in a horizontal position requires vertical forces of 100 lb. at the front and 134 lb. at the back axle. When the front axle is raised 15 in. above the back axle, the back wheel exerts a force of 137 lb. on a weighing machine. Determine the position of the centre of gravity of the motor cycle as loaded.

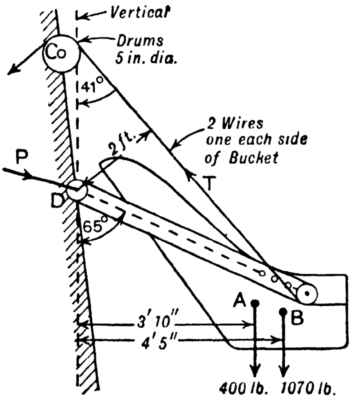


FIG. 38.

33. Fig. 38 shows the loader bucket of a concrete mixer. A is the centre of gravity of the bucket which weighs 400 lb. and B that of the mixture which weighs 1070 lb. Find the magnitude of the tension in each of the wires T and also the thrust P at each hinge D when the bucket just begins to lift. What turning moment will be required at C in lb. ft.?

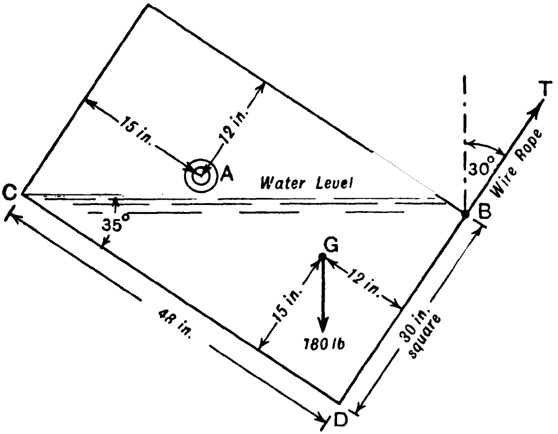


FIG. 39.

34. Fig. 39 shows the tank of a water softening plant supported on fixed trunnions at A and tilted by means of a wire rope fixed to the bottom of the tank at B. The empty tank weighs 180 lb. and its centre of gravity is at G. Water is allowed to flow into the tank, which is tilted until the water level takes up the position shown.

Find (*a*) the weight of water in the tank in this position if 1 cu. ft. of water weighs 62·3 lb.; (*b*) the position of the centre of gravity of the *water* in the tank referred to the lines DB and DC; (*c*) the tension in the wire rope T; (*d*) the magnitude and direction of the resultant thrust on the trunnions A.

CHAPTER III

RELATIVE VELOCITY. VELOCITY DIAGRAMS FOR MECHANISMS AND APPLICATIONS. INSTANTANEOUS CENTRE. EPICYCLIC GEARS

In Chapter I it was stated that motion must be considered as relative and not absolute, and must be specified in its relation to some body which is assumed for the time being to be fixed. It is now necessary to amplify this statement. In many problems the earth is regarded as fixed and the displacement of other bodies measured relatively to some point on it. In engineering problems, this may be more difficult because it is necessary to determine the magnitude and direction of the velocity and acceleration of one point on a machine relative to another point which is itself moving relative to the earth.

Relative Velocity.

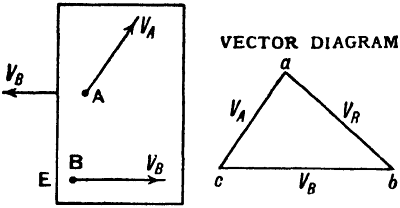
Plane motion of independent bodies in the same or parallel straight lines. For example, suppose a train A travelling due east at a velocity of $V_A = 50$ m.p.h. overtakes another train B travelling due east at $V_B = 40$ m.p.h. Since both velocities are of the same sign, the first train A will pass B at a speed of 10 m.p.h., and the relative velocity of A to B is said to be 10 m.p.h. Thus,

$$\text{relative velocity of } A \text{ to } B = (+V_A) - (+V_B) = 10 \text{ m.p.h.}$$

Now assume that B's velocity is reversed (or becomes negative) and it is travelling at $-V_B$ or 40 m.p.h. due west. In this case, A would pass a passenger in B at 90 m.p.h., or the relative velocity of A to $B = (+V_A) - (-V_B) = (50) - (-40) = 90$ m.p.h.

In both cases, the relative velocity of A to B is the difference of the velocities of A and B, taking signs into account.

Plane motion of independent bodies along non-parallel straight lines. If the bodies are not moving in the same or parallel lines the velocities can be represented vectorially and the subtraction must be done as with other vectors (p. 32).



SPACE DIAGRAM

FIG. 40.

Suppose two points A and B (Fig. 40) move over a plane rectangular surface with velocities V_A and V_B respectively relative to E, the earth. If the rectangular surface is given a velocity $-V_B$, that is, equal and opposite to V_B, then B will actually remain at rest relative to the earth and A will have a velocity compounded of V_A and V_B reversed. Therefore to B, which has been brought to rest, A will have a velocity, taken vectorially, of

$$(+V_A)+(-V_B)=V_A-V_B, \text{ or}$$
$$=ca-cb=bc+ca=ba=V_R.$$

Notice that the *order of vector lettering indicates* that the direction of motion is from the first letter, b, to the second letter, a. Thus, the vector bc, from b to c, represents the velocity of the earth E relative to B. Also the velocity of B relative to A is represented by ab.

Thus, **the velocity of A relative to B is equal to the velocity of A added vectorially to the velocity of B reversed.**

In equation form :

Velocity of A relative to B

= Velocity of A relative to E – Velocity of B relative to E. ...(1)

= Velocity of A relative to E + Velocity of E relative to B. ...(2)

The equation in its latter form is still true if any body C is taken instead of the earth, in which case,

Velocity of A relative to B

= Velocity of A relative to C + Velocity of C relative to B. ......(3)

In this form the relative velocity equation is most useful since the point C can be chosen to simplify any given problem.

Example 1. *One ship A is steaming due north at 40 knots and another, B, is due east at 30 knots. At noon, A is 8 nautical miles and B 16 nautical miles from the intersection of their courses.*

Determine (a) the relative velocity of A to B,
(b) the nearest distance between A and B,
(c) the time when A and B are nearest together,
(d) the positions of A and B when they are nearest together.

(*a*) The relative velocity of A to B is given by the equation

Velocity of A relative to B = Velocity of A + Velocity of B reversed.

In Fig. 41 let A and B mark the positions in the space diagram of the ships at noon. Construct the vector triangle by drawing *be* to represent the velocity of B reversed and *ea* to represent the velocity of A. Join *ba*, which gives the relative velocity of A to B.

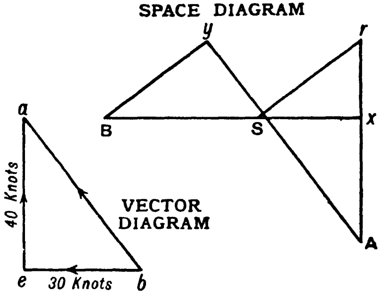


FIG. 41.

ba to scale represents **50 knots.**

(*b*) In the space diagram, draw from A a line parallel to *ba* to represent the velocity of A as its motion would appear to B if the latter were at rest. A perpendicular from B meets this line at y. Measure By to scale, as this measures the nearest distance of A to B.

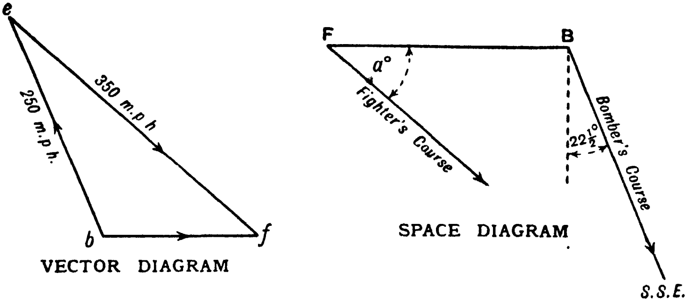
By measurement, **By = 8 nautical miles.**

(*c*) Now measure Ay and it will be found to represent 16 miles. Divide Ay by the relative velocity of A to B, which gives $\frac{16}{50}$. Thus, time to travel relatively from A to $y = \frac{16}{50}$ hour or 19·2 min. This gives the time after noon when A is nearest to B.

(*d*) In 19·2 min. A will have reached r, where $Ar = \frac{19{\cdot}2}{60} \times 40 = 12{\cdot}8$ miles.

Similarly, B will have reached S, where $\mathrm{BS} = \frac{19{\cdot}2}{60} \times 30 = 9{\cdot}6$ miles.

It will also be noticed that **rS** is equal and parallel to By.



VECTOR DIAGRAM

SPACE DIAGRAM

FIG. 42.

Example 2. *A fighter F sights a bomber B 30 miles away in a direction due east. The bomber is flying on a bearing of* 157½° *true at* 250 *m.p.h. If the fighter's speed is* 350 *m.p.h., find the course the fighter should take to reach the bomber in the shortest possible time.*

Let F and B (Fig. 42) represent the positions of the fighter and bomber respectively. If B is imagined to be at rest, F's velocity relative to B must be in the direction FB in order that B may be reached in the shortest possible time. But the velocity of F relative to B = Velocity of F relative to E + Velocity of E relative to B, where E represents the earth. Of these quantities the direction of F relative to B is known, the magnitude of F relative to E and both the magnitude and direction of E relative to B are known. Hence draw *be* to represent the velocity of B reversed (that is, E relative to B), and with centre *e* and radius corresponding to 350 m.p.h. describe an arc cutting the line from *b* parallel to FB at *f*. Join *ef*. Then *ef* represents the absolute velocity of F relative to the earth in magnitude and direction, because $bf = be + ef$.

F should therefore set off from F in the direction parallel to *ef*.

By measurement **∠a = 41 degrees.**

Hence the fighter's course will be **on a bearing of 131° true.**

Plane motion of two bodies or points on a rigid link. In the theory of machines it is often necessary to consider the motion of two points on one rigid part or link of a mechanism. These points are a fixed distance apart, and their motion may be very different but cannot be independent. Thus, the velocity of each of the two points relative to the other can have no component along the line joining them and can only be perpendicular to this line. This means

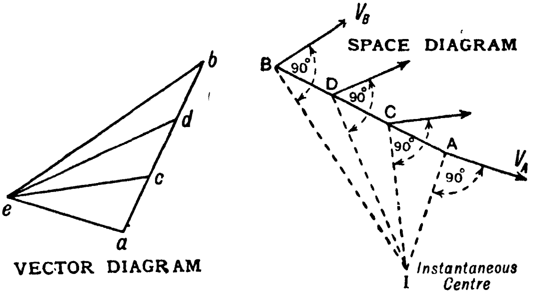


FIG. 43.

that if A and B are two fixed points on a link moving in one plane, then the relative velocity of A to B and of B to A are both perpendicular to the line AB.

Let AB, Fig. 43, represent the link of a machine and V_A and V_B respectively the directions of the velocities of A and B. Suppose also that the magnitude of V_A is known and the magnitude of V_B is required. In other words, the velocity of B relative to the earth E is required.

Using equation (3), p. 62,

$$\text{Velocity of B relative to E} = \text{Velocity of B relative to A} + \text{Velocity of A relative to E},$$

or

$$V_B = \text{Velocity of B relative to A} + V_A.$$

The directions of these three velocities are all known because the velocity of B relative to A is at right angles to the line AB. In addition, the magnitude of V_A is known, and thus the magnitudes of the other velocities can be determined. In other words, of the six quantities involved, namely, three magnitudes and three directions, so long as four are known, it is possible to find the two remaining quantities from the vector triangle. Therefore, from *e* draw *ea* to scale to represent V_A in magnitude and direction. Add the vector *ab*, which represents the direction of the velocity of B relative to A, and fix the point *b* by drawing from *e* a line parallel to V_B. Thus, $eb = ea + ab$, and *eb* will give to scale the magnitude of V_B, while *ab* will give the magnitude and direction of B relative to A, and similarly *ba* of A relative to B.

If, in the above example, the magnitude of V_B, instead of the direction, were known, the point b may be fixed by describing an arc with e as centre and radius equal to V_B and then joining e to the two points of intersection. In this way the two possible directions of V_B are obtained. It should be noted that the scale representation of V_B must not be less than the perpendicular distance from e on to ab. This perpendicular distance represents the component velocity of V_B in the direction AB.

Velocity of other points on the link. If C and D are two points on AB respectively at one-third and two-thirds the distance from A to B, their velocities can be obtained by dividing the vector ab in the same proportions, for example, so that $\frac{\text{AC}}{\text{AB}} = \frac{ac}{ab} = \frac{1}{3}$, and then joining e to c and d. Then ec and ed represent completely V_C and V_D respectively. Being on the line AB, both C and D will have the same component velocities as A and B in the direction AB, but, relatively to A, their velocities will be proportional to their distances from A.

Angular velocity of a link. To find the angular velocity of the link AB as it turns in an anticlockwise direction about A, divide the linear relative velocity of B to A by the length of the straight line AB.

Instantaneous centre. Instantaneous centre is the name given to the point about which rotation can be considered to be taking place at any particular instant and with a particular configuration of the mechanism.

In Fig. 43 draw perpendiculars from B and A respectively at right angles to the directions of the velocities of B and A and meeting at I. For a very small displacement of the link, I is called the **instantaneous centre of rotation** for the given position of the link. At the instant taken, A is turning about some point in the line AI, while B is turning about a point in the line BI. Hence I, being the point of intersection, must be the instantaneous centre for the whole link.

For any position of a moving link this centre can easily be found unless the motion of the points on the link happens to be almost parallel. Usually, as the link moves so the centre changes. The locus or path of the instantaneous centre of rotation as the link moves is called a **centrode.**

Alternative method. **Solution by the instantaneous centre method.** The problem solved (Fig. 43) by relative velocity methods can also be solved by using the centre of rotation I. Any point on the link AB, Fig. 43, will also be turning instantaneously about I. For example, C and D will be moving respectively at right angles to CI and DI and with velocities proportional to those distances. It should be noticed that these directions are respectively parallel to *ec* and *ed* in the vector diagram, and also that the triangle IAB is similar to *eab* but rotated through a right angle relative to it. Therefore, if the magnitude of the velocity of one point on the link is known about I, that of any other point can be determined. Since the triangles are similar and V_A is known and V_B is required, then

$$\frac{V_B}{V_A} = \frac{eb}{ea} = \frac{\mathrm{IB}}{\mathrm{IA}} \quad \text{or} \quad V_B = V_A \cdot \frac{\mathrm{IB}}{\mathrm{IA}}.$$

Similarly, $$V_C = V_A \cdot \frac{\mathrm{IC}}{\mathrm{IA}}, \quad \text{and} \quad V_D = V_A \cdot \frac{\mathrm{ID}}{\mathrm{IA}}.$$

Also, if ω is the angular velocity of the link momentarily about I, then

$$\omega = \frac{V_A}{\mathrm{IA}} = \frac{V_B}{\mathrm{IB}} = \frac{V_C}{\mathrm{IC}} = \frac{V_D}{\mathrm{ID}}.$$

The chief defect of this method of determining velocities is that, when the instantaneous centre is obtained graphically, in some cases it cannot be determined accurately because the perpendiculars to the directions of the velocities may be nearly parallel. Thus, large-scale drawings may be necessary for accuracy.

MECHANISMS

A mechanism may be described as an arrangement of links with pairs of rotating units or sliding parts designed collectively to transmit motion. A **rotating pair** consists of two units together transmitting rotation, and a **sliding pair** is two units, one sliding over the other under directional restraint, and together transmitting linear motion.

Examples of mechanisms.

(1) *Slider crank chain.* The mechanism of the ordinary reciprocating engine contains (*a*) one sliding pair, namely the crosshead and guides, (*b*) three rotating pairs, the crank and crankshaft and

the crankshaft bearing ; the crank pin and the big end of the connecting rod ; the gudgeon pin and the small end of the connecting rod.

(2) *Quadric cycle chain.* Consists of three links pin-jointed at their extremities, with the ends of the first and third link pin-jointed to a fixed unit. This contains four rotating pairs, one fixed and three movable links.

(3) *Double slider crank chain.* This is the mechanism of the donkey engine, containing a crank and connecting-rod mechanism as in the slider crank chain, with a gudgeon pin and slide operating in a circular guide.

(4) *Oscillating cylinder mechanism.* This consists of the crank and connecting-rod mechanism, as in the slider crank chain, with the gudgeon pin directly pin-jointed to the piston, which provides a sliding pair with the cylinder which is itself part of a rotating pair.

When the velocity vector triangle for one link has been drawn it is possible to determine the motion of a second link joined to or paired with it. The process can be continued until the velocity triangle for every link of the mechanism has been drawn and thus the velocity of every point found. The method can be best understood by means of worked examples.

Example 1. Quadric cycle chain. *Consider a mechanism (Fig. 44) consisting of a crank BC rotating at a given angular velocity about B, connected by the coupling rod CD, which causes the crank AD to move with variable angular velocity about A. It is required to determine the velocity of D, the velocity of E where* $CE = \frac{1}{3}CD$ *and the angular velocity of AD for the given position of the mechanism.*

The equation from which each vector triangle can be drawn should first be written down. The required relative velocity should be placed on the left-hand side and the given velocity and the relative velocity between the ends of the link on the right-hand side of the equation. Thus,

Velocity of D relative to B = Velocity of D relative to C
+ Velocity of C relative to B.

Here the point C is introduced into the equation because the velocity of C is known.

Draw *bc* parallel to V_C to represent the velocity of C, and a line through *c* at right angles to CD and a line through *b* parallel to V_D

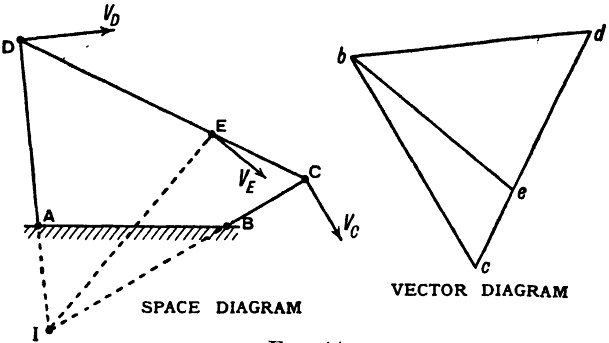


FIG. 44.

and at right angles to AD to meet the line from c at d. Then bd represents the velocity of D. The instantaneous angular velocity of AD will be given by V_D/AD. Divide dc in the ratio 2 : 1 to determine the position of e. Join be, which represents V_E.

Alternatively, the instantaneous centre method may be used to find V_D. Produce DA and CB respectively at right angles to V_D and V_E to meet at I, the instantaneous centre for the link CD. Join I to E. Then, if ω is the angular velocity of CD about I,

$$\omega = V_D/\text{ID} = V_C/\text{IC} = V_E/\text{IE}.$$

$$\therefore \quad V_D = V_C \,.\, \text{ID}/\text{IC} \quad \text{and} \quad V_E = V_C \,.\, \text{IE}/\text{IC}.$$

In this example, if AD = BC and DC = AB, the mechanism becomes similar to the familiar coupling rod and cranks of a locomotive.

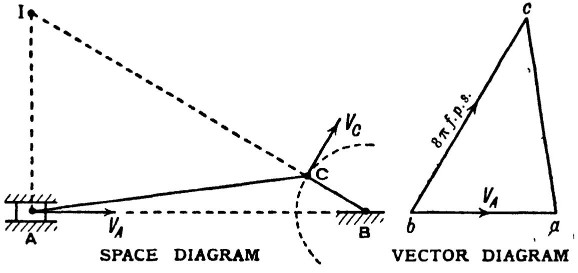
Example 2. Slider crank chain. *In the mechanism of a simple reciprocating engine (Fig. 45), the crank BC rotates at 240 r.p.m. Determine the velocity of the crosshead A and the angular velocity of C about A, if the crank is one foot long and the connecting rod AC is four feet long, in the position when the crank makes an angle of 30° with the inner dead centre.*

Method I. In Fig. 45 draw to a suitable scale the outline or space diagram of the mechanism. Write down the velocity equation.

Velocity of A relative to B = Velocity of A relative to C
+ Velocity of C relative to B.

$$\text{Velocity of C relative to B} = \frac{2\pi NR}{60} = \frac{2\pi \times 240 \times 1}{60}$$

$$= 8\pi \text{ ft. per sec.}$$



SPACE DIAGRAM VECTOR DIAGRAM

FIG. 45.

B may be regarded as fixed relative to the earth, so draw *bc* at right angles to BC in the direction V_C to represent 8π ft. per sec.

Then draw *ca* perpendicular to AC and *ba* parallel to V_A, meeting in *a*.

By measurement, *ba* representing $V_A = 15{\cdot}6$,

ca representing the relative velocity of A to C

$= 22$ ft. per sec.

$$\therefore \text{ the angular velocity of C to A} = \frac{ca}{\text{CA}} = \frac{22}{4} = 5{\cdot}5 \text{ radians/sec.}$$

Method II. To find V_A. Produce BC and draw a perpendicular to AB to meet it at I, the instantaneous centre of rotation for the link AC. IC and IA are respectively perpendicular to V_C and V_A. Measure IC and IA. Let ω be the angular velocity of AC about I.

$$\omega = \frac{V_A}{\text{IA}} = \frac{V_C}{\text{IC}}, \quad \text{or} \quad V_A = V_C\,.\,\text{IA}/\text{IC} = 8\pi\,.\,22/35$$
$$= 15{\cdot}7.$$

Example 3. *Fig. 46 shows a machine in outline in which A is a crank shaft and pivot D is fixed. Crank AB revolves at 60 r.p.m. and is connected by rod BC with rod EC which pivots about D. E drives the crosshead F through link EF. Determine the velocity of F for the given position.*

The problem may be solved in steps by determining (1) the velocity of C, (2) the velocity of E, and finally, (3) the velocity of F.

The velocity of B or $V_B = 2\pi RN$ or $2\pi \times \frac{1}{2} \times 60$ or 60π ft. per min. Points A and D are both fixed, and the velocity equation may be written down :

Velocity of C relative to A = Velocity of C relative to B
+ Velocity of B relative to A.

Of these quantities, the velocity of B relative to A is equal to 60π ft. per min. at right angles to AB ; the velocity of C relative to B is at right angles to BC, but the magnitude is unknown ; and the velocity of C relative to A is of unknown magnitude but is at right angles to CD. Accordingly, in the vector velocity diagram draw ab to scale to represent 60π ft. per min. ; from b and a draw lines respectively at right angles to BC and DC to meet at c. Hence velocity of C $= ac =$ **140 ft. per min.**

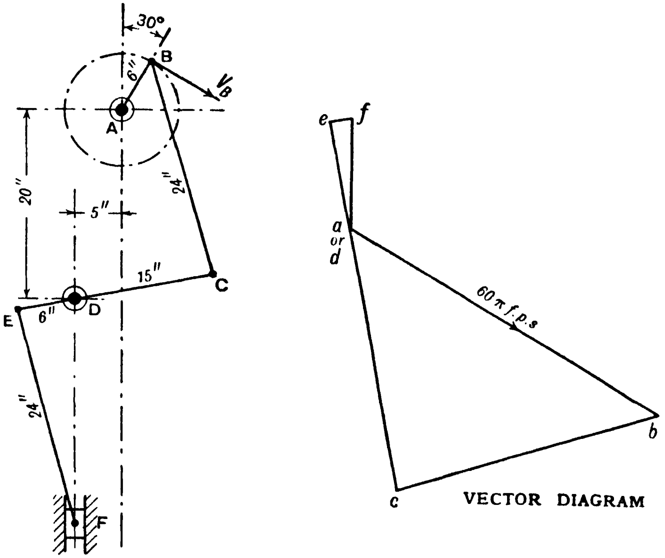
Similarly, to find the velocity of E, since CE is one link pivoted at D, then E will move in the opposite direction to C with a reduced velocity, the reduction obviously being in the ratio of 15 : 6. Therefore, from a in the line ac, but in the opposite direction, draw ae so that

$$ac : ae :: 15 : 6.$$

Thus, velocity of E $= ae =$ **56 ft. per min.**

To find the velocity of F, use the equation :

$$\text{Velocity of F relative to D} = \text{Velocity of F relative to E} + \text{Velocity of E relative to D}.$$



SPACE DIAGRAM

VECTOR DIAGRAM

Fig. 46.

The velocity of E relative to D is the same as that relative to A or *ea*. To find the magnitudes of the remaining velocities of the equation, complete the vector velocity triangle *aef*, in which *af* is drawn parallel to the motion of the crosshead F, and *ef* at right angles to the link EF to represent the relative motion between the ends of that link.

$$\text{Velocity of F} = af = 60 \text{ ft. per min.}$$

Simple and compound wheel trains. In a **simple wheel train** of toothed wheels each axle of the mechanism carries only one wheel, as in Fig. 47. In the case of a **compound train** of toothed wheels

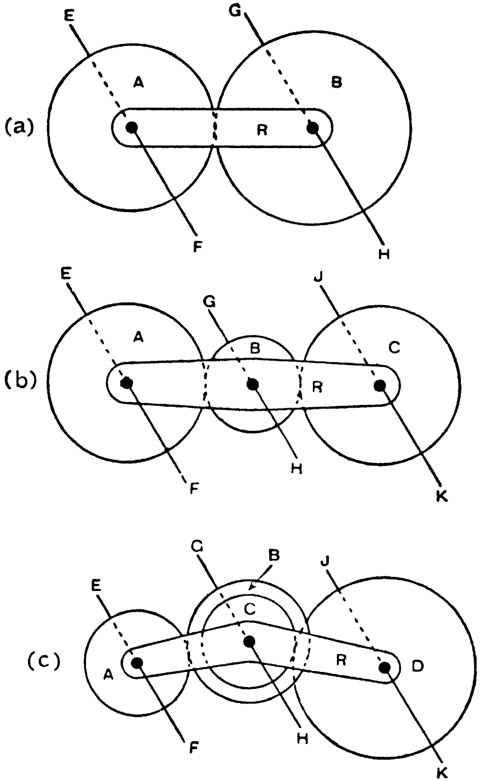


FIG. 47.

each axle except the first and last carries two wheels (Fig. 47, *c*). The velocity ratios of the axles are constant, being dependent solely on the number of teeth on the wheels meshing together, and the ratios can easily be determined as shown on p. 13, or in Vol. I, p. 171.

Both simple and compound trains of toothed wheels have wide applications in industrial machines.

Epicyclic gears. When the arm carrying the axles of a train of wheels itself rotates and one of the wheels is fixed, the resulting arrangement is called an **epicyclic gear,** because every point on a moving wheel traces out one of the cycloidal curves as it turns about the fixed wheel. In such cases at least one axis revolves about another axis which is fixed. Thus in Fig. 47 (*a*, *b*) the axis GH (and JK) might revolve about the axis EF, wheel A being fixed. In other words, the arm carrying the axles itself revolves.

When the axes of the first and last wheels coincide, the arrangement is called a **reverted wheel train.** Obviously in this case the axle is divided into two, the epicyclic gear fitting in between the two parts. An example of this type of train is the **differential bevel gear** used on the driving axles of motor-cars to permit of the driving wheels rotating at different speeds in going round a curve. With **bevel wheel trains** the problem of calculating velocity ratios is essentially the same as for ordinary wheel trains, whether simple, compound, epicyclic or reverted.

Epicyclic and reverted trains are of very great importance, inasmuch as they provide a simple compact method of obtaining a high velocity ratio between two shafts.

In calculating the velocity ratios of the various axes, wheels and arm of an epicyclic gear, the problem would at first appear to be very difficult ; but if the following points are remembered, the principle underlying the method remains the same for all the countless arrangements and combinations of this type of gear.

(1) Rotary motion can be either positive or negative, that is, either clockwise or anticlockwise.

(2) All motions are relative motions.

(3) The relative motion of *two* parts of a machine is the same in magnitude, no matter whether measured from either part or from

some independent part, and no matter which part of the machine is considered fixed.

To solve problems on epicyclic gears, first assume one part fixed and determine the relative motion between the various parts. Work is saved if the fixed part is carefully chosen. Usually it is best to assume that the arm of the gear is fixed. Once this relative motion between the parts is determined and tabulated, the whole of the results may be subjected to the processes of addition, subtraction, multiplication and division, provided the same arithmetical process is applied to the relative velocity of each part. The following examples will help to make clear the principles involved.

Example 1. *Wheel A and wheel B are connected by the arm R in the gear shown in Fig.* 47, *a, and they have respectively* 50 *and* 25 *teeth. Construct a table to give the motion of the other two parts when R, A and B are each, in turn, considered fixed. Also find how many revolutions R will make if B makes* 60 *r.p.m. in an anticlockwise direction.*

Line	Procedure	Condition	Motion of		
			A	R	B
1		R fixed	+ 1	0	− 2
2	Subtract 1 from line 1	A fixed	0	− 1	− 3
3	Add 2 to line 1	C fixed	+ 3	+ 2	0
4	Multiplying line 2 by 20	A fixed	0	− 20	− 60

Clockwise motion is here considered positive ; anticlockwise motion is considered negative.

In line 1, R is considered fixed and A is given + 1 rev., and thus B will make − 2 rev. Subtracting 1 from the results in line 1 gives the motion of R and B when A is considered fixed ; and thus line 2 is formed. In line 3, B is brought to rest by adding 2 to line 1, and the corresponding motion of A and R is determined. Line 4 is obtained by multiplying line 2 by 20, which is necessary to give B a speed of − 60 r.p.m. Thus A is at rest and R is revolving at − 20 r.p.m.

By manipulating the lines, any desired motion can be given to a pair of elements of a gear (usually one fixed and the other moving at the desired speed) and the motion of the others easily obtained. The results obtained in Example 1 can readily be checked on a simple model epicyclic gear.

Example 2. *A reverted train used as a reduction gear is outlined in Fig.* 48. *The wheel G is fixed and has* 60 *teeth, while D and E are keyed to a shaft A, which in turn is carried by an arm rotating with the driving shaft B. D has* 15 *teeth and gears with G, while E has* 16 *teeth and gears with F, which is keyed to the driven shaft C. F has* 59 *teeth. If shaft B makes* 300 *r.p.m., calculate the speed and direction of rotation of shaft* C.

Line	Procedure	Condition	Motion of				
			B	G	D	E	F and C
1		B fixed	0	+1	$-\frac{60}{15}$	$-\frac{60}{15}$	$+\frac{60}{15}\times\frac{16}{59}=1\frac{5}{59}$
2	Subtract 1 from line 1	G fixed	−1	0	−5	−5	$\frac{5}{59}$
3	Multiply line 2 by 300	G fixed B's speed 300 r.p.m.	−300	0	−1500	−1500	$\frac{5}{59}\times 300=25{\cdot}4$

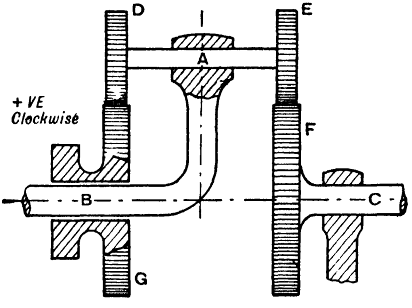


FIG. 48.

Assume that rotation is positive when clockwise as viewed in the direction of the arrow (Fig. 48).

Obtain *line 1* in the table above by assuming the shaft B and its arm are at rest and G is given 1 rev. clockwise.

Line 2 is obtained by subtracting 1 from each speed in line 1, which fixes G, and gives F a motion of $(\frac{60}{15}\times\frac{16}{59}-1)$ or $\frac{5}{59}$ rev.

Line 3 is obtained by multiplying line 2 by 300. The speed of wheel F and shaft C is thus 25·4 r.p.m. and opposite to the motion of B.

SPECIAL APPLICATION OF EPICYCLIC GEARS

Motor-car differential gear. The back axle drive of a motor-car makes it necessary for one of the rear wheels to revolve at a different speed from the other, and the ratio of these speeds must be elastic over the range of cornering, so much so that one wheel, the inner, must on certain occasions and under certain conditions be almost stationary while the outer wheel runs comparatively fast while turning the corner. In any case, the wheels are normally the same in diameter and equidistant from the line of the main engine drive, while the wheel velocity will depend on the radius of curvature at which each wheel is turning. The front wheels are not affected in this way, because they are separately mounted on steel axles and are not connected with the power transmission from the engine, except in front-wheel drive trucks.

The problem is solved by the use of a form of epicyclic gear known as the **differential gear** (Fig. 49). The wheel A is driven direct from the engine shaft and gears with B, which is free to rotate on one half of the back axle C. To this half of the back axle and to the other half D are keyed the two equal wheels E and F. Attached to B is a

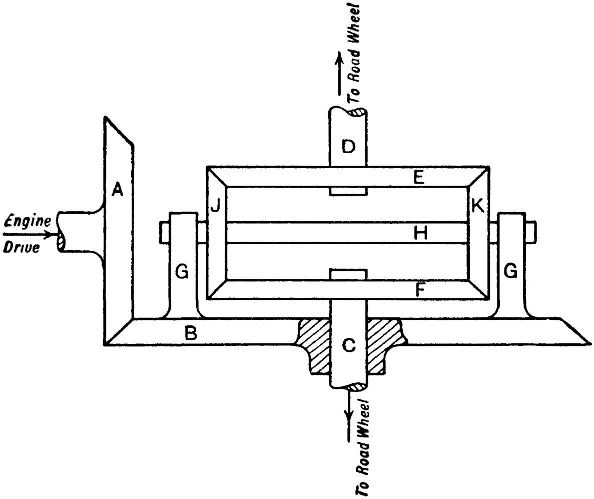


FIG. 49. DIFFERENTIAL GEAR.

bearing G carrying a shaft H to which are attached the equal wheels J and K, free to rotate on the shaft H.

The behaviour of the mechanism is best considered by drawing up the velocity table for B, E, F, J and K as follows :

Condition	B	F	E	Procedure
B fixed. Give $+1$ to E	0	-1	$+1$	
Give $+n$ to E - -	0	$-n$	$+n$	
Give $+x$ to B - -	x	$x-n$	$x+n$	Add x

It can be seen that as the speed of B is directly related to the engine speed according to the teeth relation between A and B, *any decrease or increase* of the speed of E, say by x, will produce a corresponding *increase or decrease* to its complementary wheel F without change of engine speed. Thus the road wheels connected to E and F will run at equal speeds on a straight road, but will readily alter speed to meet cornering as required.

Applications. From the Principles of Work and Power for the ideal or perfect machine (or where the friction is negligible, p. 8), a knowledge of the velocity ratio also gives the force ratio in a machine, because the force ratio is the reciprocal of the velocity ratio and *vice versa.* An unknown force acting at a certain point in a machine can thus be calculated from the vector velocity diagram. The same principle can be extended to a part of a jointed structure by imagining a small movement of a point in a structure arising from the elastic yielding of one of its members, as when the member is supposed to be replaced by a stiff coiled spring (see p. 15). Also data obtained from the vector velocity diagram enable the vector acceleration diagram to be constructed for the same configuration of the machine, and when the acceleration is known it is possible to deduce the forces involved. These forces are considerable in high-speed machinery and engines, but such work is beyond the scope of this book.

Similarly, a knowledge of the velocity ratios in operation in an epicyclic gear enables the corresponding force ratios to be determined.

EXERCISES ON CHAPTER III

1. What is the meaning of the term "relative velocity"? How is the relative velocity obtained in the case of the motion of two vehicles moving along the same or parallel straight lines?

2. Describe a method of finding the instantaneous relative velocity of two bodies which at a given instant are moving in directions making a known angle with each other.

3. Find the velocity of a point on the rim of a wheel 10 ft. in diameter which is rotating in its bearings at 10 radians per second. If the velocity increases with an acceleration of 1 radian per second per second, find the vector change of velocity after one second.

4. An aeroplane's speed is 240 knots and it is travelling due north on a calm day. What would be the actual course or track of the machine if a breeze from a bearing of 225° true at 45 knots suddenly springs up? Measure the drift, that is, the angle of deviation due to the wind.

5. A fighter has an air speed of 450 knots and is overtaking a bomber flying on the same course at 300 knots. If the fighter turns away when 100 yd. from the bomber, for how many seconds before this was the fighter within 1000 yd. range of the bomber?

6. How long will it take an aeroplane to traverse from A to B if A is 30 miles north of B and 300 miles from it if the air speed of the aeroplane is 250 knots? The wind velocity is 35 knots from a bearing of 315° true.

7. Find the shortest time it would take a swimmer whose water speed is $2\frac{1}{4}$ m.p.h. to cross a stream 200 yd. wide if the river is flowing uniformly at $1\frac{1}{2}$ m.p.h. What direction should the swimmer take?

8. Two ships, the speeds of which are 20 and 25 knots respectively, are timed to leave harbour at noon. Their courses are straight and the angle between them is 75°. Find (*a*) the speed of the second ship relative to the first, (*b*) when the first ship will be 20 nautical miles from the second, and the bearing of the second from the first at that time.

9. The straight-line tracks of two aeroplanes cross at the same altitude at right angles. At noon, the first is 20 miles from the crossing and its speed is 240 knots, while the second is 24 miles from the crossing and its speed is 360 knots. When will the two planes be nearest together, and how far will each have travelled since noon?

10. A destroyer, the maximum speed of which is 40 knots, sights due west an enemy vessel which is travelling due north at 20 knots. Find the course the destroyer must take to intercept the enemy as soon as possible, and its relative velocity to that of the enemy. How long will it take to intercept if the enemy is $17\frac{1}{2}$ miles away?

11. Two trains of equal length are approaching a level crossing or crossover at the rate of 40 and 50 ft. per sec. respectively. Find the greatest length the trains can have, in order to avoid a collision, if the distances

of the engines from the crossing are respectively 1200 yd. and 1600 yd.

12. A piston rod rotates a crank at 2 revs. per sec. through a crosshead and connecting rod. The crank is 1 ft. long and the rod 5 ft. When the crank has turned through 40° from its inner dead centre, find (*a*) the crosshead speed, (*b*) the position of the instantaneous centre of the connecting rod, (*c*) the angular velocity of the connecting rod, (*d*) the turning moment on the crank shaft if the piston thrust is 1 ton.

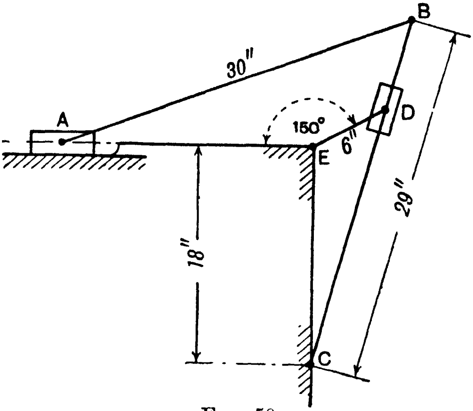


FIG. 50.

13. Find the relative velocities of the piston and valve of a steam engine when the crank is 60° from its inner dead centre and the engine speed is 120 r.p.m. The crank radius is 10 in., the connecting rod 40 in. long, eccentric radius $2\frac{1}{2}$ in., eccentric rod 40 in. long, and the angle of advance of the eccentric 30°.

14. Find the position of the instantaneous centres for all the links of the mechanism shown in Fig. 46 (p. 71) when the crank AB is in the position shown.

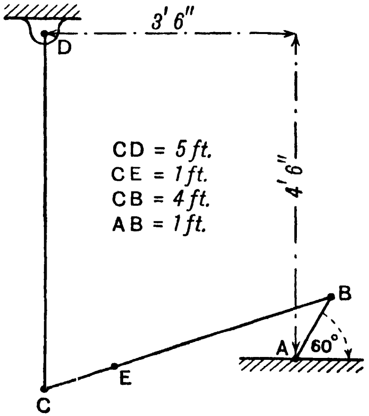


FIG. 51.

15. Fig. 50 shows a quick-return motion for a slotting machine. The block D rotates about the fixed point E and at the same time slides along rod BC which oscillates about the fixed point C. B drives the crosshead A by means of rod AB. The slide A carries the tool. If ED rotates uniformly at 30 r.p.m., find the speed of A.

16. In Fig. 51 the crank AB is 1 ft. long and rotates at 300 r.p.m. Find the angular velocities of the links BC and CD if these are respectively 4 ft. and 5 ft. long. Link CD oscillates about D. Also find the linear velocity of the point E which is 1 ft. from C.

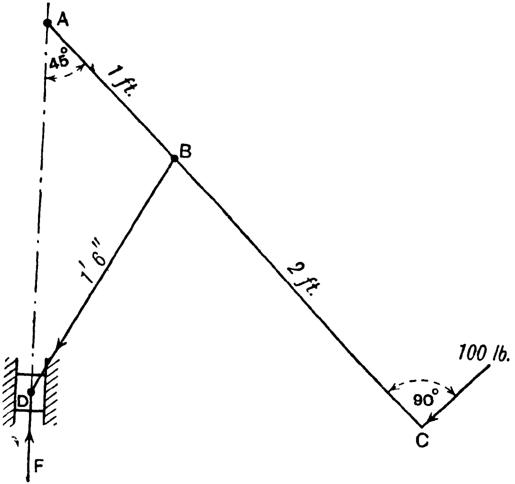

A
45°
1 ft.
B
1' 6"
2 ft.
100 lb.
90°
C
D
F

FIG. 52.

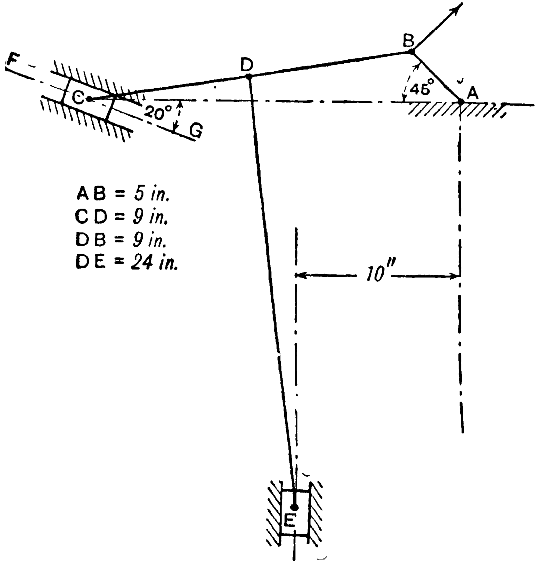

B
F
D
C
20°
G
45°
A
AB = 5 in.
CD = 9 in.
DB = 9 in.
DE = 24 in.
10"
E

FIG. 53.

17. For the simple press shown in Fig. 52, calculate the velocity of D if C is moving at right angles to AC at a speed of 1 ft. per sec. If a force of 100 lb. is applied at C, find the force F necessary to restore equilibrium for this position of the machine if friction is neglected.

18. In Fig. 53 the crank AB rotates at 240 r.p.m., C slides in the direction FG, and E slides vertically. The point D is the mid-point of BC. When the angle BAC is 45°, draw the vector velocity diagram for the mechanism and find the velocities of C and E.

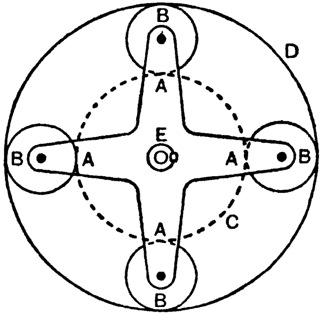


FIG. 54.

19. The wheels A, B and C in Fig. 47, *b* (p. 72), have respectively 30, 20 and 50 teeth. If wheel A is fixed, find the speeds of B and C if R rotates clockwise at 60 r.p.m.

20. In Fig. 47, *c* (p. 72), the wheels A, B, C, D have respectively 30, 50, 20 and 60 teeth. When A is fixed and R revolves clockwise at 120 r.p.m., calculate the speed of wheels B, C and D. Also find the speeds of R, A, B and C if D is fixed and C makes 200 r.p.m. anticlockwise.

21. In the epicyclic gear shown in Fig. 54, the wheels B, C and D have respectively 20, 60 and 100 teeth. The arms A are keyed to the shaft E and the planet wheels B are free to rotate at the ends of the arms. If C is fixed, find the velocity ratio between D and A.

22. A motor epicyclic reduction gear is shown in Fig. 55. Wheel B is keyed to the motor shaft, C and D are cast together and free to turn on the end of arm A. The annular wheel D is fixed and does not rotate, while the annular wheel E is keyed to the machine shaft. Find the reduction ratio for the gear if the numbers of teeth on the wheels are as indicated on the figure. Also find the force ratio at the pitch circles of the wheels B and E if friction is neglected.

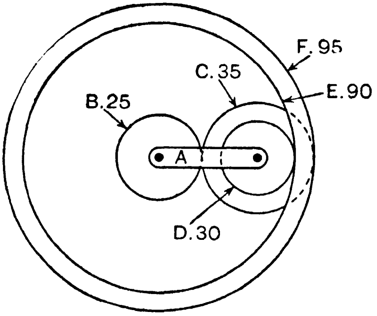


FIG. 55.

CHAPTER IV

STRENGTH AND ELASTICITY OF MATERIALS

THE effectiveness of any machine or structure depends upon a selection of suitable materials and a knowledge, possessed by the designer, of the behaviour of these materials under given conditions. Modern design, with the manufacturing conditions imposed upon it, demands constant supervision of the materials employed, and the metallurgist co-operates with the engineer to produce metals and alloys suited to the special requirements of any particular design. This supervision covers the methods employed in manufacture of the metal, its physical and chemical structure, the heat treatment which it shall receive, and the mechanical testing of representative specimens under something approaching overload conditions. Investigations are carried out on the effects of fatigue, alternation of stresses, impact loading and temperature changes, in addition to tests on the material under static loading until fracture occurs.

The purpose of this chapter is not to discuss the metallurgical aspect of these tests, but to set out the mechanical tests of the material as generally applied in the routine supervision of materials.

These tests generally take the form of (*a*) loading a specimen to destruction by tension, (*b*) some form of impact test, (*c*) an investigation into the natural hardness of the material. Supplementary tests may be carried out in compression, bending and shear, and the material may be subjected to fatigue and rapid alternation of load tests.

Behaviour of a material under load. When a material is subjected to the action of external forces, or **load**, the physical structure immediately adjusts itself to resist these forces and the material is said to be **stressed.** Invariably this stress is associated with some form of dimensional deformation, which may be temporary over the period of loading, or permanent if the material is unable to recover from the deformation when the effect of the external force is removed. This deformation is called the **strain**, and it cannot be dissociated from a condition of stress.

The extent to which the strain or deformation can be taken without assuming a permanent character is dependent upon the **elasticity** of the material, and elasticity may be described as the property a material possesses of resisting strain. The property is shown in the ability possessed by the material of recovering its original dimensions after the straining forces are removed.

Hooke's Law states that during the period of complete elasticity the strain is directly proportional to the stress.

It must be understood that the elasticity of a material does not generally break down completely at a specific load, but that the material is perfectly elastic up to a certain stress condition, after which as a rule the elasticity gradually fails and the material takes permanent deformation, although some residual elasticity is retained almost to the point of fracture.

Nature of deformation. During the period of stressing and straining in which Hooke's Law holds (that is, the stress and strain are proportional and elasticity is perfect) the strain is called the **elastic deformation.** At the end of this period the material yields, or the **limit of proportionality** is reached and the deformation becomes wholly or partially permanent. The material is then wholly or partially plastic, or mouldable, and the deformation after the **yield point** is known as **plastic deformation.**

The **yield point** is defined as the **load per square inch at which a distinctly visible increase occurs in the gauge length observed** either (*a*) by using dividers after the removal of the load, or (*b*) when there is a distinct movement of the testing machine lever as the load is steadily increased, or (*c*) when there is a drop in the gauge finger in the case of a hydraulic testing machine. In commercial tests, the yield point is often referred to as the " commercial elastic limit ".

The elastic limit, or limit of proportionality, is that point at which the extensions cease to be proportional to the loads ; that is, it is the point on the load *v.* extension and the stress *v.* strain diagrams where the graph ceases to be a straight line and becomes less steeply inclined or perhaps curved.

Fig. 56 shows diagrammatically the behaviour of three common metals tested in tension to fracture. It will be noticed that in the naturally ductile soft copper, practically the whole of the deforma-

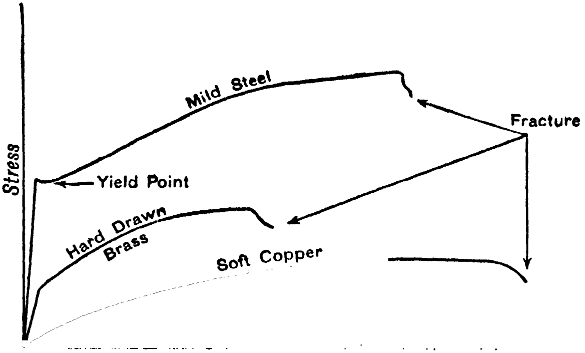


FIG. 56.

tion is plastic, whereas in mild steel there is a definite change-over from the elastic to the plastic deformation at the yield point.

MEASUREMENT OF STRESS, STRAIN AND ELASTICITY

The intensity of stress in a material subjected to external forces is measured as the average load or force per unit area of the cross-section under load, and the fractional strain is the average deformation per unit length. Thus :

$$\text{Intensity of stress} = \frac{\text{Load}}{\text{Area of cross-section}},$$

$$\text{Fractional strain} = \frac{\text{Deformation}}{\text{Original dimension}}.$$

These quantities will, in accordance with practice, be referred to in the future simply as stress and strain. **The modulus of elasticity,** or **Young's modulus,** is the ratio between the mean intensity of stress and the fractional strain taken during the period of elastic deformation. It is to some extent a measure of the elastic properties of the material and is expressed in the same units as those adopted for stress. Thus :

$$\text{Modulus of Elasticity (E)} = \frac{\text{stress}}{\text{strain}}.$$

The modulus of elasticity is obtained by plotting a graph of load *v.* extension for the material and multiplying the slope of this graph by the ratio l/A, where l is the gauge length and A the original area of cross-section of the specimen. In this way the representative mean value of stress divided by strain over the range of proportionality is best obtained.

Tensile and compressive stress produces a strain in its own direction and also an opposite kind of strain in every direction perpendicular to its own. For example, a tie bar subject to tensile stress extends in the direction of its length but contracts laterally. The ratio $\dfrac{\text{lateral strain}}{\text{longitudinal strain}}$ is usually constant for a material within the elastic limit, and is known as **Poisson's Ratio,** after S. D. Poisson (1781–1840), who first used the ratio when investigating the increase in volume of a rod in tension. For many metals the value of the ratio is about $\frac{1}{4}$, and for india-rubber about $\frac{1}{3}$. Frequently, the ratio is denoted by the symbol $1/m$, and it is an important factor in the relations which exist between the various moduli of elasticity for materials. These moduli are **Young's Modulus**, the **Bulk Modulus** and the **Modulus of Rigidity.**

Example 1. *A steel column* 4 *in. in diameter and* 20 *in. in length carries a thrust of* 94 *tons which remains axial during the loading. Calculate* (*a*) *the intensity of stress,* (*b*) *the shortening of the column, when* $E = 13{,}000$ *tons per sq. in.*

$$\text{Area of cross-section} = \pi \times 2^2 = 12{\cdot}56 \text{ sq. in.}$$

$$\text{Stress} = \frac{\text{Load}}{\text{Cross-sectional area}} = \frac{94}{12{\cdot}56} = \mathbf{7{\cdot}48 \text{ tons per sq. in.}}$$

$$E = \frac{\text{stress}}{\text{strain}} = \frac{7{\cdot}484}{x} = 13{,}000 \text{ tons per sq. in.}$$

$$x = \frac{7{\cdot}484}{13{,}000} = 0{\cdot}000576.$$

$$\text{Strain} = x = \frac{\text{deformation}}{\text{original length}}.$$

$$\text{Deformation} = 0{\cdot}000576 \times 20 = \mathbf{0{\cdot}0115 \text{ in.}}$$

Example 2. *A hexagonal tie rod of* $\frac{3}{4}$ *in. side and* 50 *in. in length is found to lengthen* 0·04 *in. under load. If the modulus of elasticity of the material is* 13,700 *tons per sq. in., find the load carried by the rod.*

$$\begin{aligned}\text{Area of cross-section} &= 6 \times \tfrac{3}{4} \times \tfrac{1}{2} \times \tfrac{3}{4} \cos 60^\circ \\ &= 1{\cdot}461 \text{ sq. in.}\end{aligned}$$

$$E=\frac{\text{stress}}{\text{strain}}=\frac{L}{A}\times\frac{l}{e}=\frac{L}{1{\cdot}461}\times\frac{50}{0{\cdot}04}.$$

$$L=\frac{13{,}700\times 1{\cdot}461\times 0{\cdot}04}{50}=\mathbf{16\ tons.}$$

Example 3. *A steel tie bar* 2 *in. in diameter is subjected to compressive stress, in the direction of its* 3 *in. length, by an axial load. The length of the bar is shortened by* 0·0036 *in. and the diameter increases by* 0·00062 *in. Calculate Poisson's Ratio for the material.*

$$\text{Longitudinal strain}=\frac{0{\cdot}0036}{3}=0{\cdot}0012.$$

$$\text{Lateral strain}=\frac{0{\cdot}00062}{2}=0{\cdot}00031.$$

$$\text{Poisson's Ratio}=\frac{0{\cdot}00031}{0{\cdot}0012}=\mathbf{0{\cdot}26.}$$

Example 4. *A bar* 1½ *in. in diameter and* 2 *ft. in length is subjected to a tensile stress of* 20 *tons per sq. in. Calculate the extension of the bar if* $E=13{,}500$ *tons per sq. in. and the probable diametrical reduction if Poisson's Ratio is* 0·25.

$$E=\frac{L}{A}\times\frac{l}{e}=\frac{\text{stress}}{\text{strain}}.$$

$$13{,}500=\frac{20}{\text{strain}}.$$

$$\text{Extension}=24\times\frac{20}{13{,}500}=\mathbf{0{\cdot}036\ in.}$$

$$\text{Poisson's Ratio}=\frac{\text{lateral strain}}{\text{longitudinal strain}}$$

$$0{\cdot}25=\frac{s\times 24}{0{\cdot}0356}\quad\therefore\ s=0{\cdot}00037.$$

$$\text{Diametrical reduction}=0{\cdot}00037\times 1{\cdot}5=\mathbf{0{\cdot}00056\ in.}$$

Composite sections. When a unit in a machine or structure is made up of two or more materials, such as reinforced concrete or a bimetal arrangement, it is assumed that the strains in the constituents are equal. For example, a reinforced concrete column will carry a load in such a way that the strain suffered by the concrete is equal to that given to the reinforcement. Since the moduli of elasticity vary, the stresses set up will be unequal.

Example 1. *A reinforced concrete pillar* 10 *in. square is reinforced by* 12 *steel bars each* ½ *in. diameter. If the length of the column is* 8 *ft., find the stresses in the concrete and reinforcement when the load is* 20 *tons. E for steel* = 13,500, *for concrete,* 1,000 *tons per sq. in.*

$$\text{Area of cross-section, reinforcement} = 12 \times \pi \times \tfrac{1}{16} = 2\cdot355 \text{ sq. in.}$$

$$\text{Area of cross-section, concrete} = 100 - 2\cdot355 = 97\cdot645 \text{ sq. in.}$$

Let f_c be stress in the concrete and f_r that in the reinforcement.

Then, for equal strains, since $E = \dfrac{\text{stress}}{\text{strain}}$,

$$\frac{f_c}{E_c} = \frac{f_r}{E_r}, \quad \text{or} \quad \frac{f_r}{13\cdot5} = \frac{f_c}{1};$$

$$f_r - 13\cdot5 f_c.$$

Now, load = stress × area.

∴ load carried by concrete $= f_c \times 97\cdot645$.

load carried by reinforcement $= 2\cdot355 f_r$.

$$(97\cdot645 + 13\cdot5 \times 2\cdot355) f_c = 20,$$

$$\mathbf{f_r = 0\cdot154 \text{ tons per sq. in.},}$$

$$\mathbf{f_c = 2\cdot08 \text{ tons per sq. in.}}$$

Example 2. *A hydraulic buffer plunger is* 10 *in. in diameter and takes an axial thrust of* 300 *tons. It is made of steel with a brass liner* 1 *in. in thickness shrunk and keyed to the steel core. If the ratio of the moduli for steel and brass are as* 13 : 8, *calculate the stresses set up in the core and the liner. What proportion of the thrust is taken by each?*

$$\text{Area of core section} = \pi \times 4^2 = 50\cdot24 \text{ sq. in.}$$

$$\text{Area of liner section} = \pi\{5^2 - 4^2\} = 28\cdot26 \text{ sq. in.}$$

$$\text{For equal strains, } \frac{f_c}{E_c} = \frac{f_e}{E_e}; \quad \text{or} \quad 8f_c = 13f_e.$$

Load taken by core + Load taken by liner = Total load.

$$50\cdot24 f_c + 28\cdot26 \times \tfrac{8}{13} f_c = 300,$$

$$67\cdot63 f_c = 300,$$

$$\mathbf{f_c = 4\cdot44 \text{ tons per sq. in.}}$$

$$\mathbf{f_e \doteq 2\cdot73 \text{ tons per sq. in.}}$$

Thrust taken by core $= 4\cdot436 \times 50\cdot24 =$ **222·8 tons.**

Thrust taken by liner $= 2\cdot73 \times 28\cdot26 =$ **77·2 tons.**

Stresses due to temperature. It is common practice to take advantage of the expansion and contraction of metals under the influence of heat when, for example, tyres have to be shrunk on to wheels, liners on to rams or plungers, and as a remedy when seizure occurs between shafts and bearings. In addition to this, temperature changes affect members of a structure and components in a machine by causing expansion or contraction against the resistance due to the rigidity of the structure or machine. This expansion or contraction may be regarded as a strain, and since stress and strain are simultaneous, if there is resistance to the expansion or contraction, stress will be set up in the member concerned.

The extent of the expansion or contraction is dependent upon two factors : (1) the change in temperature, (2) the coefficient of linear expansion of the material (Vol. II, p. 10). If these two factors are known, the amount of expansion or contraction can be calculated. The **coefficient of linear expansion** (α) is the expansion of unit length of a material during a temperature rise of 1 centigrade (or fahrenheit) degree.

Thus, for a member of length l in. the expansion will be $l\alpha(t_2 - t_1)$, where α is the coefficient of linear expansion and $t_2 - t_1$ the change in temperature in centigrade degrees.

Applying this to $E = \dfrac{\text{stress}}{\text{strain}}$, the strain expressed as a fraction of the length is $(t_2 - t_1)\alpha$, and the corresponding stress due to temperature change is

$$\mathbf{E\alpha(t_2 - t_1).}$$

The ultimate tensile strength of the carbon steels falls, at first, as the temperature increases, but then rises to a maximum at about 360° C. Above this temperature the ultimate tensile strength falls rapidly as the temperature increases. With wrought and cast iron the strength increases and reaches a maximum at about 360° C., then falls continuously with further rise in temperature. The ultimate tensile strength of copper and most non-ferrous metals decreases rapidly as the temperature rises.

It is important to realise that steam-operated plant uses steels for construction operating between 350° C. and 600° C., while steel used for oil refineries and chemical plant may have even higher operational temperatures. As the engineer constantly strives after increased thermal efficiency and a greater unit of power for every unit of weight, greater demands are likely to be made on the supply of materials capable of performing their functions under high-temperature conditions. Carbon and alloy steels are used for high-temperature service, and the effects of composition, method of manufacture and variation of physical condition as a result of heat treatment are of the greatest importance. Above 400° C. the strength properties of carbon steel are such that prolonged application of stress produces continuous slight plastic flow or **creep**, and there is no definite breaking strength. Thus the important criterion for steel under stress at high temperatures is the degree of resistance to creep, which is very sensitively measured by the rate of creep extension under tensile stress in inches per inch length of specimen per hour. Creep strains of the order of 10^{-6} inch per inch can be measured. Molybdenum appears to be the most promising element to increase creep resistance in alloy steels, while in carbon steels it has been found that no advantage has been obtained by increasing the carbon content above 0·4 per cent. Molybdenum steel would appear to form the best basis for investigation and research towards improvement. Research has also shown that the inclusion of one or more other elements makes the steel more effective over certain ranges of temperature. A vanadium-molybdenum steel has already given promising results.

Investigations are also being carried out on the corrosion produced on various alloy steels in flue gases and superheated steam. It is found that there is a marked increase in the rate of corrosion above 500° C. towards 600° C. The increase of sulphur dioxide is found to increase the amount of corrosion, being especially marked at the lower temperature. Evidence has been obtained that an alloy steel with about 5 per cent. of chromium and some molybdenum gives increased resistance to corrosion at high temperatures. The table (p. 115) lists many of the different metallic and other elements incorporated in the various types of steels, and their effects.

The addition of various elements to cast iron, combined with correct methods of manufacture and heat treatment, also gives additional strength and resistance to heat, corrosion, erosion and wear. For example, "Niresist" iron was developed primarily to resist corrosion and heat, and contains nickel (14 per cent.), copper (7 per cent.) and chromium (2 per cent.). "Nicrosilal" iron was developed for heat resistance and contains nickel (18 per cent.), chromium (2 per cent.) and silicon (5 per cent.). Both these irons have an improved resistance to creep, the latter especially at higher temperatures between 600° C. and 850° C. "Nihard" is a white cast iron, developed to provide resistance to wear, and contains nickel (4·5 per cent.) and chromium (1·5 per cent.). It may be mentioned here that "Nomag" iron, developed for its non-magnetic qualities, contains nickel (10 per cent.) and manganese (5 per cent.).

Example 1. *A steel tie rod is rigidly fixed at its ends, is* 10 *ft. in length and* 1¼ *in. in diameter. Calculate the tensile stress set up by a fall in temperature of* 10° *C. and the load equivalent to this stress. Since rigidity is, in practice, impossible, calculate also the load equivalent if the supports yield* 0·004 *in.* $E = 13{,}000$ *tons per sq. in. and* α *for steel* $= 0{\cdot}00001$.

(*a*) Rigid supports : natural contraction prevented.

$$\text{Contraction per inch length} = \alpha(t_2 - t_1)$$
$$= 0{\cdot}00001 \times 10 = 0{\cdot}0001 \text{ in.}$$
$$\text{Stress set up} = E \times \text{contraction per inch}$$
$$= 13{,}000 \times 0{\cdot}0001 = \mathbf{1{\cdot}3 \text{ tons per sq. in.}}$$
$$\text{Load equivalent} = 1{\cdot}3 \times \left\{\frac{\pi \times (1\tfrac{1}{4})^2}{4}\right\} = \mathbf{1{\cdot}59 \text{ tons.}}$$

(*b*) Yielding supports :

$$\text{Contraction prevented} = 120 \times 0{\cdot}0001 - 0{\cdot}004 = 0{\cdot}008 \text{ in.}$$
$$\text{Stress} = E \times \text{strain} = \frac{13{,}000 \times 0{\cdot}008}{120} = \mathbf{0{\cdot}87 \text{ tons per sq. in.}}$$
$$\text{Load equivalent} = \mathbf{1{\cdot}06 \text{ tons.}}$$

Example 2. *A copper rod* 1¼ *in. in diameter is sheathed by a steel tube* 1¼ *in. bore and* 2½ *in. external diameter. Find the stress set up by a temperature rise of* 140° *C. if there is no buckling over the length and the rod and tube are rigidly secured together at their ends at the lower temperature.* E *for steel* $= 13{,}000$ *tons per sq. in.* E *for copper* $= 6{,}500$ *tons per sq. in.* α *for steel* $= 0{\cdot}00001$ *and* α *for copper* $= 0{\cdot}000017$ *per* 1° *C.*

Before this question can be worked, the premises must be examined. They are :

(1) The copper will not reach its full free expansion.

(2) The steel will be extended to an extent equal to the actual expansion of the copper.

(3) The total pull in the steel will equal the total thrust in the copper.

The effective strain will be 0·000017 − 0·00001

$$= 0{\cdot}000007 \text{ per degree.}$$

$$\left.\begin{array}{l}\text{If } S_s = \text{strain in the steel} \\ \quad S_c = \quad ,, \quad ,, \quad \text{copper}\end{array}\right\} S_s + S_c = 140(0{\cdot}000007).$$

$$\text{Stress in the steel} \quad = 13{,}000\ S_s.$$

$$\text{Stress in the copper} = 6{,}500\ S_c,$$

and since pull in the steel = thrust in the copper,

$$13{,}000 \,.\, S_s \left\{\frac{\pi}{4}(2{\cdot}5^2 - 1{\cdot}25^2)\right\} = 6{,}500 \,.\, S_c \left\{\frac{\pi}{4}(1{\cdot}25)^2\right\},$$

$$2S_s(3{\cdot}75)(1{\cdot}25) = S_c(1{\cdot}25)^2,$$

$$6S_s = S_c\,;$$

but

$$S_s + S_c = 140(0{\cdot}000007)\,;$$

$$\therefore\ 7S_s = 140(0{\cdot}000007),$$

$$S_s = 0{\cdot}00014 \text{ and } S_c = 0{\cdot}00084.$$

Stress in the steel = 13,000 × 0·00014 = **1·82 tons per sq. in.**

Stress in the copper = 6,500 × 0·00084 = **5·46 tons per sq. in.**

Example 3. *Two copper blocks each* 1½ *in. in thickness are secured by four steel bolts and nuts* 1 *in. diameter fitted with large-diame er washers to prevent indentation. Calculate the increase in stress in the bolts due to a temperature rise of* 200° *C., using the values of E and α given in Example* 2.

In this case the large-diameter washers have the effect of making the compressive strain in the copper negligibly small, and the resultant temperature strain on the steel bolts is 0·000017 − 0·00001 per ° C.

$$\text{Total strain} = 200 \times 0{\cdot}000007$$

$$= 0{\cdot}0014.$$

$$\text{Increase in stress} = 13{,}500 \times 0{\cdot}0014$$

$$= \mathbf{18{\cdot}9 \text{ tons per sq. in.}}$$

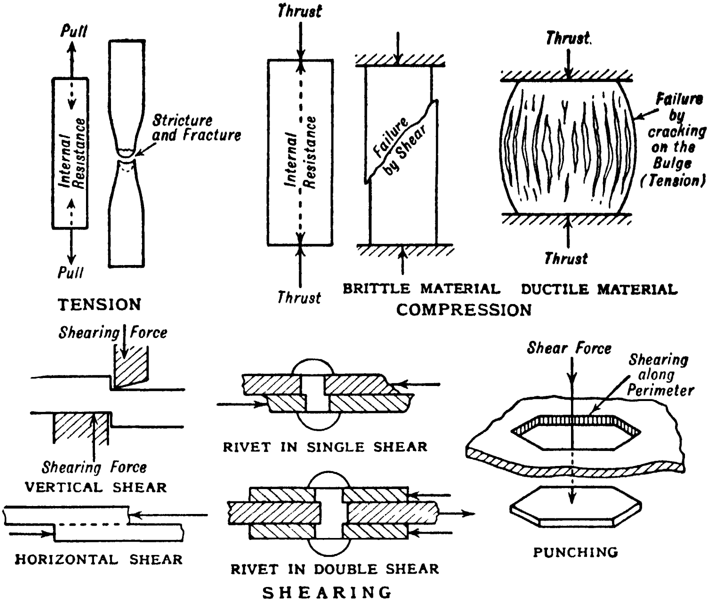


FIG. 57.

Methods of failure. The failure of a material under load may occur in one of several ways dependent upon the strength of the material and the manner of loading. The primary causes of failure are the destruction of the material under (*a*) **tension,** or direct pull, (*b*) **compression,** or direct thrust, (*c*) **shearing,** or the forcing of one portion of the material to slide over the other. In the case of compression, the actual cause of failure can usually be identified with tension or shearing, although the application of the force is direct thrust.

Fig. 57 illustrates examples of these methods of failure. The student is advised to give particular attention to the identification of modes of failure in those machine and structural components with which he is familiar.

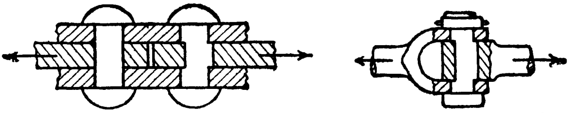
It frequently happens that conditions of loading introduce a variety of stresses in the material, and failure takes place either in the locality most heavily stressed or where the material is least able to resist the particular stress set up.

Stress distribution in beams, cantilevers and shafts. The distribution of stress over the sections of beams, cantilevers and shafts depends upon the conditions of loading and the shape of the sections. This distribution is treated in the sections of the book devoted to beams and shafts, and it need only be realised at this stage that failure can always be traced to one of the primary methods of failure taking effect at the weakest portion of the section, when regarded from its resistance to the stresses set up by the loading. This general statement also applies to struts, where the stresses due to direct loading may be augmented by those due to buckling.

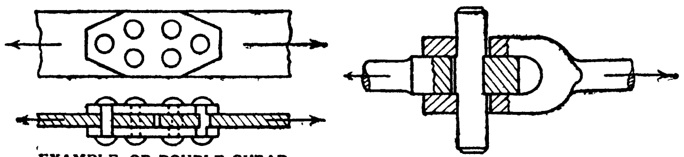
Multiple shear. When a material is subjected to shear stress the distribution may be such that more than one cross-section or surface is subjected to the stress at the same time. If failure is to occur, fracture or sliding will have to take place on each stressed area (Fig. 58), and the material is said to be in **double or treble shear** according to the number of surfaces resisting the shear.

The commonest example of multiple shear is in the form of double shear. This is found in many forms of riveted and pin joints, cottered and bolted connections and other machine and structural components.

The area resisting shear is generally taken as double or treble the single shear area according to the number of surfaces involved, but in certain Board of Trade regulations for riveted joints in double shear, 1·75 is specified instead of 2. The reason for this safety factor is to make due allowance for badly fitted rivets and the bending which may be induced.



RIVET OF BUTT JOINT IN DOUBLE SHEAR PIN JOINT IN DOUBLE SHEAR



EXAMPLE OF DOUBLE SHEAR ON EACH OF THREE RIVETS COTTERED JOINT IN DOUBLE SHEAR

Fig. 58.

Modulus of rigidity and angular strain. The previous discussion of the elastic constant for a material has been confined to a strain measured on the linear dimensions, that is, strain associated with tensile or compressive stress. The relation between stress and strain for this linear deformation is the **Modulus of Elasticity.**

When the strain, as in the case of shear, becomes the deformation of a surface, a further modulus can be obtained known as the **modulus of rigidity** (C), or, as it is sometimes called, the **modulus of transverse elasticity.**

Consider the angular strain produced by a shear stress. Suppose a block of the material (Fig. 59) be rigidly attached at OP, and a force F produces a shear stress of f tons per sq. in. in the surface QR.

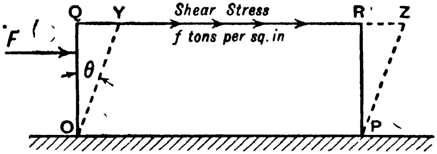


FIG. 59.

The angular displacement will be θ, and therefore the resultant shear strain expressed as an angle, QOY or θ radians.

Since θ is usually small,

$$\theta \doteq \sin\theta = \frac{QY}{OY},$$

and

$$\text{Modulus of Rigidity } (C) = \frac{\text{shear stress}}{\text{angular strain}}.$$

$$C = \frac{f}{\theta} = \frac{f}{\sin\theta} = \frac{f\,OY}{QY}$$

Resilience. When a material is subjected to tensile or compressive stresses *within its elastic limit,* energy is supplied to the material and stored in it, to be expended in the process of recovery after the straining forces are removed. According to the principle of conservation of energy, the energy stored must be equal to the work done in elastic deformation, that is, the mean or average force applied times the extension or compression produced.

Since the average force for elastic straining will be one-half the maximum applied,

$$\text{Work done} = \tfrac{1}{2} \times \text{force} \times \text{deformation} ;$$

and since $$E = \frac{\text{stress}}{\text{strain}} = \frac{\text{load} \times \text{original length}}{\text{area} \times \text{deformation}} ,$$

$$\text{Deformation} = \frac{\text{load} \times \text{original length}}{\text{Area} \times E} = \frac{\text{stress} \times l}{E} .$$

$$\begin{aligned}\text{Hence, work done} &= \tfrac{1}{2} \text{ force} \times \frac{\text{stress} \times l}{E} \\ &= \frac{\tfrac{1}{2} \text{ stress} \times \text{area} \times \text{stress} \times l}{E} \\ &= \frac{(\text{stress})^2 \times \text{volume}}{2E} .\end{aligned}$$

When this stress is that required to strain the material to its limit of proportionality, this maximum elastic work done or energy stored is known as the **Proof Resilience.**

$$\textbf{Proof Resilience} = \frac{f^2 V}{2E} ,$$

where f is the stress at the elastic limit, E is Young's modulus and V is the volume.

In the case of materials of varied section, the resilience of the various parts may be obtained and added to find the total. The area under the load *v.* extension diagram gives the work done in fracturing the specimen.

The rate of loading has a big effect on the behaviour of a metal on the plastic portion of the extension. Also, it is found that if at some point of the plastic strain the load is removed, a part of the strain disappears. When the loading is immediately resumed and increased at the former rate, a new yield point is obtained at or near the stress previously reached before the load was removed. Again, if the reloading is delayed for an hour or so, a new yield point is obtained at an appreciably higher stress than before, fracture occurs under a higher load and the total extension is reduced. A process of hardening was going on during the hour interval—a process referred to as "hardening due to permanent set". A more marked hardening, due to overstrain, occurs, and the yield point is still further raised, if the loading is stopped, then maintained constant for some hours, and then resumed. The hardening effect in each case is removed by annealing. Elasticity is lost by the overstraining, but rest and heating both tend to restore it.

When the load W is suddenly applied to a bar, the load is constant over a very brief interval, and the instantaneous stress must be twice that obtained when the loading is increasing at a uniform rate and the average load is $\frac{1}{2}W$. With sudden loading, a state of vibration is set up, the instantaneous stress producing an instantaneous strain x/l, twice that which remains when the vibrations are gradually damped down and the load becomes stationary. The work done is therefore W times x. Thus, finally, the stress and strain become the same as with gradual loading.

If the load W falls unimpeded through a distance h and strikes a collar fitted to a bar and is brought to rest after producing an instantaneous extension x, the work done on the bar is $W(h+x)$. This work done is absorbed by the bar, and the instantaneous stress f produced can be found by equating the work done to the expression for resilience and solving the resulting quadratic equation.

Thus,
$$W(h+x)=\frac{f^2 \times \text{volume}}{2E}, \quad \text{.......................(1)}$$

or, since
$$x=\frac{fl}{E}, \quad \text{..................................(2)}$$

then
$$W\left(h+\frac{fl}{E}\right)=\frac{f^2 \times \text{volume}}{2E}. \quad \text{.......................(3)}$$

When f is determined, expression (2) can be used to find the instantaneous extension x.

Example 1. *Find the force necessary to punch a circular hole $2\frac{1}{2}$ in. in diameter through plate $\frac{3}{4}$ in. in thickness, if the resistance to shear is 21 tons per sq. in.*

$$\text{Area resisting shear} = \text{circumference} \times \text{thickness of plate}$$
$$= \pi \times 2{\cdot}5 \times 0{\cdot}75 \text{ sq. in.}$$
$$\text{Required force} = \pi \times 2{\cdot}5 \times 0{\cdot}75 \times 21 \text{ tons}$$
$$= \mathbf{123{\cdot}7\ tons.}$$

Example 2. *A pin is used in a fork and eye coupling to effect the joint. Calculate the necessary pin diameter to resist a direct pull of* 36 *tons when the safe shear stress is* 8 *tons per sq. in. Take a double shear-factor of* 1·75.

$$\text{Total area resisting shear} = \frac{\text{pull}}{\text{safe stress}} = \frac{36}{8} \text{ sq. in.}$$

$$\text{Area of cross-section of pin} = \frac{36}{8 \times 1{\cdot}75} \text{ sq. in.}$$

$$\text{Diameter of pin} = \sqrt{\frac{36 \times 4}{\pi \times 8 \times 1{\cdot}75}} = \mathbf{1{\cdot}81, \text{ or say } 1\tfrac{7}{8} \text{ in.}}$$

Example 3. *A steel slide* 4 *in. wide and* 2 *in. in depth is to be faced by fitting a gun-metal strip* ½ *inch in thickness on its outer surface. The bimetal slide is then used as one of the vertical slides for a tupp which exerts a force of* 15,000 *lb. on a stop attached to the slide. Assuming equal strains, calculate the portion of the load carried by the gun-metal facing, and the minimum diameter of four gun-metal set screws used to secure the strip to the slide. E for steel* = 13,500 *tons per sq. in. E for gun-metal* = 8,500 *tons per sq. in. Safe stress in screws* = 3 *tons per sq. in.*

Area of slide section = 8 sq. in.

Area of strip section $= 4 \times \frac{1}{2} = 2$ sq. in.

For equal strains, $\dfrac{f_s}{E_s} = \dfrac{f_g}{E_g}$,

$$\frac{f_s}{13{,}500} = \frac{f_g}{8{,}500},$$

$$f_s = \tfrac{135}{85} f_g$$

$$8 \times \tfrac{135}{85} f_g + 2 \times f_g = 15{,}000,$$

$$14{\cdot}77 f_g = 15{,}000,$$

$$f_g = 1{,}016 \text{ lb. per sq. in.},$$

$$f_s = \tfrac{135}{85} f_g = 1{,}614 \text{ lb. per sq. in.}$$

Load on slide $= 8f_s = 12{,}912$ lb. approx.

Load on strip = $2f_g$ = 2,032 lb. approx.

Effective force tending to cause the strip to separate by sliding from the slide = 12,912 − 2,032 = 10,880 lb.

Let d in. be the core diameter of each screw.

Then, area resisting shear $= \frac{4\pi d^2}{4}$.

Thus, $\pi d^2 \times 3 \times 2{,}240 = 10{,}880,$

$$d = 0{\cdot}718 \text{ in.}$$

If Whitworth screws are used, the core diameter is :

$$\text{Crest diameter} - \frac{1{\cdot}28}{\text{No. of threads per in.}}.$$

Adopting $\frac{7}{8}$ in. diameter screws :

$$d = \text{core diameter} = 0{\cdot}875 - \frac{1{\cdot}28}{9} = 0{\cdot}732 \text{ in.}$$

Ans. $\frac{7}{8}$ in. diameter Whitworth screws.

Example 4. *Calculate the angular strain, and the linear strain on the top surface, of a steel block* 2 *in. wide and* 1 *in. in depth subjected to a shearing force of* 27 *tons. Modulus of rigidity* = 12,000,000 *lb. per sq. in.*

$$\text{Intensity of shear stress} = \frac{27 \times 2{,}240}{2 \times 1} = 30{,}240 \text{ lb. per sq. in.}$$

$$C = \frac{\text{stress}}{\text{angular strain}} = 12{,}000{,}000 \text{ lb. per sq. in.}$$

$$\theta = \text{angular strain} = \frac{30{,}240}{12 \times 10^6} = \mathbf{0{\cdot}00252 \text{ radians.}}$$

Linear displacement on the top surface $= x$,

$$x = \text{depth} \times \sin\theta,$$

and since $\sin\theta$ almost equals θ,

$$x = 1 \times 0{\cdot}00252 = \mathbf{0{\cdot}00252 \text{ in.}}$$

Example 5. *The linear displacement, relative to the base which is fixed, on the top surface of a block of material* 7 *in. wide and* 10 *in. in depth is found to be* 0·025 *in. If this displacement is the result of a shear stress of* 14 *tons per sq. in., calculate* (*a*) *the angular strain,* (*b*) *the modulus of rigidity of the material.*

$$\text{Angular strain} = \sin^{-1}\frac{0{\cdot}025}{10} = \mathbf{0{\cdot}0025 \text{ rad.}}$$

$$\text{Modulus of rigidity, } C = \frac{\text{stress}}{\text{angular strain}} = \frac{14}{0{\cdot}0025}$$

$$= \mathbf{5{,}600 \text{ tons per sq. in.}}$$

Example 6. *During a tension test a steel specimen* $\frac{1}{4}$ *sq. in. in cross-section,* 2 *in. gauge length, stretches* 0·002 *in. under a direct pull of* 7,500 *lb. Calculate the work done in straining the specimen. If the elastic limit for this material is reached under a load of* 11,000 *lb., find* (*a*) *the extension at the elastic limit,* (*b*) *the stress at the elastic limit,* (*c*) *the modulus of elasticity,* (*d*) *the proof resilience.*

$$\text{Work done in straining} = \tfrac{1}{2}\ \text{load} \times \text{extension}$$
$$= \tfrac{1}{2} \times 7{,}500 \times 0{\cdot}002$$
$$= \mathbf{7{\cdot}5\ in.\ lb.}$$

(*a*) Extension at elastic limit $= \dfrac{11{,}000}{7{,}500} \times 0{\cdot}002 = \mathbf{0{\cdot}0029\ in.}$

(*b*) Stress at elastic limit $= 11{,}000 \times 4 = \mathbf{44{,}000\ lb.\ per\ sq.\ in.}$

(*c*) Modulus of elasticity $= \dfrac{\text{stress}}{\text{strain}} = \dfrac{7{,}500 \times 4}{\frac{1}{2} \times 0{\cdot}002}$

$$= \mathbf{30{,}000{,}000\ lb.\ per\ sq.\ in.}$$

(*d*) Proof resilience $= \dfrac{f^2 V}{2E} = \dfrac{44{,}000 \times 44{,}000 \times 1}{2 \times 2 \times 30{,}000{,}000}$

$$= \mathbf{16{\cdot}13\ in.\ lb.}$$

Example 7. *Calculate the maximum elastic strain energy which may be absorbed by, and the stress set up in, a brass buffer* 5 *in. in diameter and* 10 *in. in length, if the modulus of elasticity is* 6,000 *tons per sq. in. and the compression at the elastic limit is* 0·009 *in.*

$$\text{Strain at the elastic limit} = \frac{0{\cdot}009}{10} = 0{\cdot}0009.$$

$$\text{Stress} = E \times \text{strain}$$
$$= 6{,}000 \times 0{\cdot}0009 = \mathbf{5{\cdot}4\ tons\ per\ sq.\ in.}$$

Maximum energy at the elastic limit, or proof resilience

$$= \frac{f^2 V}{2E} = \frac{(5{\cdot}4)^2 \times \pi \times 5^2 \times 10}{4 \times 2 \times 6{,}000} = \mathbf{0{\cdot}48\ in.\ tons.}$$

Example 8. *A rod* 2 *in. diameter and* 16 *in. in length is turned down to* 1 *in. diameter for* 7 *in. of its length, and carries an axial load of* 26 *tons. Calculate the elastic strain energy stored in the rod if* $E = 13{,}000$ *tons per sq. in.*

Volume of full portion $= \pi \times 1^2 \times 9 = 28{\cdot}26$ cu. in. $= V_1$.

Volume of reduced portion $= \pi \times (\tfrac{1}{2})^2 \times 7 = 5{\cdot}5$ cu. in. $= V_2$.

f_1 = stress in full portion $= \dfrac{26}{28{\cdot}26} = 0{\cdot}92$ tons per sq. in.

f_2 = stress in reduced portion $= \dfrac{26}{5{\cdot}5} = 4{\cdot}73$ tons per sq. in.

Total elastic strain energy $= \dfrac{f_1^2 V_1}{2E} + \dfrac{f_2^2 V_2}{2E}$

$$= \frac{0{\cdot}92^2 \times 28{\cdot}26}{26{,}000} + \frac{4{\cdot}73^2 \times 5{\cdot}5}{26{,}000}$$

$$= \frac{23{\cdot}92 + 123{\cdot}1}{26{,}000}$$

= **0·00567 inch tons.**

Fluctuating loads. The cases where stresses fluctuate repeatedly between different limits become a question of great interest from the practical and scientific points of view. As is to be expected, a part of a machine or structure or a test piece will fracture under a much lower maximum stress when subjected to a varying load than when subjected to a static load. Much research work has been done on this subject, the first being undertaken by A. Wöhler. His experiments included the alternate loading and unloading of specimens in tension, bending and torsion. The general results of his experiments and those of others showed that the maximum stress at which fracture will occur depends to a large extent on the static ultimate strength as well as the range or fluctuation of stress. A stress well below the ultimate strength, as measured in the ordinary tensile test, will suffice to produce fracture if it be removed and restored often enough ; the smaller the fluctuation the greater the number of repetitions required. The fracture is said to be due to **fatigue** of the metal. This loss of strength due to fatigue should not be confused with deterioration of strength due to shock or impact. Strength against shock is to some extent measured in the impact testing machine (see p. 140).

The number of definite stress cycles (N) applied at a definite frequency or regularity which a material can sustain without fracture is termed the **endurance of the material.** Endurance tests on a basis of not less than 10^7 reversals of stress are desirable. Frequency may be defined as the number of complete reversals or fluctuations per second.

Graphs of S v. N are plotted, where S is the greatest stress to which the material is subjected during the cycle. A number of empirical formulae have been devised to connect (*a*) the maximum stress at fracture after a large number of repetitions of stress, (*b*) the ultimate strength of the material under static conditions, and (*c*) the range of stress. Where it is warranted, this new maximum stress must be adopted for design purposes, and a suitable factor of safety adopted to obtain the safe working stress. The **factor of safety** may be defined as $\frac{\text{stress at fracture}}{\text{safe working stress}}\cdot$ Perhaps the best known of these formulae is **Gerber's formula,** namely,

$$f_m = \frac{\Delta}{2} + \sqrt{f^2 - n\Delta f},$$

where f_m is the probable new maximum stress at fracture after many repetitions of stress, Δ is the fluctuation or range of stress in terms of f_m, f is the ultimate static stress, and n is a factor depending on the material varying from 2·5 down to 1·4. $n = 1{\cdot}5$ for ductile iron and steel and up to 2·3 for harder and more brittle varieties.

The **Launhardt-Weyrauch formula** is also used and gives a new safe working stress under fatigue conditions.

New working stress =

$$\tfrac{2}{3}\text{ safe static or dead load stress }\left(1 + \tfrac{1}{2}\,\frac{\text{minimum load}}{\text{maximum load}}\right)\cdot$$

A formula which aims at giving the equivalent dead load when combined dead and live loads have to be considered is :

Equivalent dead load = maximum load + variation in load.

Example 1. *Use Gerber's formula in the following cases to obtain the safe working stress for a mild steel member if a factor of safety of* 6 *is employed. The ultimate static strength is* 28 *tons per sq. in. Take* $n = 1{\cdot}5$.

(*a*) *A stress varying between* 4 *tons per sq. in. tensile and zero.*

(*b*) *A stress varying between* 4 *tons per sq. in. tensile and* 4 *tons per sq. in. compressive.*

(*c*) *A steady dead load stress of* 4 *tons per sq. in. tensile and a varying stress ranging between* 5 *tons per sq. in. compressive and* 2 *tons per sq. in. tensile.*

(*a*) Here $\Delta = \left(\dfrac{\text{maximum} - \text{minimum stress}}{\text{maximum stress}}\right) f_m$

$$= \left(\frac{4-0}{4}\right) f_m = f_m.$$

$$\therefore\ f_m = \frac{\Delta}{2} + \sqrt{f^2 - nf\Delta} = \tfrac{1}{2} f_m + \sqrt{28^2 - 1{\cdot}5\,.\,28\,.\,f_m}.$$

$\tfrac{1}{4} f_m{}^2 = 28^2 - 1{\cdot}5\,.\,28\,.\,f_m$, whence $f_m = 16{\cdot}95$ tons per sq. in.

With a factor of safety of 6, safe working stress $= \dfrac{16{\cdot}95}{6}$

$=2{\cdot}82$ **tons per sq. in.**

(*b*) Here $\Delta = \dfrac{4 - -4}{4} f_m = 2 f_m$;

$\therefore\ f_m = f_m + \sqrt{28^2 - 1{\cdot}5\,.\,f_m\,.\,28}$, whence

$f_m = 18\tfrac{2}{3}$ tons per sq. in.

Safe working stress $= \dfrac{18\frac{2}{3}}{6}$, or **3·11 tons per sq. in.**

(*c*) Here $\Delta = \dfrac{6 - -1}{6} f_m = \dfrac{7}{6} f_m.$

$\therefore\ f_m = \tfrac{7}{12} f_m + \sqrt{28^2 - 1{\cdot}5 f_m \times \tfrac{7}{6} \times 28}$, whence

$f_m = 15{\cdot}2$ tons per sq. in.

Safe working stress $= \dfrac{15{\cdot}2}{6}$, or **2·5 tons per sq. in.**

Example 2. *A member of a structure carries a tensile dead load of 9 tons and a live load which fluctuates between 6 tons tensile and 5 tons compressive. The safe static stress is to be taken as 7 tons per sq. in. Calculate (a) the working stress, (b) the equivalent dead load.*

Using the Launhardt-Weyrauch formula,

$$\text{Working stress} = \tfrac{2}{3}\ \text{safe static stress}\left(1 + \tfrac{1}{2}\,\frac{\text{minimum load}}{\text{maximum load}}\right)$$

$= \tfrac{2}{3}$ of $7\left(1 + \tfrac{1}{2} \times \tfrac{4}{15}\right) = 5{\cdot}3$ **tons per sq. in.**

Equivalent dead load = maximum load + variation

$= (9 + 6) \qquad + (15 - 4)$

$= \quad 15 \qquad + \quad 11 \quad = 26$ **tons.**

Riveted joints. Riveting was, until recent years, the recognised method of joining flat plates, but modern practice in shipbuilding and structural engineering is to replace, at any rate, some of these riveted joints by welded joints.

In spite of the introduction of welding, riveted joints still remain an important consideration in structural and engineering design, and their possible failure, strength and construction are applications of that portion of mechanics known as strength of materials.

Preparation of materials. Plates for riveting are generally rolled, cleaned of scale and oxide, and the surfaces to be lapped or butted are fitted and burred. The rivet holes may be either punched or drilled, but in the best work the latter method is invariably used. The rivets are usually drop forgings from metal of the correct composition ; heating is carried to a temperature introducing ductility in the rivet, but not sufficiently high to cause burning or excessive oxidation.

The process of closing the rivet is generally done " hot ", with the result that the cooling of the rivet places it under an initial temperature stress.

Types of joint. Two main types of joint are in general use (Fig. 60), (1) **lap joints** and (2) **butt joints.** The butt joints may be fitted with

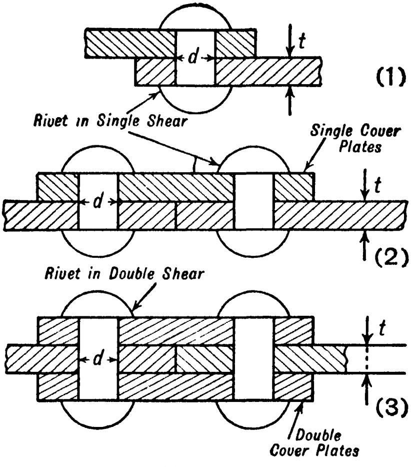


FIG. 60.

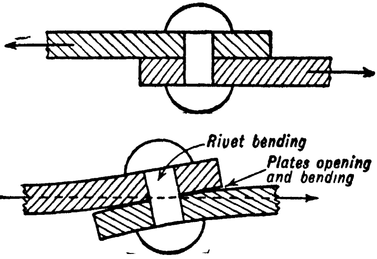


FIG. 61.

either single or double cover plates, and in both lap and butt joints several lines of rivets may be used to give the requisite strength. The principal disadvantage of lap joints is that the forces tending to fracture the joint are not in line, and thus the rivets are subjected to bending in addition to shear (Fig. 61).

Terms and proportions of joints. The following terms are used in the design of riveted joints, and the recognised proportions are shown in Fig. 62.

The *pitch* (p) is the distance between the centres of two consecutive rivets in a line of rivets. The thickness (t) of the joined plates is generally related to the diameter (d) of the rivet by the expression $\mathbf{d} = \mathbf{1{\cdot}2}\sqrt{\mathbf{t}}$; but this formula is not altogether satisfactory as it tends to give too large a rivet for the thicker plates. Fig. 63 illustrates this point, where the full-line stepped graph gives the rivet diameters for plate thicknesses from $\frac{1}{4}$ in. to 1 in. as recommended by the National Boiler and General Insurance Co. Ltd. The dotted stepped graph gives sizes of rivets much used in the shipyards, while the

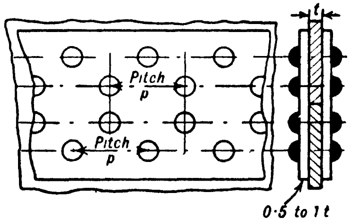


FIG. 62.

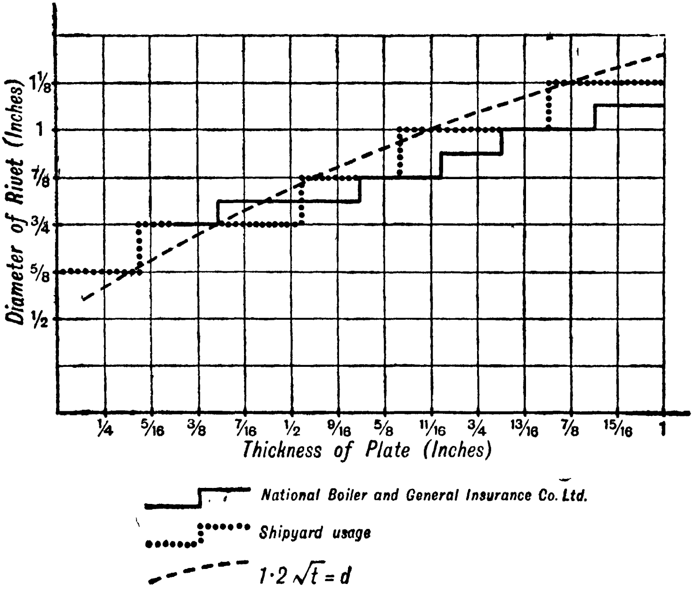


FIG. 63.

curved chain line gives rivet diameters as calculated from the formula $d = 1{\cdot}2\sqrt{t}$.

Large rivets are difficult and costly to snap down and to close satisfactorily. Although calculations for design are usually based on rivet diameters, the more logical procedure would be to work on the diameter of the drilled hole which the rivet has to fill when properly closed. This hole is usually drilled $\frac{1}{16}$ in. greater than the rivet diameter to give the necessary clearance for the insertion of the red-hot rivet. The thickness of cover-plates in butt joints varies between $t/2$ and t.

Design of riveted joints. A riveted joint may fail by one or more of the following: (*a*) shearing of rivets, (*b*) tearing of plates, (*c*) crushing of the rivets or deformation of the holes, or (*d*) fracture at the margin of the plates. Some methods of failure are not likely to be exhibited if the proportions existing between plate thickness and rivet diameter shown in Fig. 63 are followed.

The method of design is to determine first the distribution of the rivets and the type of joint to be used. Secondly, this joint is carefully considered and all possible methods of failure defined. Other factors affect the design; for example, in water-tight work the pitch must not exceed 4 to $4\frac{1}{2}$ diameters, and for oil-tight work 3 to $3\frac{1}{2}$ diameters. Bearing in mind such special considerations as these, it is then general to design the joint to have equal strength in all methods of failure, using the known safe stresses of the materials employed.

Example. Single riveted lap joint. *Design in $\frac{1}{2}$-inch plate the single riveted lap joint shown in Fig. 64. The safe stresses to be employed are: tensile strength of the plate f_t, 5 tons per sq. in., shearing strength of rivet material f_s, 4 tons per sq. in.*

As has been explained earlier, this type of joint introduces rivet bending, and the safe stresses must be taken to eliminate danger of failure from this cause. The joint may fail by shearing the rivets or tearing the plate.

Diameter of rivet $d = 1{\cdot}2\sqrt{t} = 1{\cdot}2\sqrt{0{\cdot}5} = 0{\cdot}8484$ in., say $\frac{13}{16}$ **in. rivets.**

Consider one pitch length p in which:

$$\text{Strength of one rivet in shear} = \frac{\pi d^2 f_s}{4}.$$

$$\text{Strength of cut plate in tension} = (p - d)t \, . \, f_t.$$

$$\text{For equal strengths } \frac{\pi d^2 f_s}{4} = (p - d)t \, . \, f_t,$$

$$\frac{\pi \times (\frac{13}{16})^2 \times 4}{4} = (p - \tfrac{13}{16})\tfrac{1}{2} \times 5,$$

$$p = 1{\cdot}65 \text{ in., say, pitch } 1\tfrac{3}{4} \text{ in.}$$

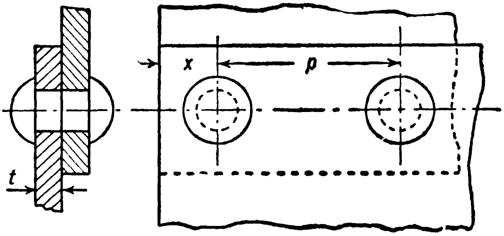


FIG. 64.

Efficiency of a joint. The efficiency of a riveted joint is the ratio between the strength of the joint under its weakest condition and that of the uncut plate. The efficiency is generally computed on a single pitch length of the joint, and the fact that a riveted joint is not as strong as the uncut plate involves, and is compensated by, using plate of greater thickness than would be necessary if no joint were needed.

The efficiency of the lap joint in the example on p. 106

$$=\frac{\text{strength on a single pitch length}}{\text{strength of uncut plate}}$$

$$=\frac{(p-d)t\,.\,f_t}{p\,.\,t\,.\,f_t}=\frac{p-d}{p}=\frac{1{\cdot}75-1}{1{\cdot}75}=43 \text{ per cent.}$$

Resistance to crushing of the rivet or plate. It may generally be assumed that, if the rivet diameter is $1{\cdot}2\sqrt{t}$, there is little likelihood of failure due to rivet crushing; but this cannot be taken for granted in the case of the plate, and designs must take into consideration bearing or crushing stresses, f_b, in the perforated plate. This is particularly necessary when the plates are of poor quality ferrous metal or non-ferrous or ductile material. In the case of steel, the resistance to crushing is approximately $1\frac{3}{4}$ times that offered to rivet shear, that is f_b is equal to $1\frac{3}{4}f_s$.

In *single shear*, the strength of one rivet is $\frac{\pi d^2 f_s}{4}$, and the resistance of the plate to crushing is $d\,.\,t\,.\,f_b$, that is, the stress times the projected area of the rivet hole, $d \times t$.

Hence for equality : $$\frac{\pi d^2 f_s}{4}=d\,.\,t\,.\,f_b,$$

and since $$f_b=1\tfrac{3}{4}f_s,$$

$$d=2{\cdot}23t.$$

It follows that, *in single shear, if d is less than* $2{\cdot}23t$ *the crushing consideration may be neglected : but if the joint exhibits double shear, crushing must be considered when the double-shear factor is* 2 *and d is greater than* $1{\cdot}11t$. *Similarly, if the double-shear factor is* $1{\cdot}75$, *d must not be greater than* $1{\cdot}27t$ *in order that crushing may be neglected.*

In the foregoing example the rivets are in single shear and d is less than $2 \cdot 23t$, so that crushing as a possible form of failure is neglected in the design.

Butt joints. The great disadvantage of lap joints lies in the fact that the forces acting on the joined plates are not in alignment. This leads to a tendency towards plate and rivet bend with joint opening as shown in Fig. 61. Particularly for joints resisting fluid pressure, butt joints are therefore preferable to lap joints. Single or double cover-plates may be used; the former place the rivet in single shear and the latter, in addition to making a more effective joint, give the advantage of rivets in double shear.

Single riveted butt joint. *Design a single riveted butt joint to join two* $\frac{3}{8}$ *in. steel plates (Fig.* 65) *; allowable stresses* $f_t = 5$, $f_s = 4$, $f_b = 8$ *tons per sq. in. Use a double-shear factor of* 1·75.

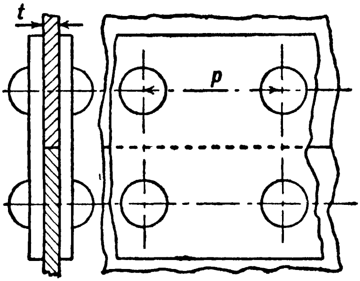


FIG. 65.

Diameter of rivet $= 1 \cdot 2\sqrt{t} = 1 \cdot 2\sqrt{0 \cdot 375} = 0 \cdot 73$ in.: adopt $\frac{3}{4}$ **in. diam. rivet.**

Consider a pitch length p.

(1) Strength of one rivet in double shear $= \dfrac{1 \cdot 75\pi d^2 f_s}{4}$.

(2) Strength of cut plate $= (p - d)t \, . \, f_t$.

(3) Strength against crushing $= d \, . \, t \, . \, f_b$.

For equal strengths in (1) and (2),

$$\frac{1 \cdot 75\pi d^2 f_s}{4} = (p - d)tf_t,$$

$$\frac{1 \cdot 75\pi \times (\frac{3}{4})^2 \times 4}{4} = (p - \tfrac{3}{4})\tfrac{3}{8} \times 5,$$

$$p = 2 \cdot 4 \text{ in.}$$

For equal strengths in (2) and (3),

$$(p-d)t \,.\, f_t = d \,.\, t \,.\, f_b,$$

$$(p-\tfrac{3}{4})\tfrac{3}{8} \times 5 = \tfrac{3}{4} \times \tfrac{3}{8} \times 8,$$

$$p = 1{\cdot}95 \text{ in.}$$

Adopt the lower of the two calculated pitches, that is, $p = 1{\cdot}95$ in.; say **2 in. pitch.**

Efficiency of joint against the crushing or greatest weakness

$$= \frac{\text{resistance to crushing}}{\text{strength of uncut plate}} = \frac{d \,.\, t \,.\, f_b}{p \,.\, t \,.\, f_t}$$

$$= \frac{d \,.\, f_b}{p \,.\, f_t} = \frac{\frac{3}{4} \times 8}{2 \times 5} = \textbf{60 per cent.}$$

Example. *Find the maximum pull which may be sustained by the spliced tie bar shown in Fig.* 66 *if the rivets are* $\frac{7}{8}$ *in. in diameter and the permissible working stresses are* $f_t = 5$, $f_s = 4$ *and* $f_b = 8$ *tons per sq. in. Use a double-shear factor of* 2.

There are two possible failures of the splice: (1) across AA, (2) across BB.

To fail across AA, the plate, less two rivet holes, together with one rivet in double shear must fail, while across BB the plate less one rivet hole must fail.

(1) Working strength across AA $= (4 - 2d)t \,.\, f_t + \dfrac{2\pi d^2 f_s}{4}$

$$= (4 - 1\tfrac{3}{4})\tfrac{1}{2} \times 5 + \frac{2\pi(\frac{7}{8})^2 \times 4}{4}$$

$$= 5\tfrac{5}{8} + \frac{49\pi}{32} = \textbf{10·44 tons.}$$

(2) Working strength across BB

$$= (4 - \tfrac{7}{8})\tfrac{1}{2} \times 5 = \textbf{7·81 tons.}$$

Working strength of three rivets in

$$\text{double shear} = \frac{3 \times 2\pi d^2 f_s}{4}$$

$$= \textbf{14·4 tons.}$$

Resistance to bearing on three rivets

$$= 3d \,.\, t \,.\, f_b$$

$$= 3 \times \tfrac{7}{8} \times \tfrac{1}{2} \times 8 = \textbf{10·5 tons.}$$

∴ **Maximum available pull = 7·81 tons.**

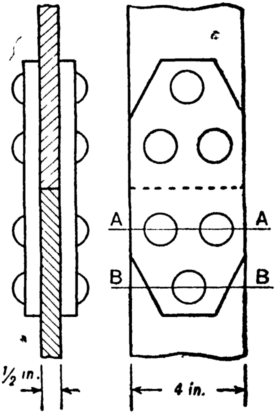


FIG. 66.

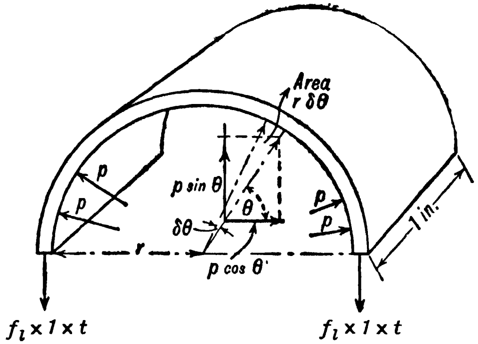


FIG. 67.

Thin cylinders. This treatment of thin cylinders is confined to shells in which the thickness of metal is small when compared with the diameter of the cylinder. When this is the case, the stress over the metal comprising the shell may be considered uniform. Suppose the cylinder (Fig. 67) be sealed across its diameter by a flat plate and the semi-cylinder so formed be subjected to an internal pressure of p lb. per sq. in. Then if the diameter of the cylinder is d in., the stress set up in a longitudinal seam is f_l lb. per sq. in. and the thickness of the material is t in., the total load on one-inch length of the sealing plate is pd lb. This may be proved mathematically as follows :

Pressure on a small elemental area of unit depth $= pr\,\delta\theta$.

$$\text{Sum of the vertical components} = \int_0^{\pi} pr \sin\theta\,\delta\theta$$

$$= \Big[-pr\cos\theta\Big]_0^{\pi}$$

$$= 2pr = pd.$$

$$\text{Sum of the horizontal components} = \int_0^{\pi} pr\cos\theta\,\delta\theta$$

$$= \Big[pr\sin\theta\Big]_0^{\pi} = 0.$$

pd is equal to the resistance of two longitudinal seams, that is, $pd = 2f_l t$.

It follows, since $pd=2f_l t$, that the stress f_l in a longitudinal seam or hoop stress is $\mathbf{f_l=\dfrac{pd}{2t}}$.

Now consider the cylinder closed at its ends. The force on the ends in the direction of the length of the boiler is $\dfrac{\pi d^2 p}{4}$, and the resistance of a circumferential seam is $\pi d f_h t$, where f_h is the longitudinal stress.

Hence
$$\frac{\pi d^2 p}{4}=\pi d f_h t.$$

$$\text{Longitudinal stress}=\mathbf{f_h=\frac{pd}{4t}}.$$

Example 1. *Find the thickness of steel plate required for the shell of a Lancashire boiler* 8 *ft. in diameter if the stress is not to exceed* 4 *tons per sq. in. under a pressure of* 160 *lb. per sq. in.*

The hoop stress is twice the longitudinal stress; hence the boiler must be designed so that the stress in a longitudinal seam does not exceed 4 tons per sq. in.

$$f_l=\frac{pd}{2t}=\frac{160\times 8\times 12}{2t}=4\times 2{,}240 \text{ lb. per sq. in.}$$

$$t=\frac{160\times 8\times 12}{2\times 4\times 2{,}240}=0{\cdot}86 \text{ in.; say } \tfrac{7}{8} \text{ in.}$$

Example 2. *What pressure may be maintained in a fire tube cylindrical boiler* 6 *ft. in diameter, metal* $\frac{3}{4}$ *in. in thickness, if the longitudinal stress is not to exceed* 6 *tons per sq. in.?*

$$f_h=\frac{pd}{4t}=\frac{p\times 6\times 12}{4\times \frac{3}{4}}=6\times 2{,}240 \text{ lb. per sq. in.}$$

$$24p=6\times 2{,}240,$$

$$\mathbf{p=560 \text{ lb. per sq. in.}}$$

Example 3. *Design a triple-riveted butt joint* (Fig. 68) *for the longitudinal seam of a boiler, internal pressure* 200 *lb. per sq. in. and diameter* 6 *ft. The allowable stresses are* $f_t=6$, $f_s=4$, $f_b=8$ *tons per sq. in. What is the efficiency of the joint?*

$$f_l=\frac{pd}{2t}=\frac{200\times 6\times 12}{2t}=6\times 2{,}240 \text{ lb. per sq. in.}$$

$$\mathbf{t=0{\cdot}54 \text{ in.}}$$

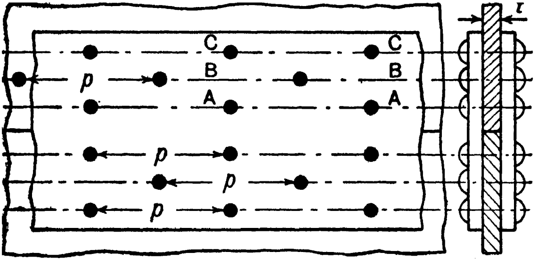


FIG. 68.

Assume an approximate joint efficiency of 80 per cent. and adopt $\frac{5}{8}$ in. thickness of plate.

Diameter of rivet $= 1{\cdot}2\sqrt{t} = 1{\cdot}2\sqrt{0{\cdot}625} = 0{\cdot}95$: adopt a rivet diameter of 1 in.

Consider a pitch length p in., double cover-plates and a double-shear factor of 1·75.

Failure at AA: a length of plate $(p-d)$ and two rivets in double shear must fail.

Failure at BB: a length of plate $(p-d)$ and one rivet in double shear must fail.

Failure at CC: a length of plate $(p-d)$ must fail, or finally *three rivets* must fail in double shear or bearing.

It follows that the strength of three rivets or failure at CC must determine the dimensions of the joint.

(1) Strength of three rivets in double shear $= \dfrac{\pi \times 1^2 \times 4}{4}(3 \times 1{\cdot}75)$

$$= 16{\cdot}5 \text{ tons.}$$

Number of rivets = three per pitch length.

(2) Strength of plate at CC against tension

$$= (p-d)t\,.\,f_t = (p-1)\tfrac{5}{8} \times 6 \text{ tons.}$$

(3) Strength of three rivets against bearing $= 3d\,.\,t\,.\,f_b$

$$= 3 \times 1 \times \tfrac{5}{8} \times 8 = 15 \text{ tons.}$$

∴ bearing is the determining factor, since the strength in (3) is less than that in (1).

Equating (2) and (3),

$$(p-1)\tfrac{5}{8} \times 6 = 15,$$

$$p = 5 \text{ in.}$$

Dimensions of joint. Plate thickness = $\frac{5}{8}$ **in.**

Rivet diameter = **1 in.**

Pitch = **5 in**

Minimum efficiency of the joint.

$$\text{Efficiency} = \frac{\text{strength of plate at CC}}{\text{strength of uncut plate}}$$

$$= \frac{(p-d)t\,.\,f_t}{p\,.\,t\,.\,f_t} = \frac{p-d}{p} = \frac{4}{5} = \mathbf{80\ per\ cent.}$$

Example 4. *A high-pressure container* 4 *ft. in diameter and* 6 *ft. in length is to be designed to resist an internal pressure of* 80 *atmospheres. Design quadruple-riveted butt joints for the longitudinal seams and triple-riveted butt joints for the circumferential seam. The plate used is an alloy steel offering a resistance to tearing of* 20 *tons per sq. in., and rivets are employed having a shear resistance of* 7 *tons per sq. in. and bearing resistance of* 12 *tons per sq. in. Use a double-shear factor of* 1·75 *and follow the recommendations of the National Boiler and General Insurance Co. Ltd. for rivet diameters.*

$$\text{Internal pressure} = 80 \times 14{\cdot}7 = 1{,}176 \text{ lb. per sq. in.}$$

Let the plate thickness be t in.; then for the longitudinal seam

$$f_l = \frac{pd}{2t} = \frac{1{,}176 \times 4 \times 12}{2t},$$

$$\frac{1{,}176 \times 4 \times 12}{2t} = 20 \times 2{,}240,$$

$$t = 0{\cdot}63 \text{ in.}$$

Allow an approximate joint efficiency of 80 per cent. and adopt

$$t = \tfrac{10}{8} \times 0{\cdot}63, \text{ say } \tfrac{13}{16} \text{ in.}$$

Diameter of rivet from Fig. 63 is **1 inch.**

Employ a quadruple-riveted butt joint (Fig. 69) for the longitudinal seam and a triple-riveted butt joint (Fig. 68) for the circumferential seam.

Longitudinal seam. Consider a pitch length p; then failure may occur as follows:

Along AA: by tearing plate $(p-d)$ and double-shearing 3 rivets;

BB: by tearing plate $(p-d)$ and double-shearing 2 rivets;

CC: by tearing plate $(p-d)$ and double-shearing 1 rivet;

DD: by tearing plate $(p-d)$;

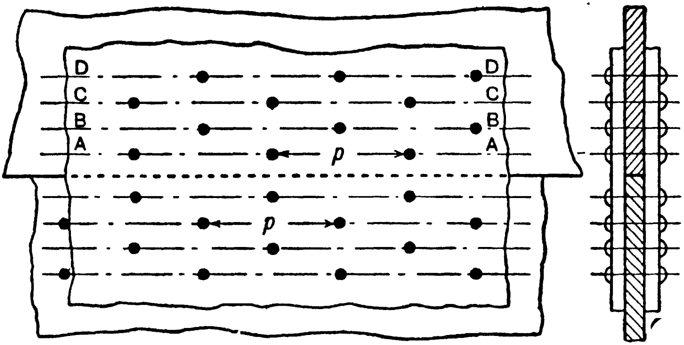


FIG. 69.

or by the failure of 4 rivets in double shear, or alternatively by their failure in bearing.

(1) Strength of plate at DD $= (p - d)tf_t = (p - 1)\frac{13}{16} \times 20.$

(2) Strength of 4 rivets in double shear $= \dfrac{4 \times 1{\cdot}75\pi d^2}{4} f_s$

$$= \frac{4 \times 1{\cdot}75\pi \times 1^2}{4} \times \frac{7}{1} = 38{\cdot}4 \text{ tons}$$

(3) Strength of 4 rivets in bearing $= 4d \, . \, t \, . \, f_b$

$$= 4 \times 1 \times \tfrac{13}{16} \times 12 = 39 \text{ tons.}$$

Equating (1) and (2),

$$(p - 1)\tfrac{13}{16} \times 20 = 38{\cdot}4,$$

$$p = 3{\cdot}4 \text{ in.}; \text{ say } 3\tfrac{1}{2} \text{ in.}$$

Dimensions of longitudinal seam : **pitch $= 3\frac{1}{2}$ in.**
rivet diameter $= 1$ in.
plate thickness $= \frac{13}{16}$ in.

Circumferential seam.

(1) Strength of plate at CC (Fig. 69) $= (p - d)tf_t$.

(2) Strength of 3 rivets in double shear $= \dfrac{1{\cdot}75 \times 3 \times \pi d^2}{4} \times f_s$

$$= \frac{1{\cdot}75 \times 3 \times \pi \times 1^2}{4} \times \frac{7}{1} = 28{\cdot}8 \text{ tons.}$$

(3) Strength of 3 rivets in bearing $= 3 \times d \times t \times f_b$

$$= 3 \times 1 \times \tfrac{13}{16} \times 12 = 29{\cdot}25 \text{ tons.}$$

Hence

$$(p-d)\,t f_t = 28{\cdot}8,$$

$$(p-1{\cdot})\tfrac{13}{16}\times 20 = 28{\cdot}8,$$

$$p = 2{\cdot}75 \text{ in.}$$

Dimensions of circumferential seam : pitch = 2·75 in.,
rivet diameter = 1 in.
plate thickness = $\tfrac{13}{16}$ in.

Alloy steels. Considerable use is now being made of alloy steels in which one of the rarer metals is alloyed in small proportions with ordinary steel to give the alloy steel so formed specific properties. The following table shows the effect on steels of alloying certain elements, and the special uses to which the resultant steels can be adapted.

Element in alloy steels	Type of steel in which the element is used
Nickel	Case-hardening and nitriding steels. High tensile, rust- and acid-resisting, heat-resisting and non-magnetic steels.
Manganese	High-tensile structural, low alloy, spring, non-magnetic, high coefficient of expansion.
Chromium	Corrosion-resisting structural, medium and high-tensile, case-hardening and nitriding, spring and ball race, magnetic and high-speed tool, rust- and acid-resisting, heat-resisting and high coefficient of expansion.
Molybdenum	Heat-resisting, high-tensile, nitriding, high-speed tool.
Tungsten	Magnet, high-speed tool, heat-resisting.
Cobalt	Cobalt magnet, high-speed tool.
Vanadium	High-speed tool, spring, medium tensile.
Copper	Structural.
Aluminium	Nitriding.
Silicon	Heat-resisting.

EXERCISES ON CHAPTER IV

Stress, Strain and Modulus of Elasticity.

1. Define mean intensity of stress. A steel bar $2\frac{1}{2}$ in. in diameter carries an axial pull of 20 tons. Calculate the mean intensity of tensile stress.

2. A wire 100 in. in length stretches 0·01 in. under a pull of 14 lb. Calculate the fractional strain and the load which would produce an extension of 0·07 in.

3. A column which is too short for buckling to occur, is 15 in. in length and compresses 0·001 in. under load. Find the fractional strain. What is the mean intensity of stress if the Modulus of Elasticity (E) is 30×10^6 lb. per sq. in.?

4. What is Hooke's Law? A steel tie bar 10 ft. long and $1\frac{1}{2}$ in. in diameter carries a pull of 18 tons. Find (*a*) the mean intensity of stress, (*b*) the total stretch. $E = 13{,}500$ tons per sq. in.

5. A resultant tensile load of 6 tons acts axially along a rod $\frac{3}{4}$ in. in diameter and 40 in. long. Assuming the load is distributed uniformly over the section, find (*a*) the increase in length, and (*b*) the mean intensity of tensile stress. $E = 12{,}000$ tons per sq. in.

6. A steel boiler-stay supports a surface of 240 sq. in., which is subjected to a gauge pressure of 180 lb. per sq. in. If the mean intensity of tensile stress in the stay is not to exceed 10,000 lb. per sq. in., calculate the necessary diameter of the stay.

7. A hollow cylindrical cast iron strut 8 ft. long receives an axial load of 1,000 tons. If the internal and external radii of the cross-section of the strut are respectively 8 in. and 12 in., calculate the mean compressive stress and the shortening of the column due to the load. $E = 6{,}500$ tons per sq. in.

8. A cast iron column 7 ft. in length, external diameter 6 in., internal diameter 4 in., is known to shorten 0·007 in. under a load. Find the magnitude of the load if $E = 7{,}000$ tons per sq. in.

9. A steel column of I section has flanges each $3 \times \frac{3}{4}$ in. and a web 4 in. $\times \frac{1}{2}$ in. and is subjected to a uniform compressive stress of 6 tons per sq. in. on a cross-section. Calculate the load and the shortening produced if the column is 7 ft. long and E is 13,500 tons per sq. in.

10. A tie rod in a roof structure 12 ft. in length is found to stretch 0·022 in. under a certain load. If E is 13,500 tons per sq. in. and the rod is 2 in. in diameter, find the load.

11. If the value of Young's Modulus for brass is 14×10^6 lb. per sq. in., calculate the load which would cause an extension of 0·02 in. on a bar 10 ft. long and $1\frac{1}{2}$ in. in diameter.

12. Compare the greatest loads which two bars each 30 ft. long can carry if the stretch is not to exceed 0·1 in. and E is 13,500 tons per sq. in. One bar has a diameter of 4 in. and the other has a square cross-section of 4 in. side.

13. A copper rod 3 ft. long is to carry a tensile load of 2,500 lb. Find the necessary diameter of cross-section if the extension is not to exceed 0·01 in. and E for copper is 15×10^6 lb. per sq. in.

14. In a roof truss a tie is to be of T section and is to carry a load of 40 tons. If the safe working stress permissible is 7 tons per sq. in., obtain the dimensions in whole inches of a suitable section if the metal is ⅝ in. in thickness. Allow for one rivet of $\frac{11}{16}$ in. in diameter in the leg of the T.

15. A balloon is stationary 10,000 ft. above the earth. What is the greatest intensity of stress existing in a steel cable which is hanging vertically from the balloon and just reaching the earth? Weight of steel is 495 lb. per cu. ft. Calculate the maximum stress in the cable. How much cable must hang from the balloon if the maximum stress is not to exceed 20,000 lb. per sq. in.?

16. A specimen of steel of gauge length 8 in. and 0·497 in. in diameter gave the following results when tested in tension :

Load in tons - -	1	1·6	2	2·4	2·8	3·2	3·6	4·0
Extension in thousandths of an inch or mils - -	3·1	5	6	7	8·4	9·8	10·9	12

Plot a graph of load *v.* extension and use it to determine the Modulus of Elasticity of the specimen.

17. With the aid of a graph determine the modulus of elasticity of brass from the following data obtained from a specimen of gauge length 4 in. and cross-sectional area of specimen 0·194 sq. in.

Load in lb. -	0	700	1200	1900	2300	3000
Extension in mils	0	1	1·8	2·4	3·4	5

18. Calculate the value of E, assuming the following data are representative of the material. Length of tie rod, 10 ft. ; cross-sectional area, 3¼ sq. in. ; axial pull, 25 tons ; stretch, 0·07 in.

19. What is Poisson's Ratio? If within the elastic limit a 1 in. diameter rod 8 in. long stretches 0·01 in. for a certain load and the diameter decreases by 0·00036 in., calculate the value of Poisson's Ratio.

20. A cylindrical bar of steel 10 in. long and ¾ in. in diameter stretches 0·00302 in. under a pull of 4,000 lb. while the diameter decreases by 0·000057. Find (*a*) Young's Modulus, (*b*) Poisson's Ratio.

21. A specimen of indiarubber 8·3 in. long and 0·25 in. diameter subjected to an axial load of $\frac{3}{4}$ lb. stretches $\frac{3}{4}$ in. while the diameter decreases by 0·0075 in. Calculate the value of Young's Modulus and Poisson's Ratio.

Composite Struts and Ties.

22. A ferro-concrete column is 14 in. square in cross-section ; the main reinforcement consists of four longitudinal 2 in. diameter mild steel rods, one rod being placed close to each angle of the cross-section. The ratio of the moduli of steel and concrete is 15. If a gross compressive load of 60 tons is supported by this column, what is the gross load and compressive stress in (*a*) the concrete, (*b*) the reinforcing bars. What is the final strain of the block if E for steel is 30,000,000 lb. per sq. in.?

23. Design a reinforced concrete column to take an axial load of 70 tons. Take the ratio of the moduli of elasticity of steel and concrete as 15, and the outside diameter of the column as 14 in. Neglect the increased stress in the concrete usually allowed owing to the presence of binding reinforcement. Stress in concrete to be 600 lb. per sq. in.

24. In order to strengthen a wooden joist 4 in. × 10 in., which is in tension, two steel plates each $\frac{1}{4}$ in. × 9 in. are bolted one on each side of the joist. If the total direct tensile load on the composite joist is 6 tons, find the stress intensity in the wood and steel if the modulus of elasticity of the wood is one-twentieth of that of the steel. (Neglect the loss of material due to the bolt holes.)

25. A weight of 300 lb. is suspended by two vertical wires, one of which is known to stretch $\frac{1}{10}$ in. per 100 lb. of pull and the other is 0·1 in. diameter and 10 ft. long and is made of steel. Find what load is carried by each of the wires. E for steel $= 30 \times 10^6$ lb. per sq. in.

26. Calculate the modulus of elasticity of a steel-cored aluminium cable by treating it as a composite tie in which the strains on both component materials are equal. The cable consists of 26 aluminium wires each of diameter 0·09 in. and 7 steel wires each of diameter 0·07 in. E for steel $= 30 \times 10^6$ lb. per sq. in. ; E for aluminium $= 10 \times 10^6$ lb. per sq. in.

27. A cylindrical bar of copper 1 in. in diameter is to be sheathed with a tube of steel fitting tightly round it so as to form a compound bar of copper and steel. Find the thickness of the steel sheathing required in order that, when the compound bar is subjected to an axial pull, the steel shall carry $\frac{3}{4}$ of the total load. E for copper $= 6{,}000$ tons per sq. in. ; E for steel $= 13{,}000$ tons per sq. in.

Stress due to change of temperature.

28. The temperature of a steel bar $1\frac{1}{2}$ in. in diameter and 12 ft. long is raised through 60 centigrade degrees while the ends are gripped as rigidly as possible. If the grips yield 0·05 in. after cooling, calculate the final stress and the total load on the bar. α for steel $= 0{\cdot}000011$ per centigrade degree. E for steel $= 30{,}000{,}000$ lb. per sq. in.

29. A steel rod 2 in. in diameter and 10 ft. long is heated through 80 centigrade degrees in temperature, but, after cooling, the distance between the supports increases by only 0·04 in. Calculate the strain and stress in the bar caused by the temperature rise and the total load on it. Take α and E as in Question 28.

30. To what temperature must a ring be raised if it is to slip over a boss 6 in. in diameter kept at 15° C., if the ring is bored 0·025 in. smaller than the boss? What will be the approximate mean stress in the ring after cooling if the boss is assumed to be rigid and unyielding? $\alpha = 0{\cdot}0000125$ per centigrade degree. $E = 12{,}000$ tons per sq. in.

31. Define modulus of elasticity. A steel hoop is shrunk on a fly-wheel boss 20 in. in diameter in order to strengthen it. If the hoop is bored 0·015 in. smaller than the boss, to what temperature must the ring be raised in order to slip over the boss? Assume the boss is unyielding, and find the mean intensity of tensile stress in the hoop after cooling. $\alpha = 0{\cdot}000012$ per centigrade degree. $E = 13{,}000$ tons per sq. in.

32. A steel pin is ground 1·0000 in. in diameter and a brass bush is ground 0·9999 in. internal diameter measured at the same temperature (60° F.). The bush is driven on the pin and the unit is then put into service where it reaches a temperature of 420° F. What is the quality of the fit between the bush and the pin at this temperature? α for brass is 0·0000105 per Fahr. degree ; α for steel is 0·000006. At what temperature do the two components have equal diameters?

33. Calculate the internal dimensions of a brass bridle required to slip over a rectangular bar 3 in. × 2 in. at 0° C., when the bridle is at 300° C. What will be the approximate tensile stress in the bridle after it has been shrunk on and cooled? Assume the bar is unyielding. α for brass $= 0{\cdot}000018$ per centigrade degree. $E = 6{,}500$ tons per sq. in.

Shear Stress, Resilience, Impact.

34. What is the meaning of ultimate shear stress? Find the dimensions of a square section key which will fracture across the square section in shear under a load of 70 tons if the ultimate shear stress is 26 tons per sq. in.

35. Calculate the force required to punch a circular hole 10 in. in diameter from brass plate $\frac{3}{16}$ in. in thickness if the ultimate shear stress for brass is 14 tons per sq. in.

36. Distinguish between single and double shear. A hydraulic punching machine capable of exerting a force of 600 tons is employed to punch a hexagonal-shaped hole of side one inch through brass plate of thickness 0·064 in. Calculate the number of plates which may be punched at one operation if the ultimate shear stress of the brass is 11 tons per sq. in.

37. Determine the diameter of a pin required to suit a knuckle joint connecting two rods each of one inch diameter. The safe stress in single shear for the pin is 4 tons per sq. in. and the safe tensile stress for the rod is 5 tons per sq. in. Take the effective area for double shear to be equivalent to 1·75 times that for single shear.

38. The maximum safe compressive stress in a steel punch is 50 tons per sq. in. The ultimate shearing stress for mild steel being 28 tons per sq. in., find the diameter of the largest hole which can be punched through mild steel plate $\frac{5}{8}$ in. in thickness by this punch.

39. Find the resilience of a bar of mild steel 1 in. in diameter and 10 ft. long carrying a tensile load of 7 tons. $E = 13{,}500$ tons per sq. in.

40. Define the terms "elastic limit" and "resilience". Find the total proof resilience and the resilience per cubic inch for a bar of steel $1\frac{1}{2}$ in. in diameter and 12 ft. long if the elastic limit is reached with a stress of 18 tons per sq. in. $E = 13{,}500$ tons per sq. in.

41. Calculate the extension produced and the work expended in extension when a bar of steel 15 in. long and $1\frac{1}{2}$ in. in diameter is subjected to a pull of 6 tons. Over what length must the bar be reduced to one inch in diameter in order that the extension produced by the same load be increased by 50 per cent.? $E = 30 \times 10^6$ lb. per sq. in.

42. A bar 2 in. in diameter is subjected to a pull of 10 tons. Calculate the least length so that the bar may store 0·507 inch ton of elastic strain energy. $E = 13{,}000$ tons per sq. in.

43. A steel bar has been turned down to different diameters, namely, 2 in. and $1\frac{1}{2}$ in. for lengths of 2 ft. and 1 ft. respectively. An axial load of 5,000 lb. acts on this bar. Calculate the stress in each part of the bar and also the resilience or work stored in the bar. Neglect any effect of abrupt change of section on the distribution of stress. $E = 30 \times 10^6$ lb. per sq. in.

44. The following table gives the loads and corresponding extensions in thousandths of an inch (mils) in a tensile test on an aluminium bar 1 inch in diameter. Extensions were measured on a length of 8 in. The elastic limit was 12,400 lb.

Load (lb.) - -	0	2000	4000	6000	8000	10,000	12,400
Extension (mils) -	0	2·1	4·2	6·4	8·4	10·8	13·8

Find (*a*) the modulus of elasticity, and (*b*) the proof resilience of the bar per foot length.

45. A member, of effective length 36 in. and 1 in. in diameter, forming part of a machine, became 60 centigrade degrees hotter than the rest of the machine. Assuming the supports to yield 0·005 in. in permitting partial expansion, calculate for this member:

(*a*) the fractional strain produced; (*b*) the mean intensity of stress in tons per sq. in.; (*c*) the resilience or work stored in in. lb. units. [$E = 13{,}500$ tons per sq. in.; $\alpha = 0{\cdot}000011$ per centigrade degree.]

46. Find the extension produced in a length of 10 ft. of steel bar 1·25 in. in diameter by a sudden pull of 8 tons. If the stress at the elastic limit is 15 tons per sq. in., find the proof resilience in in. lb. per cu. in. $E = 13{,}500$ tons per sq. in.

47. A bar of steel 2 in. in diameter and 12 ft. long is subject to the action of a load of 50 lb. which falls vertically 10 in. on to a collar on the end of the bar. Calculate the elongation and the maximum instantaneous stress. $E = 30 \times 10^6$ lb. per sq. in.

48. When a load of 100 lb. falls through a distance of 6 in. on to a collar on the end of a bar 120 in. long and d in. in diameter, it sets up an instantaneous stress of 5 tons per sq. in. If E equals 30,000,000 lb. per sq. in., calculate the diameter of the bar.

49. The impact produced by a falling weight of 200 lb. is wholly taken up in stretching a steel bar $1\frac{1}{2}$ in. in diameter and 10 ft. long. Find the extension and the intensity of stress produced. The weight falls 2 in. before impact. $E = 30 \times 10^6$ lb. per sq. in.

50. Find the equivalent dead loads to the following :

(*a*) Tensile dead load of 40 tons and a tensile live load of 10 tons.
(*b*) Tensile dead load of 20 tons and a compressive live load of 20 tons.
(*c*) Tensile dead load of 15 tons and a tensile live load of 20 tons.
(*d*) Compressive dead load of 25 tons and a tensile live load of 20 tons.

If the strain is not to exceed 0·001, find the area of section required in each case. $E = 13{,}500$ tons per sq. in.

Riveted Joints and Thin Shells.

51. Find the diameter of the rivets and their pitch for a double-riveted lap joint if the thickness of the plates is $\frac{7}{8}$ in. The permissible stresses in tension, shear and bearing are respectively 5, 4 and 7 tons per sq. in. Find the efficiency of the joint.

52. Two plates each $\frac{1}{2}$ in. thick are to be joined by a double-riveted lap joint. Find the pitch and size of rivets which would be suitable. Stresses as in Question No. 51.

53. Two lengths of mild steel tie bar 12 in. wide and $\frac{5}{8}$ in. thick carrying a load of 48 tons are joined by a riveted butt joint. Fig. 70 shows the arrangement of the rivets, which are to be of 1 in. diameter. Calculate the relative strength of the joint as compared with the uncut plate if fracture is assumed to occur across one of the sections AA, BB, CC and DD. Assume a rivet in double shear to be $1\frac{3}{4}$ times as strong as one in single shear, and $\frac{5}{7}$

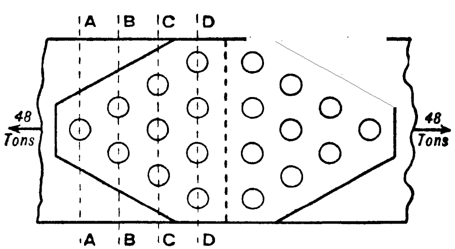


FIG. 70.

tensile stress in plate equals shear stress in rivets. What is the average shear and bearing stress on the rivets?

54. The two plates used to form a tie are of section 7 in. × $\frac{13}{16}$ in. and are to be joined by a riveted butt joint. Determine the diameter and number of the rivets and the pitch. Assume $\frac{8}{10}$ tensile stress $=$ shear stress $= \frac{4}{7}$ compressive stress.

55. A joint is to be made in a tie plate $\frac{3}{4}$ in. thick which is to take a maximum load of 25 tons. Assume a rivet in double shear to have $1\frac{3}{4}$ times the strength in single shear and take the working stresses in shear, tension and bearing as 10,000, 8,000 and 12,000 lb. per sq. in. respectively. Design the joint.

56. Determine the diameter of the rivets and the pitch suitable for a double-riveted butt joint for $\frac{3}{8}$ in. plates. Calculate the efficiency of the joint. Permissible stresses in tension, shear and bearing are respectively 5, 4 and 7 tons per sq. in.

57. Design the essential proportions for a triple-riveted butt joint for $\frac{3}{4}$ in. plates, using the stresses allowed in tension, shear and bearing as respectively 7, 5 and 9 tons per sq. in.

58. Find the circumferential and longitudinal stresses in a cylindrical boiler shell some distance from the ends, if the gauge boiler pressure is 80 lb. per sq. in., the diameter is 90 in. and the plate thickness is $\frac{1}{2}$ in.

59. What is the minimum thickness of a seamless pipe 4 in. in diameter in order that when containing a fluid under internal gauge pressure of 100 lb. per sq. in. the greatest intensity of stress does not exceed 1000 lb. per sq. in.?

60. Find the approximate thickness of plates required for a boiler shell to work at a gauge pressure of 160 lb. per sq. in.; diameter of shell 8 ft.; efficiency of riveted joints 75 per cent.; tensile stress in plates not to exceed 5 tons per sq. in.

61. A spherical vessel 6 ft. in diameter is subjected to an internal gas gauge pressure of 120 lb. per sq. in. Find the thickness of the plating required for a joint efficiency of 67 per cent. and a working stress of 6000 lb. per sq. in.

62. The joint shown in Fig. 71 is for the draught link of a heavy trailer, which is to sustain a maximum pull of 35 tons. If $t = 1$ in. find the dimensions W, D and X to resist tearing, shearing and crushing, and design the joint. Allowable safe stresses, $f_t = 3$, $f_s = 2{\cdot}5$, $f_b = 6$ tons per sq. in.

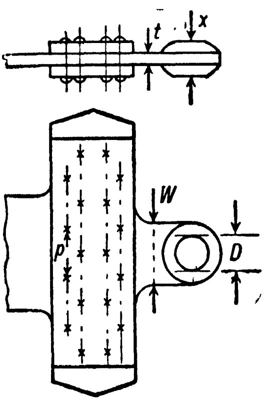


FIG. 71.

CHAPTER V

MECHANICAL TESTS ON MATERIALS

THE mechanical test of a material has for its objects the determination of quantitative values relating to its strength, elasticity and hardness, and qualitative observations taken during the test and during and after failure. The extent to which the test is taken will be governed by the information required, and the nature of the test will depend upon the particular feature or features to be examined.

When material is supplied by a manufacturer it is generally to a specification, and it is the business of those responsible for the tests to see that representative specimens conform to this specification. The specification may include a certain minimum tensile stress at fracture, a specified requirement on impact test, a certain degree of hardness, prescribed percentage elongation, a definite yield stress and strain, or it may specify certain behaviour in shear, compression, torsion or workability according to the nature of the work for which it is ordered.

It follows that tests must be devised to meet the specification, and sufficient observations and specimens taken to ensure that the delivery is on representative performance at least equal to the specification.

Apparatus. Specially designed machines can be obtained for every aspect of mechanical testing ; separate machines may be installed for tension and compression, bending, shear, torsion and hardness, or, as is often the case, a universal machine is provided which can cater for all aspects of the test. The stress is measured by accurate instruments ; if the load is applied hydraulically, by pressure gauges calibrated to read directly the load on the specimen ; or if the load is applied by gearing and a prime mover, it is measured by a weigh arm or lever balance.

Strains within the elastic period are measured with accurate multiplying instruments in which the strain is magnified by levers

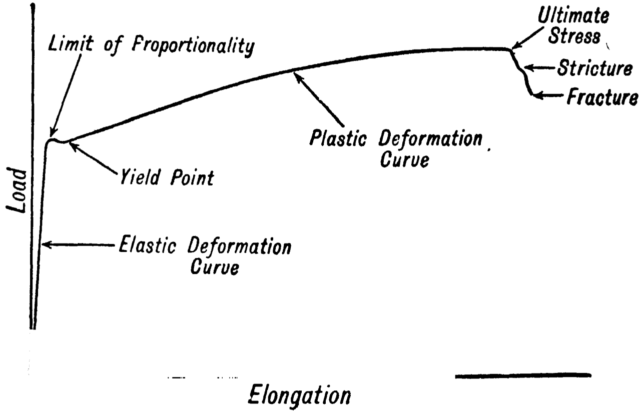


FIG. 72.

or other mechanism, and a direct reading of the deformation given. These instruments, called **extensometers** for tensile strain and **torsion meters** for torsion or twisting strain, have a small range, and strains beyond the elastic period are measured by means of vernier or micrometer instruments.

Tensile test. Before proceeding to an actual test, it is desirable that the student should familiarise himself with the terms employed and the critical points in the test. For this purpose, consider a specimen of mild steel loaded steadily until fracture occurs, that is, tested to destruction by tension (Fig. 72).

The steel specimen will suffer a steadily increasing stress, which according to Hooke's Law is proportional to the strain *until the limit of proportionality or elastic limit* is reached. This period shows a straight-line graph between stress and strain, and is known as the **elastic deformation period.** At any time in this period the load may be removed and the elastic property of the material will assert itself to restore to the specimen the dimensions it possessed before loading.

The **elastic limit** is the point at which the extensions cease to be proportional to the loads, that is, where the load *v.* extension diagram ceases to be a straight line. The **yield point** is the stress at which a distinctly visible increase occurs in the distance between the gauge points on the test-piece as observed by using dividers after the removal of the load ; or the point at which when the load is increased there is a distinct drop of the gauge finger in a hydraulic testing machine or a distinct drop of the weigh arm in a lever testing machine. Mild steel gives a clearly defined yield point ; but the yield point is not so evident in other metals, and in very ductile metals it is absent altogether.

From the yield point onwards, strain ceases to be proportional to stress and the material suffers **plastic deformation,** or is **ductile.** The period following yield point on the stress *v.* strain curve is known as the **plastic deformation period.**

This ductile period will continue to a stage when the specimen begins to decrease appreciably in cross-sectional area, or waist, and this is the period of **stricture,** which is immediately followed by fracture. The performance of the specimen after stricture is of little value, and the **ultimate stress,** or failing stress, is taken as the maximum stress reached immediately before stricture occurs.

Critical observations. The test will require observations for one, some, or all of the following :

(*a*) *Ultimate tensile stress*, or breaking stress.
(*b*) *Yield stress*, or stress at yield point.
(*c*) *Percentage yield extension*, or extension at yield point as a percentage of gauge-length.
(*d*) *Percentage fracture extension*, or extension at fracture as a percentage of gauge length.
(*e*) *Modulus of elasticity*, or the ratio of stress to strain up to the limit of proportionality or elastic limit.

When the test is of an exhaustive character, it is general to prepare a graph of stress *v.* strain *or* load *v.* extension in order to ensure that there was no unusual behaviour of the specimen during either the elastic or plastic deformation period. If the graph exhibits characteristics of an unusual type, the test is repeated on further specimens to confirm, or otherwise, the unusual behaviour as characteristic of the material from which samples have been taken.

Report on the test. The preparation of a test report should, in addition to revealing the observations taken during the test, give the following information :

(*a*) Source of the specimen and the method of its selection.

(*b*) Dimensions of the specimen and the gauge length used as a base for elongation observations.

(*c*) Type of testing machine used and extensometer employed.

(*d*) *Ultimate tensile stress* $= \dfrac{\text{Ultimate load}}{\text{Area of cross-section}}$.

(*e*) *Yield Stress* $= \dfrac{\text{Load at Yield Point}}{\text{Area of cross-section}}$.

(*f*) *Percentage yield elongation* $= \dfrac{100 \times \text{extension at yield point}}{\text{gauge length}}$.

(*g*) *Percentage fracture elongation* $= \dfrac{100 \times \text{total elongation}}{\text{gauge length}}$.

(*h*) *Modulus of elasticity* $= \dfrac{\text{stress}}{\text{strain}}$ taken at a period in the elastic deformation curve.

The report may, if necessary, be amplified by graphs of load *v.* extension for the whole test and for the elastic period separately, with photographs of the fracture and comments upon any specific features revealed by the test.

The tensile test has been dealt with more fully to illustrate the principles of testing ; the subsequent tests will be explained more briefly, emphasising features by which they differ materially from the tensile test.

Compression test. The compression test is carried out on a short stiff specimen in order that failure may be due to compression only, and not associated with any tendency the specimen may have towards buckling. The behaviour of the material will depend upon its nature and, for brittle materials such as cast iron, failure will be sudden, with fracture following closely on the first indication of failure. Ductile materials, in their varying degrees of ductility, such as iron, steel and non-ferrous metals, will collapse under the stress, and it will be difficult to assign an ultimate stress.

The value of the compression test is thus confined to brittle materials where the determination of ultimate stress is the primary aim. For other materials the test determines the yield point and the physical behaviour of the material under load.

Shearing test. Shearing tests are usually conducted with short specimens of the material, and care is taken to ensure that failure is due to pure shear and not associated with bending or flexure.

Bending or flexure test. Tests taken of a material under bending or flexure are not always taken to fracture or destruction. It is often sufficient to determine the deflection of a standard specimen under load, as the particular failing stresses can be linked by calculation with those found under tension, compression and shear. The specimen is usually square or rectangular in section, and the test is confined to central loads.

Torsion test. Torsion tests are carried out to determine the behaviour of a material under pure torsion or twisting, and the strain observations are measured as angular strain, while the equivalent to the load is a twisting couple of increasing moment. Ultimate failure is generally by shear. The main function of the test is to determine the value of the torque, or twisting couple, producing yield, together with the angular strain and its relation to the torque producing it. From this the modulus of rigidity can be deduced and strain values used in the design of components subjected to torsion.

Hardness tests. Hardness is a quality which may be defined as a resistance to abrasion or indentation of the surface, and is of considerable importance where materials are used as metal to metal contacts in slides, rotating parts, bearings and impacting parts. It is also of great importance in the manufacture of tools, jigs and gauges where hardened surfaces are subjected to wear.

The test, in its indentation form, is carried out (*a*) by indenting the surface and using a defined pressure behind an indenting tool, or (*b*) in the scleroscope form, by measuring the rebound of a small hammer from the surface when the hammer falls from a fixed height. In the indentation form the shape of the tool employed determines the calculation necessary to assess the hardness, and indenting tools are usually either in the form of balls or pyramids made of hardened steel or of diamond, according to the relative hardness of the material under test. The relationship between the hardness value and the indentation is usually taken as one involving the surface area of the indentation. Since the indentation surfaces formed by a square-based pyramid on different materials are *similar* surfaces, the hardness must be proportional to the square of a linear dimension of that surface. In the case of ball indentation the hardness is again proportional to the surface area of the segment of the sphere formed and for a given size of indenting ball must be proportional to the depth of the indentation.

Brinell tests. In this test a hardened steel ball is forced, under a fixed pressure produced by a pump, into the surface of the metal, and the diameter of the indentation is measured by a vernier microscope attached to the instrument. This diameter may then be associated with the **Brinell Hardness Number** by interpolation from a chart, or alternatively by calculation, using the following expression :

$$\text{Brinell Hardness Number} = \frac{\text{total pressure on ball}}{\text{surface area of indentation}}$$

$$= \frac{p}{2\pi r\left(r - \sqrt{r^2 - \frac{D^2}{4}}\right)},$$

where p is the pressure in lb., r is the radius of the ball in mm., D the diameter of the indentation in mm. It is usual to employ a ball of 5 mm. radius for this test.

Rockwell tests. In this case a small ball is pressed into the material by a lever-operated press with dead-weight loading. A first impression is made with a 10 kgm. weight, to be followed by a superimposed indentation with a 100 kgm. weight. The depth difference of the two indentations is taken to some suitable scale as an indication of the hardness, which is read directly from a dial graduated from 0 to 100. The ball employed is usually $\frac{1}{16}$ in. in diameter of steel, or alternatively, for very hard materials, a cone of 120° apex angle made of diamond. The former of these is used for soft metals and gives the Rockwell B scale of hardness, and the latter for hard materials the Rockwell C scale.

Vickers and Firth diamond tests. In these tests the indenting tool is a square-based diamond pyramid and the diagonal of the indentation is related to the hardness. This diagonal is measured by micro-vernier, or alternatively shown on a calibrated dial. These tests depend on the load being applied and automatically released after a specified time, and can be used for thin sheets with shallow indentations causing a minimum of disfigurement to the plate surface.

Scleroscope tests. In this test a small diamond-tipped hammer is allowed to fall from a fixed height on to the surface to be tested and the rebound is measured on a scale. The hardness is related to the rebound, and is scaled with 100 as the maximum hardness value, the hardness depending upon the proportion of the energy of the falling hammer absorbed by the material at impact. Two scales are used, one for soft materials and the other for harder varieties as in the Rockwell test.

Herbert pendulum test. This test consists of a compound pendulum involving a weight supported on a 1 mm. diameter hardened steel or diamond ball. When the pendulum rests on a specimen, an indentation related to the hardness is made, and the pendulum oscillates with a time of oscillation indicating the hardness. This period varies from a short period with soft metals to a longer period with the harder metals. The pendulum may be rolled on the surface of the specimen with a corresponding increase in hardness, and some estimate can be formed of the effect of cold working on the hardness of the material.

Relation between ultimate tensile stress and hardness number. The assessed hardness number for a material is closely associated with its ultimate tensile strength, and for rough estimation hardness may be taken as an indication of tenacity. For this purpose the ultimate tensile stress may be taken *as* 0·22 *of the Brinell number.*

Impact tests. The purpose of impact tests is to determine the shock-resisting properties of the material under test. The popular machine for this test is the **Izod machine,** in which a square-sectioned specimen is used. This specimen has a carefully prepared notch machined to a standard depth and shape across its face. The specimen is secured in the vice of the machine with the groove just projecting from the jaws. Impact is made by a pendulum, with a striking block attached, which is allowed to swing from a fixed height and strike the specimen at a fixed distance from the notch.

This pendulum will lose some of its kinetic energy in fracturing the specimen, and arrangements are made for a scale reading of this loss to be made by a pointer attached to the pendulum. The test is generally taken at each of three notches on adjacent sides of the square specimen, with the object of revealing any difference in behaviour under impact due to fibre structure or rolling of the metal. The test can be applied to heat-treated materials and gives a ready means of testing the effect of heat treatment on different sides or at different depths in the material.

One of the difficulties experienced in this test is that the material may not fracture under impact, and the specimen merely bends. Hence the test must be confined to non-ductile materials. A continental machine, the **Charpy machine,** overcomes this difficulty to a certain extent, as it arranges for the test piece to be struck opposite the groove and not immediately at its side.

The results of an impact test must be considered with the tensile test, as impact alone gives no true indication of ultimate stress. It is particularly noticeable that high-tensile materials sometimes develop brittleness, especially after heat treatment, with a consequently low Izod value. Conversely, materials with a very high Izod value sometimes exhibit low ultimates on a tensile test.

The theory of the impact machine is that the product of the difference in height between release and recovery of the pendulum and its weight is a measure in foot pounds of the energy spent in fracturing the specimen. Actually, some part of this energy is absorbed by the foundations and parts of the machine, and observations and calibration have to be based on known performances and cannot be regarded as absolute. The fracture in its ideal sense is one of pure shear, but again some portion of the energy is given to deflecting the machine itself before it asserts itself in fracturing the specimen. In spite of these difficulties some standardisation of these tests has been established. The British Standards Institution has laid down the dimensions of both pendulum and specimen, together with the distance from the notch at which impact is to occur. If these standards are adhered to and the machine is carefully calibrated and founded, the indicated Izod values can be regarded as reliable. The British Air Board specifies a round specimen of 0·45 in. in diameter with a flat ¼ in. in width to receive the impact, and this type of test piece, mainly on account of its ready manufacture, is becoming increasingly used in standard tests as an alternative to the square section, see Fig. 83, p. 142.

Fatigue of Metals

It has long been established that the ultimate strength of a material depends upon the conditions of loading and the nature of the load. Stresses which vary between compression and tension, or are rapidly repeated and relieved, produce a condition in the material known as **fatigue.** This condition has a direct influence on the ultimate stress, and apparatus has been designed to provide rapid alternations of stress in order that material may be tested under fatigue conditions.

Among the earlier investigators of the phenomena of fatigue was Wohler, and he designed a testing machine, in which the specimen with its axis in a horizontal position was rotated at about 2000 revolutions per minute while a load was suspended from it. This had the effect of rapidly alternating the stress between compression and tension, and provided a test of the fatiguing effect on the ultimate strength of the metal. The test is now carried out on a modification of the Wohler machine; and by means of a weigh arm similar to that of a single-lever testing machine, the range of stress variation is determined. Rotation under alternating stress is continued until fracture occurs, and the fracturing stress is related to the ultimate tensile stress. Results of such a test are given as the number of alternations before fracture and the stress range at which the specimen was loaded. It does not appear that the speed of alternation has any marked effect on the result, but invariably the breaking stress under these conditions is considerably less than the ultimate tensile stress under static load.

Proof stress. In the earlier discussion of the behaviour of mild steel under tension, it was pointed out that the yield point was clearly defined by a momentary falling of the load on the testing machine indicator. This property of mild steel is not characteristic of the alloy steels or of the more ductile metals, and some means had to be devised with the increasing use of alloy steels to define

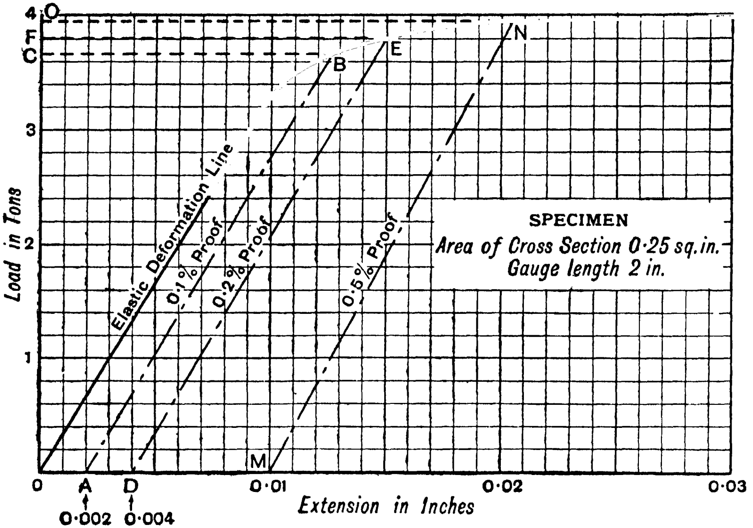


FIG. 73.

clearly the maximum limits to which the designer may use the material without the danger of premature fracture.

To meet this need, **proof stresses** were established; the proof stress is obtained by the methods described in the following paragraphs. The proof stress depends on both the extension percentage used, which is generally 0·1, 0·2 or 0·5 per cent. of the gauge length, and the slope of the elastic deformation line. The proof stress, when stated, must refer to both the stress value and the extension percentage used.

The percentage elongation actually adopted to determine the proof stress depends upon the nature of the demands to be made on the material, and is usually defined in the specification for the supply of the material.

Fig. 73 shows part of the load *v.* extension diagram for a chrome molybdenum alloy steel. The proof stresses are determined as follows. Since the gauge length is 2 inches, mark A, D and M respectively at 0·1 per cent., 0·2 per cent. and 0·5 per cent. of 2 inches along the extension axis, that is, 0·002, 0·004 and 0·01 inch of extension. Draw AB, DE and MN parallel to the elastic deformation line;

then C, F and O on the load axis indicate the proof loads, which may be converted to proof stresses by dividing by the area of cross-section, that is,

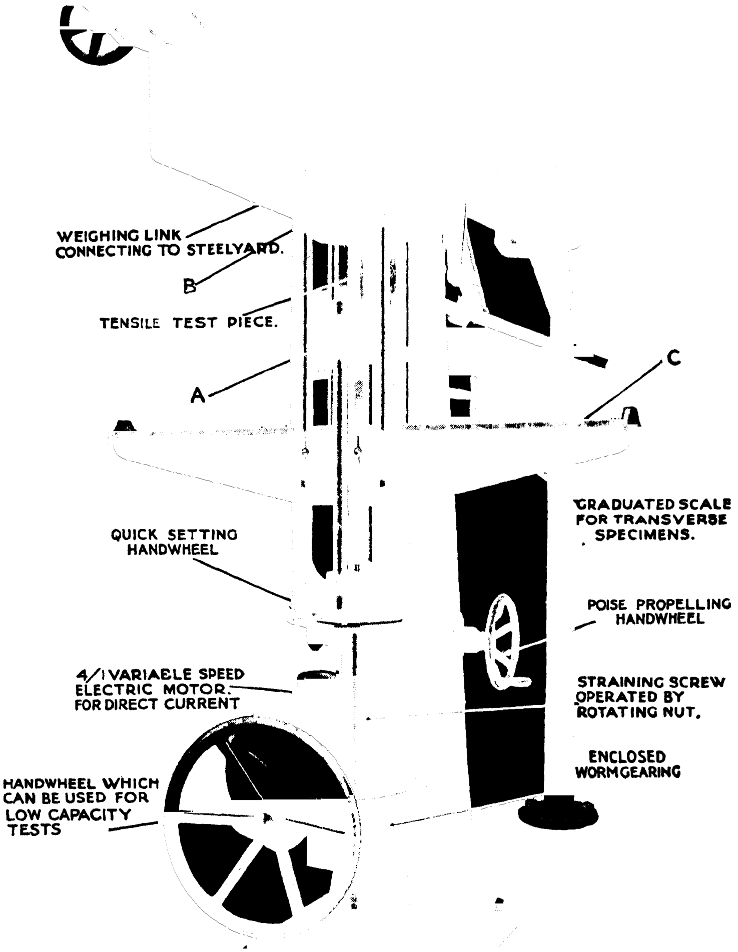
$$\text{Proof stress} = \frac{\text{Proof load}}{\text{Area of cross-section}}.$$

Factor of safety (see p. 101). The factor of safety is the ratio $\frac{\text{Ultimate stress}}{\text{Safe working stress}}$, and is an arbitrary constant fixed according to the conditions of working. Any factor of safety must bring the working stress appreciably below the elastic limit, otherwise danger of plastic deformation exists ; and it is general so to assign a factor of safety that, however strenuous the condition of loading, a good margin is established below the limit of elasticity. Proof stresses and the yield point determination provide an indication of what safety factors may be considered and define clearly the limits of elasticity in the material. The student should realise that the application of safety factors without accurate knowledge of the proof or yield stresses is apt to reduce the " factor of safety " to a " factor of ignorance ", resulting in either inadequate dimensions in design or a design far too heavy for the strength required.

TESTING MACHINES

Universal testing machine. The following description applies, in principle, to any universal testing machine, and to other machines designed to perform one or more specific tests.

Messrs. W. & T. Avery of Birmingham manufacture a universal machine capable of performing tests in tension, compression, bending, shear, hardness and torsion. This machine is built to have a maximum capacity of anything between 5 and 100 tons, but the more generally used laboratory capacity is either 5 or 10 tons. The load is applied by an electric motor geared to a worm and worm-wheel which acts as a nut for the straining screw. This screw is attached to a crosshead through which the strain is applied. Hand operation is provided as an alternative to power and to facilitate adjustments to the specimen. The actual straining by hand operation is limited to a load of about 5 tons. Return motion for the straining mechanism is achieved by a reversing switch, and a clutch


WEIGHING LINK
CONNECTING TO STEELYARD.
B
TENSILE TEST PIECE.
A
C
QUICK SETTING HANDWHEEL
GRADUATED SCALE FOR TRANSVERSE SPECIMENS.
POISE PROPELLING HANDWHEEL
4/1 VARIABLE SPEED ELECTRIC MOTOR FOR DIRECT CURRENT
STRAINING SCREW OPERATED BY ROTATING NUT.
ENCLOSED WORMGEARING
HANDWHEEL WHICH CAN BE USED FOR LOW CAPACITY TESTS

By courtesy of Messrs. W. & T. Avery, Ltd.

FIG. 74.—Universal Testing Machine.

is fitted which enables the loading to be controlled without cutting out the motor.

Weighing is done by a weighing crosshead **C** suspended from a knife-edge on the steelyard by means of a link. The poise can be moved along the steelyard by gearing until balance is obtained, and this poise may be in two parts as shown, or a single weight. The position of the poise is shown on an engraved scale fitted with a vernier and graduated to read the direct load on the specimen.

When a split poise is used, two scales are provided, one to read the load from the larger part of the poise and the other to register the more accurate phase of the loading given by the movement of the smaller poise.

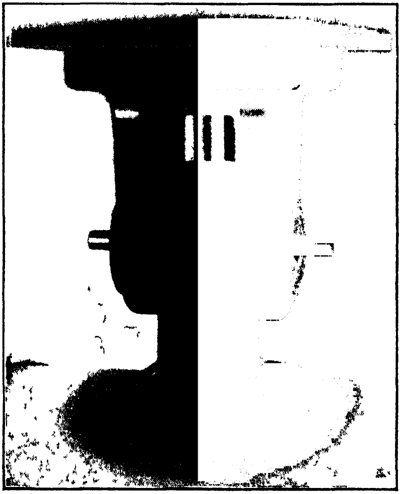
When the machine is used for torsion tests, a further scale is employed to give direct readings of the torque in pounds inches. The balance **B** is arranged to give the operator a visual indication of the movement of the steelyard while he operates the poise-gearing through a handwheel. Shock absorbers **A** are introduced to absorb the shock to the moving parts when fracture occurs.

Fig. 74 shows the machine set with the tools in position for tension, compression or bending tests. Load is applied to the specimen by the downward movement of the crosshead A and transmitted through the specimen to the weighing crosshead B, the link and the steelyard where it is balanced by the moving poise.

For *tension* tests the specimen is carried between the crossheads A and B ; whereas for *compression* tests, the specimen rests on the beam C and the load is transmitted by means of four steel rods to the weighing crosshead.

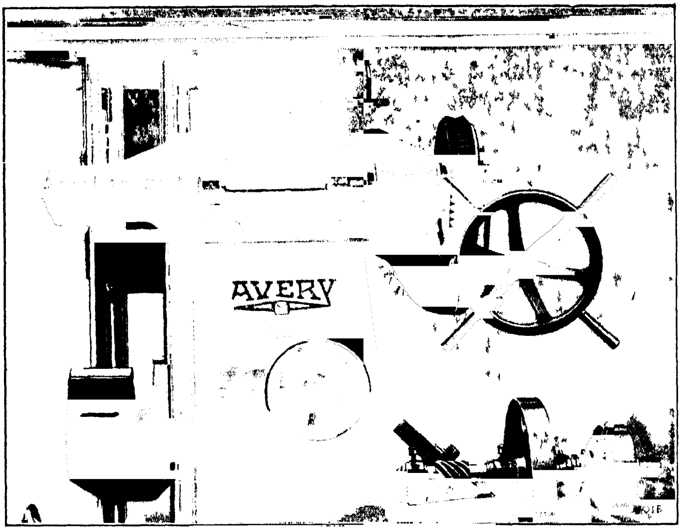
When the machine is used for *bending* tests the specimen is placed across fulcra on the beam C and the load applied to the centre of the span by a special tool placed under the straining crosshead A.

Shearing tests are carried out by introducing shear tools (Fig. 75) placed between



By courtesy of Messrs. W. & T. Avery, Ltd.

FIG. 75.—Shearing Test Tool.



By courtesy of Messrs W & T. Avery, Ltd

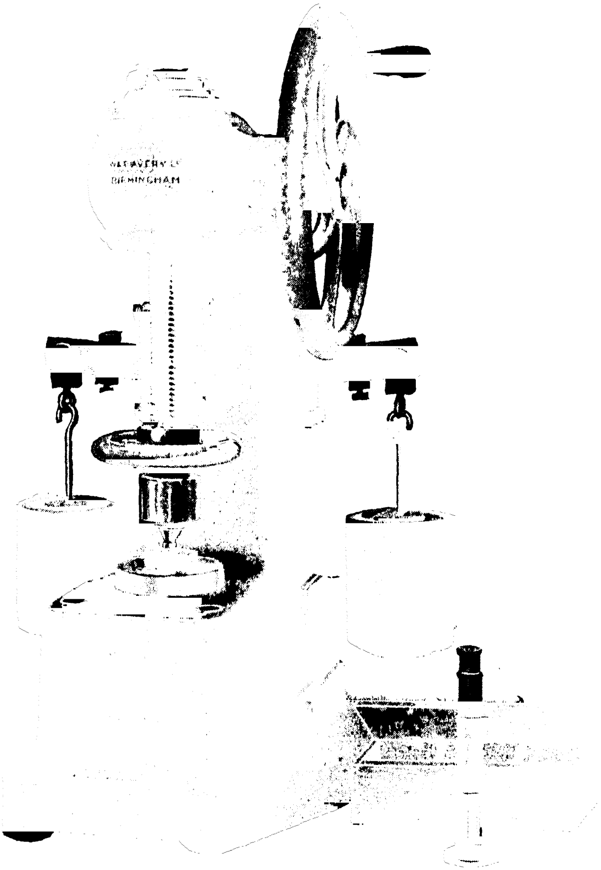
FIG. 76.—Torsion Test.

the compression surfaces of the machine. The tests are made on cylindrical specimens in double shear.

Torsion tests are performed on an attachment secured to the vertical frame of the machine (Fig. 76). The specimen is held in a holder attached to a shaft rotated by a handwheel B through spur and worm-gearing. The other end of the specimen is secured in a second holder attached to an arm the end of which is connected by a subsidiary knife-edge linked to the steelyard.

Plastic materials are subjected to very similar types of test to metals and wood, including the usual physical tests for specific gravity, coefficient of expansion, water absorption, dielectric strength, volume resistivity and power factor. Mechanical tests are taken for tensile and shear stress and the modulus of rupture. Approximate maximum values for specimens of bakelite are tensile stress 25,000 lb. per sq. in., shear stress 16,000 lb. per sq. in., and modulus of rupture 19,000 lb. per sq. in.

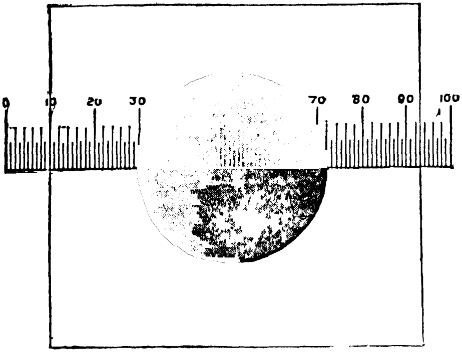
Hardness testing machines. Fig. 77 shows the Avery hardness



By courtesy of Messrs. W & T Avery, Ltd.

FIG. 77.—Hardness Testing Machine.

testing machine designed to carry out Brinell tests. For ferrous metals a load of 3000 kgm. is applied to the specimen through a steel ball 10 mm. in diameter for a period of 15 sec. When the test is carried out on non-ferrous metals, a load of 500 kgm. is used for a period of 30 sec. The Brinell hardness number is obtained by the method shown on p. 128, or by direct reference to a hardness chart after the impressed diameter has been measured by a micro-



By courtesy of Messrs. W. & T Avery, Ltd.

FIG. 78.

scope scale (Fig. 78). During the test, load is applied through the handwheel and a worm-drive until the steelyards are horizontal, thus indicating a full load. After the prescribed interval of time, the handwheel is reversed and the load removed.

Vickers hardness machine. This type of hardness testing machine is regarded as a standard instrument where reliable hardness values are required which are absolute in their interpretation. The quantitative value of hardness is determined by means of the ratio $\dfrac{\text{Load}}{\text{Area of indentation}}$, and the accuracy of the interpretation of this ratio is the result of the mechanical control of the loading and the use of a practically undeformable indenter of regular shape which gives geometrically similar impressions.

In the Vickers machine the diamond indenter is capable of indenting the hardest metals without appreciable deformation of the material or indenter. Its shape is that of a square-based pyramid of 136° interfacet angle, and the area of the indentation produced, as derived from its diagonal measurements, is applied to the above-mentioned ratio, $\dfrac{\text{Load}}{\text{Area of indentation}}$, to obtain the hardness.

A range of loads varying from 1 to 120 kgm. is available to enable the use of a suitable load for the material under test. The load is applied through a simple lever, and its application, duration of loading, and removal automatically controlled; special precautions being taken to avoid shock at all stages.

The surface indentation is clearly defined and can be measured with extreme accuracy across its diagonals. Fig. 79 shows such an impression viewed through the microscope. The hardness value is independent of the load applied, since the ratio $\frac{\text{Load}}{\text{Area of indentation}}$ is constant for homogeneous materials and is internationally recognised as the **V.P.N.** or the **Vickers Pyramid Numeral.**

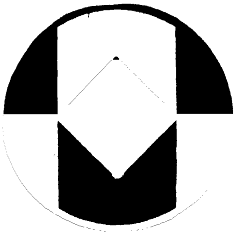


FIG. 79.

Fig. 80 illustrates the construction of the machine. This consists of a main frame **F** carrying the stage **S** with a simple lever **L** of 20 to 1 ratio, through which the load is applied to a thrust rod **Tr** and a tube **T** free to reciprocate vertically and carrying the indenting tool **D** at its lower end.

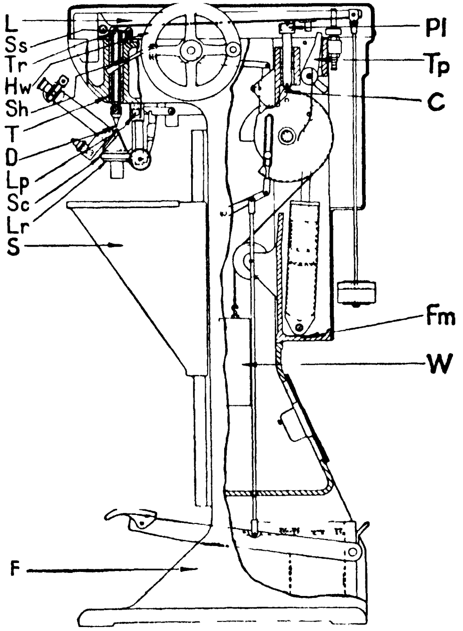
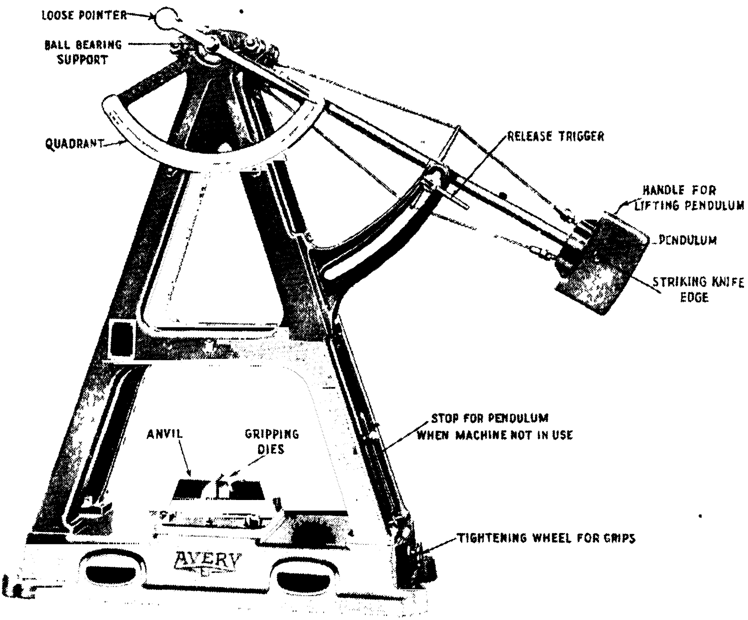


FIG. 80.—Vickers Hardness Machine.

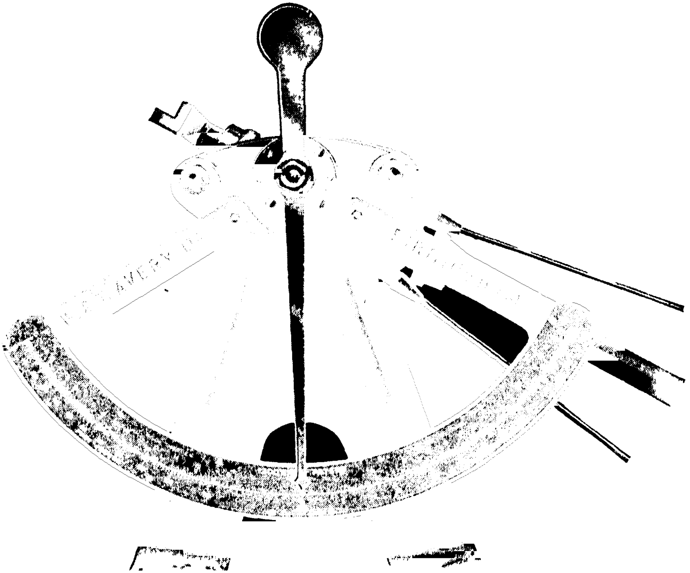
Attached to the main frame is the unit **Fm** containing the control mechanism, which consists of the plunger **Pl** operated by the rotating cam **C** applying and releasing the load. The cam is mounted on a drum rotated with the release of the starting handle **Sh** and controlled by the weight **W** attached to a flexible wire. A piston operating in an oil-filled dashpot, in which the oil displacement is regulated by an adjustable valve, controls the time cycle of the cam. The plunger carries a rubber pad at its upper end which engages a cone mounted on a beam, thus ensuring a slow and diminishing speed of application for the load. Since the cam both lowers and raises the plunger, uniformity in application and duration of loading is achieved without inertia errors and premature removal of load. Depression of the foot pedal returns the cam drum and weight to their original positions, while a tripping piece **Tp** supports the beam and drops out of action when the plunger is in its top position. The machine is then ready for further tests.

Impact testing machine. Fig. 81 shows an Izod impact testing machine manufactured by Messrs. W. & T. Avery. In this machine



By courtesy of Messrs. W. & T. Avery, Ltd.

FIG. 81.—Izod Impact Testing Machine.



By courtesy of Messrs. W. & T Avery, Ltd.

FIG. 82.

the specimen is held between the gripping dies and the pendulum raised to engage the release trigger. After release, the pendulum strikes the specimen and energy is used to cause fracture at the notch. An inverted quadrant and needle (Fig. 82) are employed to give a direct reading of the energy absorbed during the process of fracturing the specimen. This is read off from the needle position on the graduated quadrant either to a maximum of 120 ft. lb. or its equivalent in metres kilograms.

Specimens for standard tests. Fig. 83 shows the standard type of specimen used for the various tests. The British Standards Institution recommends test pieces of $\frac{1}{4}$, $\frac{1}{2}$ and $\frac{3}{4}$ sq. in. in cross-section, with gauge lengths respectively 2, 3 and $3\frac{1}{2}$ in. These dimensions satisfy the equation

$$\text{Gauge length } \mathbf{L}=4\sqrt{\mathbf{A}},$$

where A is the area of the original cross-section. It is found that

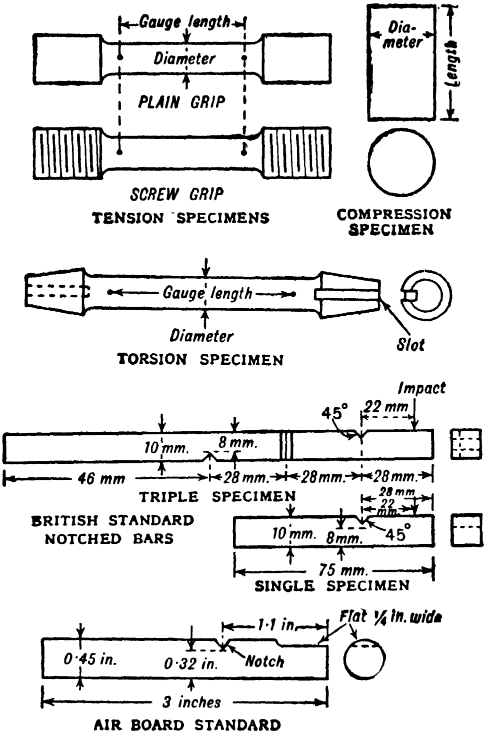


FIG. 83.—Impact Specimens.

where these sizes are used the percentage extension and the percentage diminution in area are approximately equal for the three sizes.

Experiment shows that local extension at the waist of a specimen, formed during plastic deformation, is proportional to the square root of the cross-sectional area, while the total extension, elastic and plastic, is proportional to the gauge length plus a constant times the square root of the area of cross-section.

For compression the specimen has a length which seldom exceeds $2\frac{1}{2}$ times the diameter, thus eliminating the danger of buckling under load. A good diameter for a compression specimen is 1·128 in., which gives a cross-sectional area of one square inch.

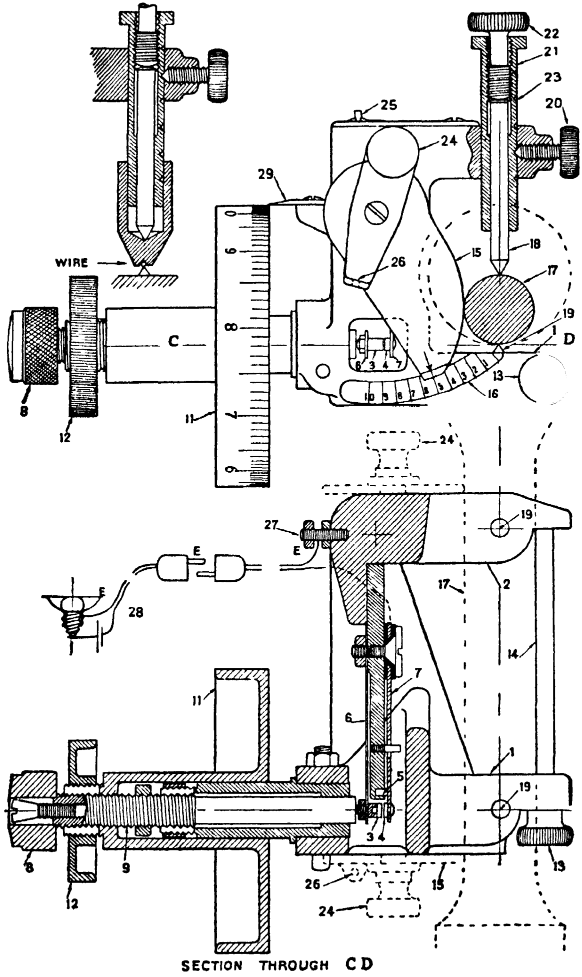
The torsion specimen has two tapered ends to fit the holders, and in these ends keyway slots are cut to prevent the rotation of the specimen. Keys in the holders fit these keyways, and the keys are so designed that the maximum torque applied will not cause them to shear.

Two forms of impact test pieces are illustrated: (*a*) the **British Standard Notched Bar,** which is a 10 mm. square test piece ; and (*b*) the **Air Board Standard Test Piece,** which is 0·45 in. in diameter; both test pieces have a 45° notch. The area of cross-section is thus the same in each type of specimen, and impact occurs at the same distance from the notch with consequently equal Izod values. When three tests are required at right angles, triple test pieces are prepared as shown in the figure, and the three impact tests are taken in sequence on the same notched bar.

Radiological tests. It is well known that X-rays are used by physicians and surgeons in the examination of the human body. The principal property of these rays is that they are able to penetrate solid matter with varying degrees of absorption. The degree of absorption varies from lead, the strongest absorber, through the denser metals to materials of an organic character such as wood, carbon and compounds consisting of very light atoms such as hydrogen, oxygen or nitrogen, which are easily penetrated by X-rays.

Industrially, the X-ray method is used to detect porosity, blowholes and irregularities in the structure of metals. The object to be examined is placed between an apparatus producing X-rays and a suitable photographic film or a fluorescent screen, and a picture is obtained of the object, not clearly defined as in a photograph, but consisting of a number of shaded areas, the intensity of the shading indicating the degree of absorption offered by the material at this area. Cracks, flaws, blow-holes and very porous areas will give high penetration and will be revealed on the sensitive film or screen, while a perfectly homogeneous material free from flaws will show a shaded area of equal intensity or density throughout.

Considerable experience is required in interpreting an X-ray photograph, and although the apparatus can be used by a trained operative, the interpretation must be given by an expert.



Messrs. Tensometer, Ltd

FIG. 84.—Hounsfield Extensometer.

Crack and surface flaw detection. The general method now adopted for the detection of surface cracks and flaws depends upon the ability to magnetise the material, and can thus be used for almost all the ferrous metals and their alloys. A notable exception is the non-magnetic austenitic type of steel. Cracks in the non-ferrous metals must still be found by microscopic examination, etching of suitable sections or X-rays.

The standard type of instrument for the magnetic detection processes is the **Johnson-Fel Crack Detector.** The specimen can be magnetised by the passage of a momentary current through it, or its exposure to an external magnetic field of sufficient strength to magnetise that portion of the specimen to be examined. When the specimen is magnetised, it is covered with a layer of detector ink carrying iron particles in suspension; then the cracks will reveal themselves by the adherence of the iron particles to their walls.

Extensometers. An extensometer is an instrument designed to measure accurately the small elastic deformations and sufficient of the plastic deformations to determine proof stresses, yield point, limit of proportionality and modulus of elasticity during a tensile test. The extensometer is always removed before the deformation takes appreciable shape, and the instrument is thus a delicate measuring device suitable for small deformations only, and must be handled and used with these restrictions firmly in mind.

Extensometers may be divided broadly into two types: (1) those relying upon *mechanical* means to magnify the small deformations; and (2) those using *optical* means for the same purpose. In the former group the measurement is made by means of levers, a calibrated scale, micrometers or dial indicators; whereas in the latter type a beam of light reflected by a mirror, or series of mirrors, is employed. The former type is employed in general mechanical tests, while the latter is almost entirely confined in its use to laboratory and research tests, where exceptional accuracy is demanded.

There are many types of instrument in each group, each with its advantages and special suitability. We have selected for description one type from each group representative of good modern practice.

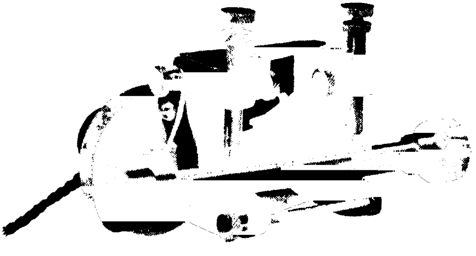
The Hounsfield Extensometer. This extensometer is of comparatively recent introduction. The essential parts are shown in Fig. 84, namely, a clamping frame (1), and a bellcrank lever (2) pivoted on the frame, the position of the pivot being marked in the diagram. These parts (1) and (2) can be attached by point contact (18) and (19) (Figs. 84 and 85) to the test piece over the usual gauge length of two inches ; the stop (14) is employed to ensure that the points (18) and (19) are exactly two inches apart. The bell crank lever moves, about its pivot, with the deformation and has a 2 : 1 arm ratio, the longer of which contacts the anvil of a 50 threads per inch micrometer screw (9), to which is attached by friction a graduated wheel (11) divided on its periphery into 100 equal parts. Thus one division on the graduated wheel represents

$$(\text{Screw}) \times (\text{Graduations}) \times (\text{Lever}) =$$

$$\frac{1}{50} \times \frac{1}{100} \times \frac{1}{2} = \frac{1}{10{,}000} = 0{\cdot}0001 \text{ in. deformation}$$

on the specimen. The micrometer screw is operated by a knurled head (8), the larger nut (12) providing the friction grip of the graduated wheel on the measuring screw. To obtain positive readings free from personal error, electrical contacts are provided which close a lamp circuit when the correct reading is obtained.

A platinum contact (4) is mounted on an insulated spring (7) pressing firmly against an insulated stop (5). The contact (3) is mounted on an earthed spring (6) pressing lightly on the micrometer anvil, and the circuit is completed through an external battery and bulb of the flash-lamp type. To obtain a reading the micrometer



Messrs. Tensometer, Ltd.

Fig. 85.

screw (9) must be screwed in against the spring (6) only until the platinum points (3) and (4) touch and the lamp *just* lights.

Plates (15) on each clamp move over a sector scale (16) graduated in tenths of an inch. These can be pre-set to the diameter of the specimen when the clamping points (18) are in position, to be finally adjusted by the screws (20) and (22) fitted to the clamping sleeves (21).

The sensitivity of this instrument is such that it will detect the expansion produced by the heat of a lighted match on a specimen of 2 in. gauge length, and it can be relied upon to give positive readings to $\frac{1}{10000}$ in. deformation, with estimated readings to an even smaller amount.

Amsler mirror extensometer. This type of extensometer is used to measure small elastic deformations with extreme accuracy and is adopted fairly generally for research tests. On each side the specimen is marked to its exact gauge length, and a pair of comparison strips of an equal length are lightly clipped, one on each side, with the sharp edges at one end placed in one of the marks of the gauge length. At the other end of each comparison strip a light lever fitted with sharp parallel knife-edges is supported, one knife-edge fitting into a notch in the comparison strip and the other into the gauge mark on the specimen. Each lever carries a plane mirror, so that the readings from the sides of the specimens can be averaged. When the gauge length suffers deformation, that of the comparison strips remains constant, with the result that the levers are tilted, the mirrors thus being given an angular movement equal to that of the lever.

The angular movement of the mirrors is measured by a stationary telescope directed at the mirror and free to rotate against a scale at right angles to the axis. The movement of the mirror allows the image of the scale to be reflected and traverse the telescope reticule by an amount which is related to the gauge length. The Marten's Mirror Extensometer is made by Alfred J. Amsler & Co. (Agents, T. C. Howden & Co., Birmingham.)

Principle of the mirror extensometer. When the specimen is deformed by an extension e, the mirror moves through an angle α with respect to its original position, where telescope and gauge line are in line, and an image of the point O on the scale can be seen through

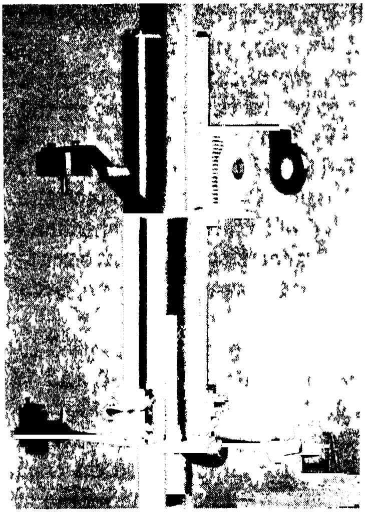


FIG. 86.

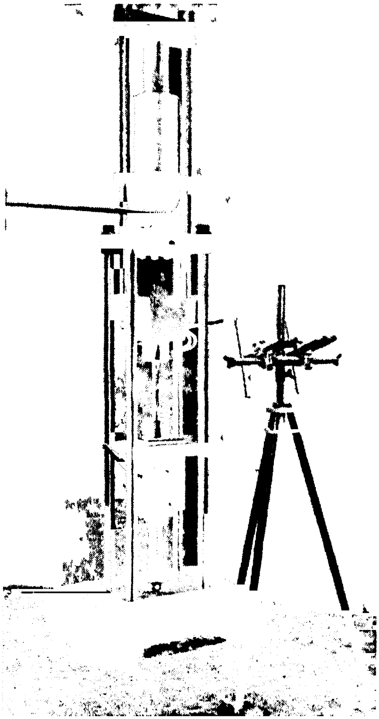


FIG. 87 —Amsler Mirror Extensometer.

the telescope Fig 88. The image of P is then seen through the telescope, where the angle subtended by the lever and mirror axis and OP is 2α. Then OP $= x$.

Let the gauge length be g and the length of the lever a.

Then $$a \sin \alpha = e, \qquad \text{......................................(1)}$$

and $$\frac{x}{y} = \tan 2\alpha. \qquad \text{..............................(2)}$$

Since α is small, then

$$\tan 2\alpha = 2 \tan \alpha = 2 \sin \alpha,$$

$$\text{or} \quad \tan \alpha = \sin \alpha.$$

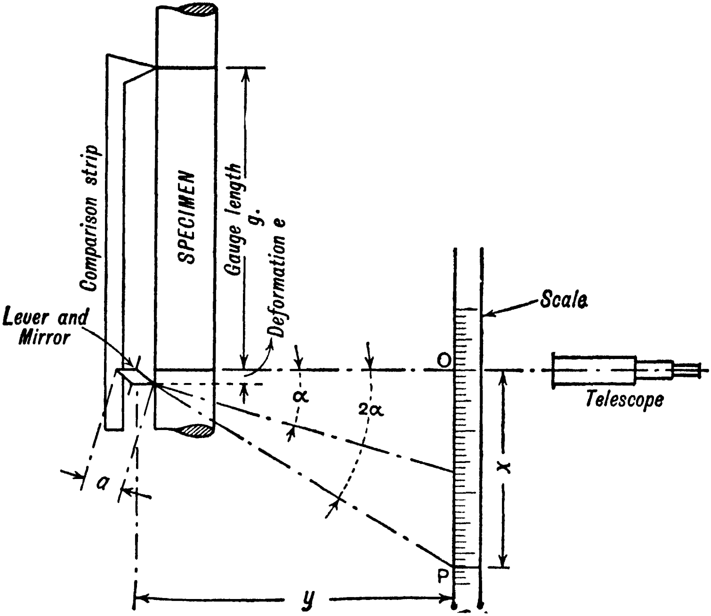


FIG. 88.—Principle of Mirror Extensometer.

Hence in (1), $a \tan \alpha = e,$

and in (2), $\frac{x}{y} = 2 \tan \alpha.$

$$\therefore e = \frac{\mathbf{ax}}{2\mathrm{y}}.$$

Thus the deformation e (Fig. 88) can be calculated from the observed values of x and y with the known length of the lever a. The distance y is so chosen that a suitable value of x is given.

Selection of extensometers. The choice of an extensometer suitable for a particular field of testing is one to which considerable thought should be given. For routine testing in the materials laboratory of a factory, the extensometer should be of a robust type giving clear and easily read observations within the limitations of accuracy demanded by the test. For this work a highly skilled operator is seldom employed and the limitations of the operator must be considered.

Technical schools and colleges generally possess several types of extensometers, and the more delicate and accurate types are only released for operation by senior students taking work of a high standard. The mirror type of extensometer and the more delicate of the mechanical types are essentially the instruments of the factory research laboratories, the more advanced technical students and other cases where very accurate results are demanded and the operators are skilled in the setting up and use of delicate instruments.

Representative Mechanical Tests

The following is a series of representative tests in which various materials are tested by straining, the observations tabulated and essential derived results obtained.

Tensile tests.

Test No. 1. Mild steel.

Diameter of specimen = 0·565 in.

Area of cross-section = 0·25 sq. in.

Gauge length = 2 in.

Observations.

Ultimate load = 8 tons.

Yield load = 5 tons.

Maximum elongation = 0·46 in.

Load in tons -	1	2	3	4	5	5·5	6·0	6·5	7·0	7·5	8·0	7·6	7·2
Elongation in inches	0·0006	0·0013	0·002	0·0025	0·003	0·035	0·055	0·08	0·1	0·16	0·39	0·44	0·46

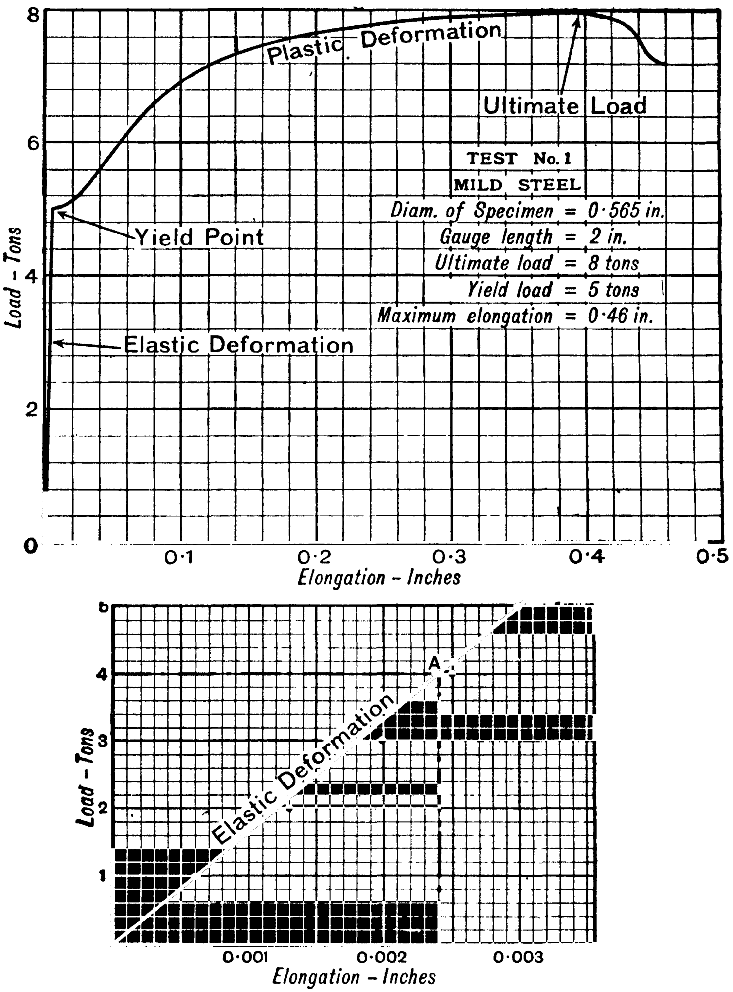

8
6
4
2
0
Load - Tons
0·1
0·2
0·3
0·4
0·5
Elongation - Inches
Plastic Deformation
Ultimate Load
TEST No. 1
MILD STEEL
Diam. of Specimen = 0·565 in.
Gauge length = 2 in.
Ultimate load = 8 tons
Yield load = 5 tons
Maximum elongation = 0·46 in.
Yield Point
Elastic Deformation
5
4
3
2
1
A
Load - Tons
Elastic Deformation
0·001
0·002
0·003
Elongation - Inches

FIG. 89.

DERIVED RESULTS.

(*a*) *Ultimate tensile stress* $=\dfrac{\text{ultimate load}}{\text{cross-sectional area}}=\dfrac{8}{0{\cdot}25}$

$=$ **32 tons per sq. in.**

(*b*) *Yield stress* $=\dfrac{\text{yield load}}{\text{cross-sectional area}}=\dfrac{5}{0{\cdot}25}=$ **20 tons per sq. in.**

(*c*) *Percentage elongation* $=\dfrac{\text{max. elongation}\times 100}{\text{gauge length}}=\dfrac{0{\cdot}46\times 100}{2}$

$=$ **23%**

(*d*) *Modulus of elasticity*, at A on the elastic curve, p. 151.

Stress $=\dfrac{4}{0{\cdot}25}=16$ tons per sq. in. Strain $=\dfrac{0{\cdot}0024}{2}=0{\cdot}0012.$

Modulus of elasticity $=\dfrac{\text{stress}}{\text{strain}}=\dfrac{16}{0{\cdot}0012}$

$=$ **13,333 tons per sq. in.**

Test No. 2. High tenacity non-ferrous alloy.

Diameter of specimen = 0·357 in.
Area of cross-section = 0·1 sq. in.
Gauge length = 2 in.

OBSERVATIONS.

Ultimate load = 5·15 tons.
Maximum elongation = 0·8 in.

Load in tons - -	0·5	1·0	1·5	2·0	2·25	2·5
Elongation in inches -	0·0009	0·0018	0·0027	0·0036	0·0045	0·006

Load in tons - -	2·75	3·0	3·25	3·5	3·75	4·0
Elongation in inches -	0·018	0·05	0·07	0·1	0·13	0·18

Load in tons - -	4·25	4·5	4·75	5·0	5·15	4·6
Elongation in inches -	0·23	0·32	0·4	0·53	0·64	0·8

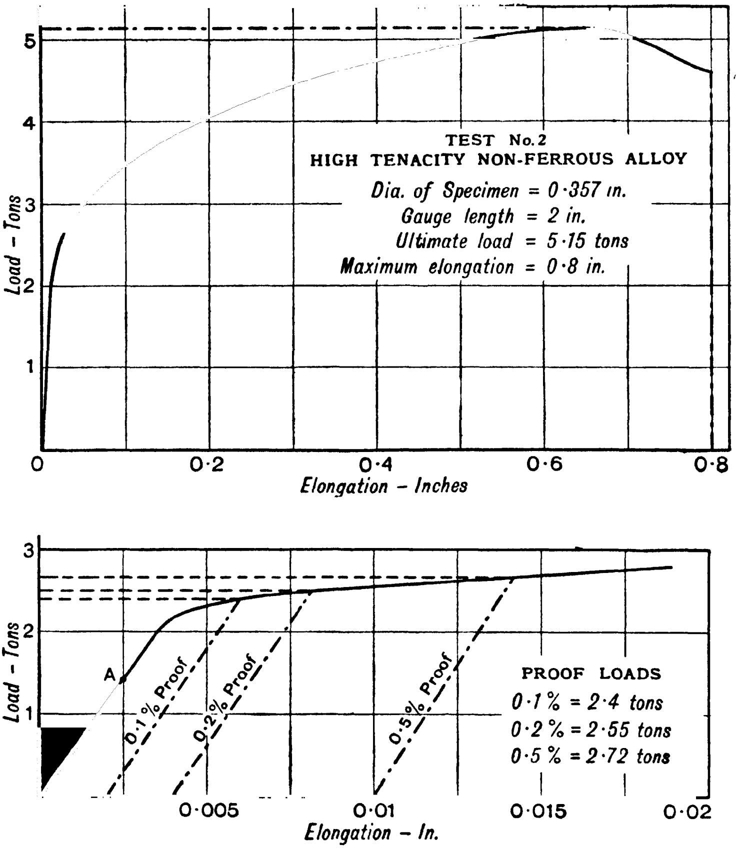

TEST No. 2
HIGH TENACITY NON-FERROUS ALLOY
Dia. of Specimen = 0·357 in.
Gauge length = 2 in.
Ultimate load = 5·15 tons
Maximum elongation = 0·8 in.
Load - Tons
Elongation - Inches
0
0·2
0·4
0·6
0·8
1
2
3
4
5
A
0·1 % Proof
0·2 % Proof
0·5 % Proof
PROOF LOADS
0·1 % = 2·4 tons
0·2 % = 2·55 tons
0·5 % = 2·72 tons
Load - Tons
Elongation - In.
0·005
0·01
0·015
0·02
1
2
3

Fig. 90.

DERIVED RESULTS.

(a) *Ultimate tensile stress:* $= \frac{5{\cdot}15}{0{\cdot}1} = 51{\cdot}5$ **tons per sq. in.**

(b) *Proof stresses:*
$$\begin{cases} 0{\cdot}1 \text{ per cent. proof} = \frac{2{\cdot}4}{0{\cdot}1} = 24 \text{ tons per sq. in.} \\ 0{\cdot}2 \text{ per cent. proof} = \frac{2{\cdot}55}{0{\cdot}1} = 25{\cdot}5 \text{ tons per sq. in.} \\ 0{\cdot}5 \text{ per cent. proof} = \frac{2{\cdot}72}{0{\cdot}1} = 27{\cdot}2 \text{ tons per sq. in.} \end{cases}$$

(c) *Percentage elongation on a 2 in. length* $= \frac{0{\cdot}8}{2} \times 100 = 40$ **per cent.**

(d) *Modulus of elasticity* at A on the elastic deformation curve.

$$\text{Stress} = \frac{1{\cdot}4}{0{\cdot}1} = 14 \text{ tons per sq. in.} \quad \text{Strain} = \frac{0{\cdot}0025}{2} = 0{\cdot}00125, \text{ p. 153.}$$

$$E = \frac{\text{stress}}{\text{strain}} = \frac{14}{0{\cdot}00125} = 11{,}200 \text{ tons per sq. in.}$$

Test No. 3. High tenacity alloy steel.

Diameter of specimen = 0·252 in.

Area of cross-section = 0·05 sq. in.

Gauge length = 2 in.

OBSERVATIONS.

The complete load *v.* elongation curve is produced by an autographic recorder, and the following observations are taken by extensometer over the elastic period and sufficiently beyond to give the proof stresses.

Load in tons	1	1·5	2·0	2·5	3·0	3·25
Elongation in inches	0·003	0·005	0·006	0·008	0·009	0·011

Load in tons	3·5	3·75	4·0	4·25	4·5	4·75
Elongation in inches	0·016	0·023	0·035	0·05	0·074	0·1

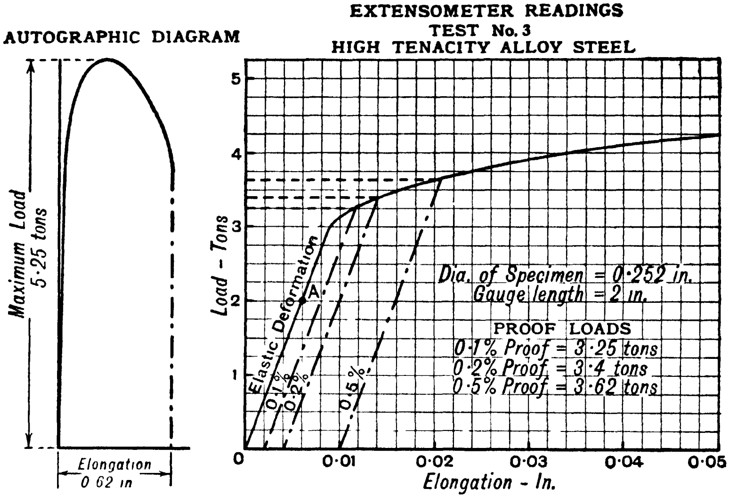


FIG. 91.

DERIVED RESULTS.

(*a*) From autographic diagram.

$$\textit{Ultimate tensile stress} = \frac{5{\cdot}25}{0{\cdot}05} = \mathbf{105\ tons\ per\ sq.\ in.}$$

$$\textit{Percentage elongation on a 2 in. length} = \frac{0{\cdot}62}{2} \times 100 = \mathbf{31\ per\ cent.}$$

(*b*) From extensometer readings :

$$\textit{Proof stresses :}\ \begin{cases} 0{\cdot}1 \text{ per cent. proof} = \dfrac{3{\cdot}25}{0{\cdot}05} = \mathbf{65\ tons\ per\ sq.\ in.} \\ 0{\cdot}2 \text{ per cent. proof} = \dfrac{3{\cdot}4}{0{\cdot}05} = \mathbf{68\ tons\ per\ sq.\ in.} \\ 0{\cdot}5 \text{ per cent. proof} = \dfrac{3{\cdot}62}{0{\cdot}05} = \mathbf{72{\cdot}4\ tons\ per\ sq.\ in.} \end{cases}$$

Modulus of elasticity. At A on the elastic deformation curve above :

$$\text{Stress} = \frac{2}{0{\cdot}05} = 40 \text{ tons per sq. in.} \quad \text{Strain} = \frac{0{\cdot}006}{2} = 0{\cdot}003.$$

$$\mathsf{E} = \frac{\text{stress}}{\text{strain}} = \frac{40}{0{\cdot}003} = \mathbf{13{,}333\ tons\ per\ sq.\ in.}$$

Test No. 4. Non-ferrous metals.

These tests have been taken of several non-ferrous metals and are largely comparative of ultimate stresses and elongation. The yield point is indefinable and although, if extensometer readings are taken, proof stresses may be obtained, these values would not be very informative.

OBSERVATIONS AND DERIVED RESULTS.

Area of cross-section $=0{\cdot}25$ sq. in.

Ultimate tensile stresses.

Aluminium alloy $=\dfrac{1{\cdot}9}{0{\cdot}25}=7{\cdot}6$ **tons per sq. in.**

Soft copper $=\dfrac{2{\cdot}92}{0{\cdot}25}=11{\cdot}68$ **tons per sq. in.**

Soft brass $=\dfrac{4{\cdot}2}{0{\cdot}25}=16{\cdot}8$ **tons per sq. in.**

Drawn brass $=\dfrac{5{\cdot}9}{0{\cdot}25}=23{\cdot}6$ **tons per sq. in.**

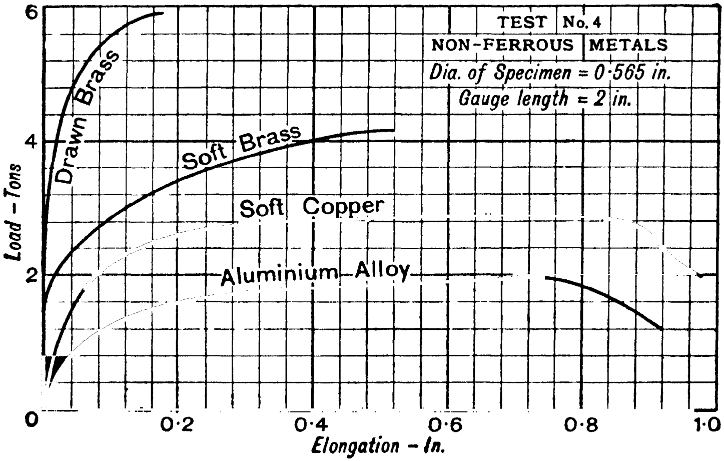


FIG. 92.

Percentage elongation on a 2 in. length.

Aluminium alloy $=\frac{0\cdot92}{2}\times 100=$ **46 per cent.**

Soft copper $=\frac{0\cdot98}{2}\times 100=$ **49 per cent.**

Soft brass $=\frac{0\cdot52}{2}\times 100=$ **26 per cent.**

Drawn brass $=\frac{0\cdot18}{2}\times 100=$ **9 per cent.**

NOTE.—The values of ultimate tensile stress and percentage elongation vary considerably with the composition and treatment of non-ferrous materials, and wide variation may be expected under test.

Compression tests.

Test No. 5.

These tests have been taken on a series of test pieces 1·128 in. in diameter and 2 in. in length. The curves are representative. Cast iron shows an almost straight line on the load *v.* compression curve until fracture ; steel gives a definite yield point followed by a slightly convex curve to fracture or failure ; the ductile metals give a convex curve turning to concave towards fracture.

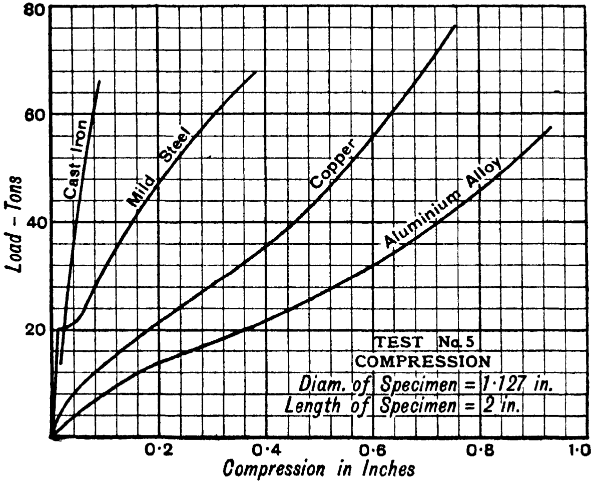


FIG. 93.

OBSERVATIONS AND DERIVED RESULTS.

Area of cross-section = 1 sq. in.

Ultimate compressive stress of cast iron = 63 tons per sq. in.

Destruction stresses for the more ductile metals.

Mild steel = 68 tons per sq. in.
Copper = 76 tons per sq. in.
Aluminium alloy = 56·5 tons per sq. in.

NOTE.—The value of the ultimate compressive stresses for the ductile metals is very unreliable, and it would be more correct to call them destruction stresses. Failure commences almost with loading in some cases, and the destruction stress is taken when the material cracks and fails physically.

Bending test.

Test No. 6. Mild steel.

This test was carried out on a centrally loaded beam of 24 in. span, depth 3 in., width $1\frac{1}{2}$ in.

OBJECTS.

(1) *To find the maximum stress set up in the material.*

(2) *To find the value of the modulus of elasticity for the steel.*

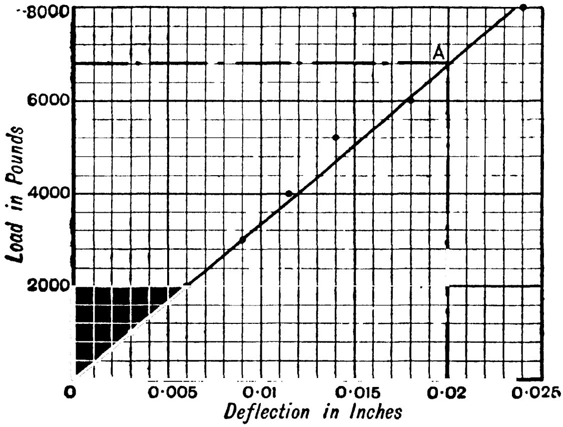


FIG. 94.

OBSERVATIONS.

Span = 24 in. ; depth = 3 in. ; width = $1\frac{1}{2}$ in.

Central load in pounds	1000	2000	3000	4000	5000	6000	7000	8000
Central deflection in inches	0·003	0·006	0·009	0·0115	0·014	0·018	0·021	0·024

DERIVED RESULTS.

(1) Maximum load = 8000 lb.

$$\text{Moment of resistance} = \frac{fbd^2}{6}. \quad \text{See p. 211.}$$

$$\text{Maximum bending moment} = \frac{WS}{4} = \frac{8000 \times 24}{4}$$

$$= 48{,}000 \text{ lb. in.}$$

$$\frac{fbd^2}{6} = 48{,}000 = f \times \frac{1{\cdot}5 \times 3^2}{6},$$

Max. stress $f = \dfrac{48{,}000 \times 6}{1{\cdot}5 \times 9} = \textbf{21,333 lb. per sq. in.}$

(2) $$\text{Deflection} = \frac{WS^3}{48E.I.},$$

but $$I = \frac{bd^3}{12} = \frac{1\frac{1}{2} \times 27}{12} = \frac{27}{8} \text{ in.}^4 \text{ units.}$$

At A on graph, p. 158, deflection = 0·02 in. = δ,

and load = 6800 lb. = W.

$$\text{Deflection} = \frac{WS^3}{48E.I.},$$

$$\delta = 0{\cdot}02 = \frac{6800 \times 24^3 \times 8}{48 \times E \times 27},$$

$$E = \frac{6800 \times 24^3 \times 8}{48 \times 27 \times 0{\cdot}02} = \textbf{29,013,000 lb. per sq. in.}$$

Rubber. Rubber is a material used to an increasing extent, both as the natural product and in its synthetic forms, in engineering design. The material does not conform to the standard conceptions of stressing and straining usually accepted in a study of strength of materials, but is a border-line material existing at normal temperatures in a state which invites consideration as a solid and a liquid at one and the same time. Its behaviour under stress obeys simultaneously the laws applicable to solids and also those applicable to viscous liquids without completely conforming to either, so that any investigation of its mechanical properties calls for a special technique founded on a knowledge of the mechanics of solids and fluids, with an adaptation to the peculiar behaviour of this material.

It has been fairly satisfactorily established that rubber does not fall completely into any of the accepted states of matter. It is not crystalline or amorphous, and is not strictly solid or liquid ; in fact its behaviour is not wholly elastic or wholly plastic.

Molecularly, rubber is regarded as being a material made up of long chains of molecules ; these chains, in the unstrained condition, assuming a curled up or spiral configuration. The application of stress tends to uncurl this configuration, and the elementary elasticity is the tendency for the strained curls to resume their normal shape on the cessation of loading. These molecular chains not only possess strong intermolecular forces holding them together and in formation, but also considerable cohesion between the chains to prevent internally slipping and shear failure.

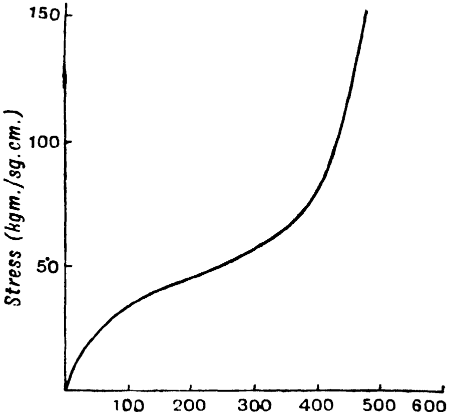


FIG. 95.

Stress v. strain curve. The stress *v.* strain curve follows Hooke's Law only for very small deformations ; there is no defined yield point, and plastic deformation is very difficult to divorce from elastic deformation. The stress-strain curve is also considerably affected by a rise or fall of temperature, and cyclic and non-cyclic stress variations. Fig. 95 shows a fairly typical curve for elongation *v.* stress.

Shear-strain diagram. The behaviour of rubber in shear more nearly approaches that of solid materials. Fig. 96 shows a typical shear *v.* strain diagram where the percentage shear strain is 100 tan ϕ and ϕ is the angular shear strain.

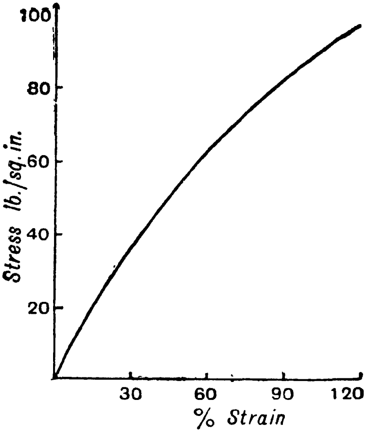


FIG. 96.

Sufficient has been said to warn the student against the application of his knowledge of strength of materials to the investigation of the performance of rubber ; and to invite him, if he has to deal with rubber, to study carefully the standard works on the subject before embarking on any tests likely to supply results for the purposes of design.

EXERCISES ON CHAPTER V

1. Give reasons why comprehensive mechanical tests are taken of the materials used in construction, and state the principal tests employed with their objects.

2. Explain the terms : " elastic deformation " ; " plastic deformation " ; " yield point " ; " limit of proportionality " ; " period of stricture " and " elastic limit " when applied to a test to destruction of a mild steel specimen in tension. Which of these terms have the same meaning?

3. Explain the terms " gauge length " and " percentage elongation " as applied to a tensile test. How are the stresses at Yield Point and the Ultimate Tensile Stress determined from the Load *v.* Elongation curve for a tension specimen? How would you determine the Yield Point if the material was mild steel, and what difference would you expect to find at this period on the graph if the material were an alloy steel?

4. How is the "modulus of elasticity" determined from a tensile test to destruction? What precautions would you observe before fixing upon the point on your graph from which this modulus could be calculated?

5. Explain the limitations of a compression test on a ductile material and state the precautions necessary in a compression test on cast iron to ensure that a true value is obtained for the ultimate compressive stress. Sketch the form of fracture for cast iron and state how failure probably occurred.

6. What is the purpose of an impact test? Why is it necessary to regard the results of this test in conjunction with the tensile test if the suitability of the material for certain purposes is being considered?

7. Explain the general theory of hardness testing, and show the reason why spherical and pyramid forms of indenting tools are used.

8. How is the Brinell test carried out? Explain the method used to obtain the Brinell number when charts are not available. What relation exists between the Brinell number and the ultimate tensile stress of the material? Can this relation be taken as reliable, or is it a relationship which may be used as a general guide?

9. Describe hardness tests other than the Brinell, and point out the advantages and disadvantages of each method when compared with the Brinell.

10. Prepare dimensioned sketches of impact test pieces (*a*) British Standard, (*b*) Air Board Standard. What are the difficulties experienced when carrying out impact tests on (*a*) materials possessing a tendency towards ductility, (*b*) materials suffering from faults in rolling such as lamination or excessively crystalline patches? How are these difficulties met?

11. What do you understand by "proof stress"? Name the specific reasons for the development of the use of proof stress in the technique of testing.

12. Describe with sketches each of the following:
   (*a*) Tensile testing machine of the hydraulic type.
   (*b*) Tensile machine of the lever type.
   (*c*) Torsion testing machine.
   (*d*) Impact machine of the Izod type.
   (*e*) Hardness testing machine depending upon indentation.

13. Describe, with detail sketches, a universal testing machine, and explain how a bending test may be performed on this machine.

14. A material is specified as follows: The minimum ultimate tensile strength to be 95 tons per sq. in. on static loading; maximum elongation on a 2 in. gauge length 28 per cent.; proof stress taken at 0·2 per cent.; 70 per cent. of ultimate tensile strength; hardness value on Rockwell "C" scale between 65 and 70. State what tests you would carry out on the material, as delivered, to satisfy yourself that the specification had been met.

15. What do you understand by " fatigue in metals "? Describe the basic tests which are carried out in order to determine the effects of alternating and repeated stresses on the ultimate tensile stress.

16. During a Brinell hardness test on a specimen of nickel chromo steel, a 10 mm. ball was used with a load of 2000 kg. Calculate the Brinell number for this steel if the diameter of the indentation was 3·12 mm.

17. A steel of Brinell hardness number 350 is to be checked for hardness. The indentation is limited to 2 mm. Find the maximum load which may be applied to a 10 mm. diameter ball.

18. Give a brief outline of the application of X-rays to the testing of materials for structural and other faults.

19. The following values of load and elongation on a 2 in. gauge length were taken for a high tenacity alloy steel specimen of 0·252 in. in diameter. Determine the proof stresses at 0·1, 0·2 and 0·5 per cent. elongation.

Load in tons	1	2	3	3·25
Elongation in inches	0·003	0·006	0·009	0·0115

Load in tons	3·5	3·75	4·0	
Elongation in inches	0·017	0·023	0·038	

CHAPTER VI

MACHINES WITH FRICTION. FRICTION IN BEARINGS. COIL FRICTION.

Friction. The lateral or tangential force which resists or tends to resist the sliding motion of one body over another is called the **force of friction.** Over a limited range of sliding speeds and normal pressures and between dry surfaces in contact, certain simple laws are approximately true. These laws are :

The force of friction F,

(1) is independent of the speed of the relative sliding motion ;

(2) is independent of the area in contact ;

(3) is proportional to the normal pressure R between the surfaces ; and

(4) varies with the nature and type of the surfaces in contact.

From (3), since F is proportional to R, then

$$F/R = \text{a constant} = \mu. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

This constant, μ, is called the **coefficient of friction** for the particular surfaces in contact. (See Vol. I, p. 103.)

When a tangential force is too weak to cause sliding, it is resisted by an equal and opposite tangential force sufficient to maintain equilibrium. This resisting force has a limiting value, and when equal to the applied force E the ratio $\frac{E}{R}$ then becomes the **coefficient of limiting friction** and is slightly higher than the value in equation (1) when motion is taking place.

Graphical interpretation. Fig. 97 shows a block of weight W resting on a horizontal table and subjected to a pull E, while at the same time the force of friction F is resisting motion and is adjusting itself to the value of E until motion takes place. At that moment, F has its maximum value. N is the normal reaction of the surface. The resultant of F and N is equal to R, and when F is a maximum, the angle between R and W is called the **angle of friction** and is denoted by ϕ. When motion takes place, and unless E and F are in the same straight line, then W and N will not be in the same straight line; and hence W and N will form a couple, equal and opposite in magnitude to the couple formed by E and F, to produce equilibrium. It is obvious from the force diagram that $F = N \tan \phi$, or $R \sin \phi$.

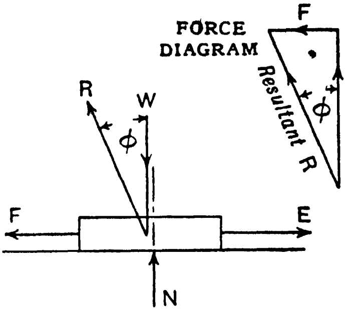


FIG. 97.

Body resting freely on a rough incline. A body will remain at rest on an inclined plane so long as the angle of the plane does not exceed the angle of friction (Fig. 98). This fact can easily be proved by experiment, and can be readily understood; because if the angle of the plane is equal to the angle of friction, the resultant R is unable to supply a component to balance W and so prevent sliding down the plane. If W is split into its two rectangular components parallel and at right angles to the plane, then, for equilibrium, $F = W \sin \theta$ and $N = W \cos \theta$. When θ is less than ϕ, then F is less than its limiting value; and when θ is greater than ϕ, then $W \sin \theta$ is greater than F, which has now reached its limiting value, $W \sin \phi$. In this case, the force producing acceleration down the plane is

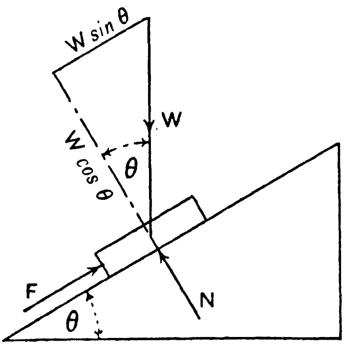


FIG. 98.

$$W \sin \theta - F = W \sin \theta - \mu N = W \sin \theta - \mu W \cos \theta$$
$$= W (\sin \theta - \mu \cos \theta). \quad \ldots\ldots\ldots(2)$$

Experimental determination of coefficient of friction. The angle of friction between any two given surfaces can easily be obtained by fitting an inclined plane with one surface and a slider with the other, and then finding the angle of inclination of the plane so that sliding without acceleration is obtained. If expression (2) is equated to zero, then $\mu = \tan\theta$, in which case θ is equal to ϕ, the angle of friction.

Example 1. *A case weighing* 10 *cwt. rests on a rough inclined plane for which* $\mu = 0{\cdot}3$. *Find* (1) *the force parallel to the plane necessary to maintain equilibrium if the angle of the plane is* 30° *to the horizontal.* (2) *the force acting parallel to the plane which will cause the case to accelerate down the plane inclined at* 45° *to the horizontal.*

(1) Force necessary to maintain equilibrium $= W(\sin\theta - \mu\cos\theta)$

$$= 1120(0{\cdot}5 - 0{\cdot}3 \times 0{\cdot}866)$$

$$= 269 \text{ lb.}$$

This force must be applied up the plane.

(2) Force available for producing acceleration $= W(\sin\theta - \mu\cos\theta)$

$$= 1120(0{\cdot}707 \times 0{\cdot}7)$$

$$= 554 \text{ lb.}$$

Example 2. *Find the greatest angle at which a conveyor belt can be inclined when it is used to move metal articles from one part of a factory to another, if the coefficient of friction between polished metal and belt has a limiting value of* $0{\cdot}28$.

If the articles are not to slide, the inclination of the belt must not exceed the angle of friction.

Angle of friction $= \tan^{-1} 0{\cdot}28 = 15^\circ\, 39'$.

Motion of a body on a rough incline. Suppose a force E is applied at an angle α to a body resting on a rough plane inclined at an angle θ (Fig. 99). When motion without acceleration is about to take place up the plane, the reaction R of the rough plane will act at an angle of friction ϕ to the normal to the plane in such a way as to give a component parallel to the plane opposing the direction of motion. The triangle of forces ABC is also shown in the figure. From the sine rule,

$$\frac{E}{\sin \mathrm{C}} = \frac{W}{\sin \mathrm{B}} = \frac{R}{\sin \mathrm{A}},$$

where $\sin \mathrm{A} = \sin\{90 - (\theta + \alpha)\} = \cos(\theta + \alpha)$, $\quad \sin \mathrm{C} = \sin(\theta + \phi)$,

and $\quad \sin \mathrm{B} = \sin\{\theta + \alpha + 90 - (\phi + \theta)\} = \sin\{90 - (\phi - \alpha)\} = \cos(\alpha - \phi)$.

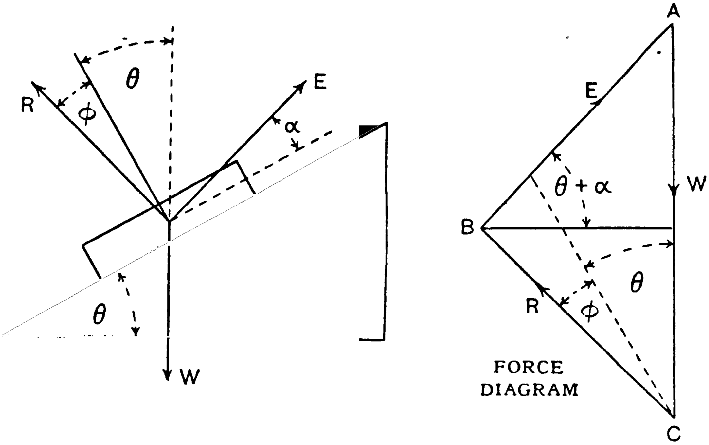


Fig. 99.

Substituting these values for the angles A, B and C, then

$$\frac{E}{\sin(\theta+\phi)}=\frac{W}{\cos(\alpha-\phi)}=\frac{R}{\cos(\theta+\alpha)}. \quad \text{..............}(3)$$

Thus,
$$E=\frac{W\sin(\theta+\phi)}{\cos(\alpha-\phi)}. \quad \text{..........................}(4)$$

The expression (4) is very important, and it may be adapted to meet the special cases of a rough incline as follows :

Case 1. Force E parallel to incline. In this case, $\alpha=0$ and (4) becomes

$$E=\frac{W\sin(\theta+\phi)}{\cos\phi}. \quad \text{..........................}(5)$$

Case 2. Force E applied horizontally. Here, $\alpha=-\theta$ and (4) becomes

$$E=\frac{W\sin(\theta+\phi)}{\cos(-\theta-\phi)}=W\tan(\theta+\phi). \quad \text{..................}(6)$$

Case 3. Minimum value of force E. In this case the denominator of (4) is a maximum and $\cos(\alpha-\phi)=1$, or $\alpha=\phi$.

Hence, minimum value of E is $W\sin(\theta+\phi)$. ....(7)

Case 4. Value of force E when used to pull a body down the plane. The angle of friction in this case is negative and (4) above becomes

$$E = \frac{W \sin(\theta - \phi)}{\cos(\alpha + \phi)} . \quad \text{.........................(8)}$$

Efficiency of a rough incline as a lifting machine. One definition of efficiency is given as the ratio, ideal effort/actual effort. Since expressions for the actual effort for the general case and the four mentioned above have now been determined in (4), (5), (6), (7), (8), and remembering for each case the ideal effort is obtained when $\phi = 0$, it is easy to calculate the efficiency.

Example 3. *Obtain the expression for the efficiency of a rough inclined plane when the effort is applied horizontally. Also determine when this efficiency is a maximum.*

Actual effort from (6) gives $E = W \tan(\theta + \phi)$.

Ideal effort occurs when $\phi = 0$, whence $E = W \tan \theta$.

Hence, $\quad$ efficiency $= \tan \theta / \tan(\theta + \phi)$. ....(9)

This expression may also be rewritten :

$$\text{Efficiency} = \frac{\sin\theta \cos(\theta + \phi)}{\cos\theta \sin(\theta + \phi)} = \frac{\sin(2\theta + \phi) - \sin\phi}{\sin(2\theta + \phi) + \sin\phi} . \quad \text{.........(10)}$$

For any given value of ϕ this has a maximum value when $\sin(2\theta + \phi)$ is a maximum, that is, when $2\theta + \phi = 90°$, or when θ, which is the slope of the plane, is $45° - \frac{1}{2}\phi$.

Substituting in (10), the expression for maximum efficiency is equal to

$$(1 - \sin\phi)/(1 + \sin\phi).$$

Example 4. *At a particular spot on an overhead transporter cable, the cable is inclined at* 20° *to the horizontal and the motive cable (which moves the load) is at an angle of* 30°. *If the load being pulled along the transporter cable, including the weight of pulley and cage, is* 10 *tons, find the tension in the motive cable, given* $\mu = 0{\cdot}007$. *What would be the tension in this cable if the load were being lowered at a uniform speed? Also, find the tension in this cable if this tension is a minimum up the plane?*

$$\phi = \tan^{-1} 0{\cdot}007 = 24', \; \alpha = 30° - 20° = 10°, \; \theta = 20°.$$

Using (4),
$$\begin{aligned} E &= W \sin(\theta + \phi)/\cos(\alpha - \phi) \\ &= 10 \sin 20° \, 24' / \cos 9° \, 36' \\ &= 3{\cdot}486/0{\cdot}986 = 3{\cdot}54 \text{ tons}. \end{aligned}$$

Using (8), $E = W \sin(\theta - \phi)/\cos(\alpha + \phi)$

$= 10 \sin 19^\circ\ 36'/\cos 10^\circ\ 24'$

$= 3{\cdot}355/0{\cdot}9836 = \mathbf{3{\cdot}41}$ **tons.**

Using (7), $E = W \sin(\theta + \phi) = 10 \sin 20^\circ\ 24' = \mathbf{3{\cdot}49}$ **tons.**

Friction of a screw. The square-threaded screw can be regarded as a rough inclined plane of square section wound around the cylindrical core of the screw to form a helix, and the expressions for mechanical advantage and efficiency can be applied. Thus, in the case of a screw jack, the screw working in a fixed nut is used to overcome an axial resistance, the screw being turned by means of an effort applied at the end of a long arm perpendicular to the axis. The angle of the thread or inclined plane is given by $\tan\theta = p/\pi d$, where d is the mean diameter of the thread of the screw and p the pitch. Since the effort is applied at right angles to the axis of the screw, case 2 of p. 167 is applicable, and remembering $\mu = \tan\phi$, expression (6) becomes

$$\frac{E}{W} = \tan(\theta + \phi) = \frac{\tan\theta + \tan\phi}{1 - \tan\theta\tan\phi} = \frac{p/\pi d + \mu}{1 - \mu p/\pi d}$$

$$= \frac{p + \mu\pi d}{\pi d - \mu p}. \quad \text{.................................(11)}$$

Expression (9) becomes,

$$\text{efficiency} = \tan\theta/\tan(\theta + \phi)$$

$$= \frac{p - \mu p^2/\pi d}{p + \mu\pi d}. \quad \text{..............................(12)}$$

When the screw jack is used for lowering, expression (8) becomes applicable, in which $\alpha = -\theta$ and

$$E/W = \tan(\theta - \phi).$$

When the angle of the plane is equal to the angle of friction, that is, when $\theta = \phi$, then $E/W = \tan(\theta - \phi) = \tan 0 = 0$. When θ is less than ϕ, then E/W becomes negative and an effort will be required to lower the load. Again, when θ is greater than ϕ, an effort will be required to prevent the load running down.

The necessity for a supporting effort thus begins when $\theta = \phi$. At this *critical value*, we have:

$$\text{Efficiency} = \tan\theta/\tan 2\phi = \tan\phi/\tan 2\phi = \tfrac{1}{2}(1 - \tan^2\phi)$$

$$= \tfrac{1}{2}(1 - \mu^2).$$

Thus, if μ has any value whatever, running down will commence when the effort is removed in such cases when the efficiency is rather less than 0·5. It will be shown later that this result is generally true with other machines.

It is also important to notice that it is better to reduce the value of ϕ by efficient lubrication than to decrease θ by reducing the pitch.

Example 5. *Calculate the efficiency and velocity ratio of a screw jack if the pitch is $\frac{1}{4}$ in., the mean diameter of the screw is 2 in. and the coefficient of friction is $\frac{1}{3}$. The effort is applied at right angles at the end of an 18-inch arm.*

$$\text{Velocity ratio} = \frac{2\pi R}{p} = 144\,\pi.$$

$$\begin{aligned}\text{Efficiency} &= \frac{p - \mu p^2/\pi d}{p + \mu \pi d}\\ &= \frac{1/\mu - p/\pi d}{1/\mu + \pi d/p} = \frac{3 - 1/8\pi}{3 + 8\pi} = 0{\cdot}105 \text{ or } 10{\cdot}5 \text{ per cent.}\end{aligned}$$

Triangular screw thread. If the cross-section of the thread is not square, the force of friction is increased by the double inclination of the thread. The axial load W has one component perpendicular to the axis of the screw and the other normal to the surface of the thread. If ψ is half the angle of the thread, the normal pressure will be $W \sec \psi$. The frictional resistance is directly proportional to this normal pressure, and thus the coefficient of friction is increased to $\mu \sec \psi$. Hence formulae (11) and (12) can be modified to suit triangular screw threads by writing $\mu \sec \psi$ for μ and taking an effective angle of friction of $\phi = \tan^{-1} \mu \sec \psi$.

Example 6. *A Vee thread has an angle of 55° and the coefficient of friction between the nut and screw is 0·21. Find the value of the effort required to exert a pressure of 500 lb. by means of a screw with this thread which has a mean diameter of 3·6 in. and a pitch of $\frac{1}{4}$ in., and a handwheel 24 in. in diameter.*

$$\begin{aligned}\phi = \text{Effective angle of friction} &= \tan^{-1} \mu \sec \psi\\ &= \tan^{-1} 0{\cdot}21 \sec 27\tfrac{1}{2}^\circ\\ &= \tan^{-1} 0{\cdot}2368 = 13^\circ\ 19'.\end{aligned}$$

$$\tan\theta = 1/(4\pi \times 3{\cdot}6) = 1/14{\cdot}4\pi = 0{\cdot}0221 \text{ and } \theta = 1^\circ\ 16'.$$

$$E = \frac{1{\cdot}8}{12} \times 500 \tan(\theta + \phi) = 75 \tan 14^\circ\ 35' = 19{\cdot}5 \text{ lb.}$$

Efficiency of a lifting machine. A considerable amount of experimental work has been given in Vol. I, Chap. VII, with the object of determining the efficiency of a machine at varying loads. The law connecting load and effort may be taken as

$$E = mL + c,$$

determinable from the graph of load *v.* effort; while the efficiency is the ratio between mechanical advantage and velocity ratio (p. 9). It therefore follows that since Mechanical advantage = Load/effort,

then $$\text{Efficiency} = M.A./V.R. = \text{Load}/(\text{effort} \times V.R.)$$

$$= \frac{L}{(mL + c)\,V.R.} = \frac{1}{(m + c/L)\,V.R.}.$$

Obviously the efficiency is a maximum when L is infinite, and the maximum efficiency is $1/m \times V.R.$

Example 7. *The law of a machine is* $E = 0{\cdot}28L + 3{\cdot}7$. *Find the maximum value of the efficiency if the velocity ratio is* 6.

Max. efficiency $= 1/(m \times V.R.) = 1/(0{\cdot}28 \times 6)$ - 59 5 per cent.

Reversal of a machine. If it be assumed that the frictional resistance in a machine is the same both normally and in reverse, and if the effect of the weight of chains and containers for the load are neglected, it can be shown that a machine can run back in the opposite direction solely under the action of the load, that is, be capable of reversal, if the efficiency exceeds a value in the region of 50 per cent.

When in a machine an effort E lb. raises a load L lb. a distance of 1 ft., the useful work done is L ft. lb. If the velocity ratio of the machine is V, then the work put into the machine is EV ft. lb., and the energy lost in friction is $EV - L$ ft. lb. In order that the machine may run down, L must be greater than $EV - L$.

Adding L to each side of the inequality,

$$2L > EV;$$

dividing each side of the inequality by $2EV$,

then $$L/EV > \tfrac{1}{2}.$$

Actually, in practice, the figure is rather less than $\tfrac{1}{2}$, because the frictional resistance is not quite as assumed above.

Friction in bearings. When a shaft turns in an ordinary cylindrical bearing, the reaction, owing to friction between the shaft and the bearing, will not pass through the centre of the shaft, but the line of reaction will be inclined to the normal at the point of contact at an angle equal to the angle of friction. Fig. 100 illustrates this point, which shows that for clockwise motion contact occurs at point X, to which the point of contact rises when motion occurs. For equilibrium, $R=W$ and the angle between the line of action of R and OX is equal to ϕ. The anticlockwise couple acting against rotation is $Wr \sin \phi$ or $Rr \sin \phi$.

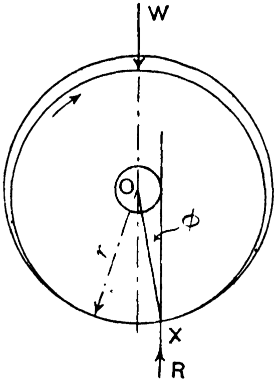


FIG. 100.

Since $\sin \phi$ is small, it may be taken as equal to $\tan \phi$ or μ; and hence

$$\begin{aligned}\text{Work done per rev. against friction} &= \text{moment of couple} \times \text{radians per rev.}\\ &= 2\pi Wr \sin \phi\\ &= 2\pi \mu r W \text{ nearly.}\end{aligned}$$

The small circle of radius $r \sin \phi$ is known as the **friction circle** for the particular shaft and the given conditions of lubrication; and when relative motion is taking place, the thrust R is tangential to this friction circle on one side or the other according to the direction of motion.

In a bearing, lubrication is perfect when a continuous film of oil separates the shaft from its bearing. This ideal condition can most nearly be obtained when oil is pumped into the bearing at a pressure greater than that due to the external forces thrusting between the shaft and the bearing. In practice, there is generally only partial separation between shaft and bearing by a film of oil. Thus, the conditions often partly correspond to friction between two solids, but the presence of an oil film in places greatly reduces the risk of seizure which is likely to occur even with low bearing pressures in the case of chemically clean, dry metal surfaces.

Ball and roller bearings (see Vol. I, pp. 108-113). The resistance offered to rolling between two bodies results from the indentation of these bodies one by the other. There is also a certain amount of sliding between the surfaces. If these surfaces are hard, the indentation, the areas in contact and the resistance to rolling are less than with softer surfaces. With the latter, the yield is considerably greater and the areas in contact, the relative sliding motion and the resistance to rolling are greatly increased.

In roller (or ball) bearings, the natural small resistance to rolling is ensured by the very hard surfaces given by a special process to the rollers (or ball bearings) and the bearing surfaces, and the use of these types of bearings is becoming more and more common for both journal and thrust bearings. The load or thrust is transmitted through a series of rollers (or ball bearings) loosely held in a cage. Cages, often made of gun metal, ensure that the rollers or balls, as the case may be, are maintained at a predetermined distance apart, and special greases are employed to reduce wear by corrosion and to exclude dust.

Coil friction. To determine the power which can be transmitted by belting or ropes, it is necessary to obtain the relation existing between the tension on the tight and slack sides of a pulley. This can be determined theoretically as follows :

Suppose Fig. 101 represents a driving pulley rotating anti-clockwise, with θ the angle of the arc of contact in radians and μ the coefficient of friction between the belt and the pulley. Consider an elemental piece of belt subtending an angle $\delta\theta$ at the pulley centre

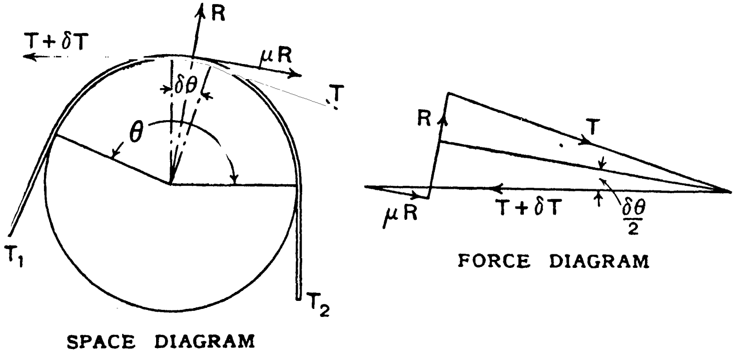


Fig. 101

and the forces acting on it. The forces are the tensions T and $T + \delta T$ at the two extremities of the portion of the belt, the reaction R and the force of friction μR opposing any possible slip and acting at right angles to R. Since the forces are in equilibrium, the force polygon will close. Resolving all the forces first tangentially, that is, parallel to μR:

$$\mu R = (T + \delta T) \cos \tfrac{1}{2}\delta\theta - T \cos \tfrac{1}{2}\delta\theta = \delta T \cos\tfrac{1}{2}\, \delta\theta.$$

Resolving radially, that is, parallel to R,

$$R = T \sin \tfrac{1}{2}\delta\theta + (T + \delta T) \sin \tfrac{1}{2}\delta\theta = 2T \sin \tfrac{1}{2}\delta\theta,$$

since $\delta T \sin \frac{1}{2}\delta\theta$ can be neglected. Also, as $\delta\theta$ is small, then $\sin \frac{1}{2}\delta\theta$ may be taken as $\frac{1}{2}\delta\theta$ in radian measure and $\cos \frac{1}{2}\delta\theta$ may be assumed to be unity.

The expressions thus become $\mu R = \delta T$ and $R = T\delta\theta$.

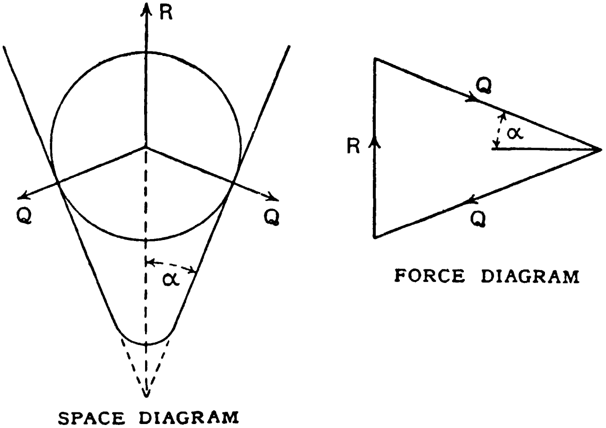
Dividing one by the other and integrating between the corresponding limits T_1 and T_2 and θ and zero,

$$\int_{T_2}^{T_1} \delta T/T = \int_0^{\theta} \mu\delta\theta, \text{ which gives } \log_e (T_1/T_2) = \mu\theta. \quad \text{.........(13)}$$

In this result, θ is in radians and hyperbolic or Naperian logarithms are used. The result may also be written,

$$T_1 = T_2\, e^{\mu\theta} \text{ and } 2{\cdot}3 \log_{10} (T_1/T_2) = \mu\theta. \quad \text{...............(14)}$$

When ropes are used, the pulleys must be grooved, and in this case (Fig. 102) the normal reaction R is replaced on each surface by



SPACE DIAGRAM

FORCE DIAGRAM

FIG. 102.

a force Q, where $Q = \frac{1}{2}R$ cosec α and 2α is the angle of the groove. The resistance to slipping is thus $2\mu Q$ or μR cosec α, so that compared with a flat belt the coefficient of friction may be considered to be increased to μ cosec α. Therefore, in the case of a rope drive, expression (14) can be written

$$2{\cdot}3 \log_{10} (T_1/T_2) = \mu\theta \operatorname{cosec} \alpha. \ldots\ldots\ldots\ldots\ldots\ldots(15)$$

Centrifugal tension in belts and ropes. When the part of a belt or rope is moving in the path of the arc of a circle, it is subjected to centrifugal tension. This is negligible at low speeds, but at 40 feet per second it becomes important, while at a belt speed of about 1700 feet per second this tension will approximate to the strength of the belt.

EXPT. 4. OBJECT. *To verify the relationship* $\log_e (T_1/T_2) = \mu\theta$ *for a belt or cord passing around a pulley or drum.*

APPARATUS. The apparatus consists of a wall-board fixed vertically with a hollow cylindrical drum screwed to the board. Three

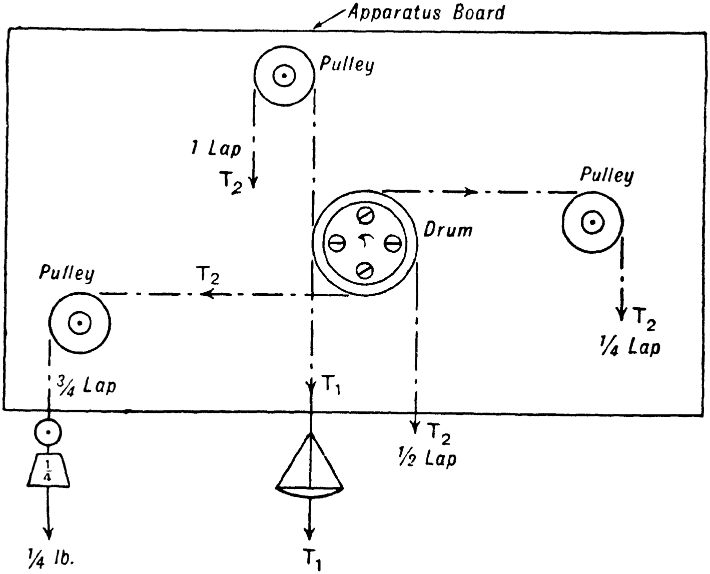


FIG. 103.

pulleys are fastened to the boards so that $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, 1, $1\frac{1}{4}$, $1\frac{1}{2}$, etc. lap can be conveniently obtained when a cord or belt is passed over the drum and weights are attached at each end.

METHOD OF PROCEDURE. In the experiment T_2 is maintained constant at 0·38 lb., and weights are added in the pan T_1 until the cord just begins to slide on the drum. The angles of contact θ are taken as $\frac{1}{2}\pi$, π, $\frac{3}{2}\pi$, etc. radians and the values of T_1 tabulated for each case. A graph of $\log_e (T_1/T_2)$ is plotted against θ and a straight line is obtained.

OBSERVATIONS. $T_2 = 0{\cdot}38$ lb.

Angle of lap in radians θ, -	$\frac{1}{2}\pi$	π	$1\frac{1}{2}\pi$	2π	$2\frac{1}{2}\pi$	3π	$3\frac{1}{2}\pi$
T_1 in lb., - - - -	0·53	0·6	0·78	0·98	1·28	1·48	2·08
$\log_e T_1/T_2$, - - -	0·33	0·46	0·7	20·95	1·21	1·36	1·70

DERIVED RESULTS. When the straight-line graph $\log_e (T_1/T_2)$ *v.* θ is obtained, the slope will be found to be 0·155. Hence, the coefficient of friction μ for this experiment is 0·155.

Horse-power transmitted by a belt.

Horse-power $= \dfrac{(T_1 - T_2)\,V}{550}$, where T_1 and T_2 are equal to the tensions on the tight and slack sides of the belt respectively, and V is the speed of the belt in feet per second. Substituting $T_2 = T_1/e^{\mu\theta}$ from (14),

$$\text{Horse-power} = \frac{T_1(1 - 1/e^{\mu\theta})\,V}{550}. \quad \ldots\ldots\ldots\ldots\ldots\ldots(16)$$

The same formula can be used for a rope drive if μ cosec α be substituted for μ, α being half the angle of the rope groove.

Example 8. *Find the width of belt necessary to transmit* 10 *H.P. if the belt speed is* 25 *ft. per sec. and* $\mu = 0{\cdot}2$. *Allow* 100 *lb. as the permissible tension per inch width of belt and the angle of contact between belt and pulley as* 180°.

$$\text{Horse-power} = (T_1 - T_2)\,V/550.$$
$$10 = (T_1 - T_2)\,25/550.$$
$$\therefore\ T_1 - T_2 = 220 \text{ lb.}$$

Using $2{\cdot}3 \log (T_1/T_2) = \mu\theta$, $\log (T_1/T_2) = 0{\cdot}2\pi/2{\cdot}3$ and $T_2 = 0{\cdot}533T_1$.

Thus, $\quad T_1 - 0{\cdot}533T_1 = 220$ and $T_1 = 471$ lb.

$\therefore$ Width of belt required $= \frac{471}{100} = 4{\cdot}7$, or say $4\frac{3}{4}$ in.

N.B.—Centrifugal tension in the above question is neglected.

Example 9. *A hemp rope is used in mooring a ship alongside a quay. How many turns of the rope must be passed around a bollard in order that a 5-ton load on the tight side may be held by a sailor using a force of* 50 *lb. on the slack side?* $\mu = 0{\cdot}4$.

Let n = no. of turns required, and use $2{\cdot}3 \log (T_1/T_2) = \mu\theta$.

Here $\theta = 2\pi n$; $\mu = 0{\cdot}4$; $T_2 = 50$ lb. ; $T_1 = 11{,}200$ lb.

Then $2{\cdot}3 \log (11{,}200/50) = 0{\cdot}4 \times 2\pi n$.

$$n = \frac{2{\cdot}3 \times 2{\cdot}35}{0{\cdot}4 \times 2\pi}$$

$$= 2{\cdot}15 \text{ or } 2\tfrac{3}{20} \text{ turns.}$$

N.B.—It is assumed that any one turn does not touch the rope in the turn adjacent to it.

Friction of a collar-bearing. Assuming that the thrust P is evenly distributed over the whole surface of the collar, the intensity of pressure

$$p = \frac{P}{\pi (r_1^2 - r_2^2)} .$$

The pressure on an elementary ring of radius x and width δx is $2\pi x p \delta x$. The moment of friction on this elementary ring is

$$2\pi x^2 \mu p \delta x,$$

and the turning moment on the whole surface is

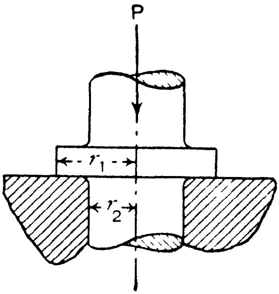


FIG. 104.

$$2\pi\mu p \int_{r_2}^{r_1} x^2 \,.\, \delta x = \tfrac{2}{3}\pi\mu p (r_1^3 - r_2^3). \quad \ldots\ldots\ldots\ldots\ldots\ldots(17)$$

Equation (17) becomes $\frac{2}{3}\pi\mu p r_1^3$ in the case of an ordinary flat pivot with no collar, where r_1 is the radius of a shaft turning in a socket of the same diameter.

Example 10. *The inside and outside diameters of a collar-bearing are* $2\frac{1}{2}$ *in. and* 4 *in. respectively, and the total load on the vertical shaft which is carried by the collar is* $\frac{1}{4}$ *ton. Taking* $\mu = 0{\cdot}1$ *and the speed of the shaft as* 90 *r.p.m., calculate the horse-power absorbed in friction.*

Intensity of pressure $p = 560/\pi (2^2 - 1\tfrac{1}{4}^2)$ lb./in.^{2}

$$= 73{\cdot}13 \text{ lb./in.}^2$$

$$
\begin{aligned}
\text{Moment of friction} = M &= \tfrac{2}{3}\pi\mu p\,(r_1{}^3 - r_2{}^3) \\
&= \tfrac{2}{3}\pi \times 0{\cdot}1 \times 73{\cdot}13\,(2^3 - 1\tfrac{1}{4}{}^3)\ \text{lb. in.} \\
&= 92{\cdot}6\ \text{lb. in.} \\
\text{Horse-power absorbed} &= 2\pi MN/33{,}000 \times 12 = 0{\cdot}13.
\end{aligned}
$$

EXERCISES ON CHAPTER VI

1. Find the force, parallel to the plane, necessary to keep a body weighing 2 tons at rest on a plane inclined at 30° to the horizontal if (*a*) the plane is smooth, (*b*) if the plane is rough and the coefficient of friction $\mu = 0{\cdot}35$.

2. A machine which weighs 30 cwt. is being lowered at a uniform speed down a plane inclined at 25° by a rope which is parallel to the plane. If $\mu = 0{\cdot}3$, find the pull in the rope. Find the pull in the rope if the rope is inclined at 40° to the horizontal.

3. A wooden crate weighing 1 ton rests on an incline of 23° and the coefficient of friction between the surfaces is 0·4. When a crane cable is attached to the crate and hauled taut, it makes an angle of 75° with the horizontal. Calculate the least tension in the cable to cause the block to slide up the plane.

4. A sliding body weighing 240 lb. is pulled with uniform velocity up a plane inclined at 20° to the horizontal. Determine the effort necessary if it acts (*a*) parallel to the surface, (*b*) horizontally, (*c*) in the direction in which the effort is least. Coefficient of friction = 0·2.

5. If a block of stone weighing 2000 lb. can just be hauled up a steel incline of 12° to the horizontal by a force of 800 lb. applied in the most effective direction, calculate the value of the coefficient of friction and the direction in which the force is applied.

6. A solid cylinder 4 in. in diameter and 8 in. long is placed with its base on a rough plane which is raised so that its inclination to the horizontal is gradually increased. If $\mu = 0{\cdot}25$, find whether the cylinder will fall over or slide down the plane. Find the value of μ for such a plane and cylinder in order that the cylinder may just be on the point of overturning as the plane is gradually raised.

7. A crane has a velocity ratio of 300, and in a test the following values of load L and effort E were observed :

Load (lb.) -	1000	2000	3000	4000	5000
Effort (lb.) -	19	28	37·5	46	54·5

Obtain the equation connecting load and effort, and plot the load and efficiency curves. Calculate the upper limiting value of the efficiency which can be obtained with this crane.

8. The law connecting E and L for a certain pulley tackle is $E = 0{\cdot}405L + 2{\cdot}5$ lb. The velocity ratio is 4 to 1. Find (*a*) the equation to the efficiency curve, (*b*) the maximum efficiency of this machine, and (*c*) the efficiency when raising a load of 100 lb.

9. A pulley wheel weighing W lb. is placed with its shaft horizontal and supported by Vee blocks. Find the minimum turning moment required to turn the pulley if the shaft is d in. in diameter, the angle of the Vee blocks is 90° and the coefficient of friction is μ.

10. In a screw jack the pitch is 1 in., mean diameter of screw $2\frac{1}{2}$ in., angle of friction 12°. Neglecting collar friction, calculate the effort necessary at a radius of 18 in. to overcome an axial resistance of 2500 lb.

11. The effort in a screw jack is applied through a handle 20 in. in length, the screw is 2 in. in mean diameter and the pitch $\frac{1}{2}$ in. If $\mu = 0{\cdot}15$ and the load rotates with the screw, calculate the force required at the end of the handle to raise a load of 18 cwt.

12. A 6 in. stop valve designed to withstand a gauge pressure of 300 lb. per sq. in. has a spindle, 2 in. in diameter, with a square threaded screw of 4 threads per inch. Determine the turning moment necessary to rotate the spindle and the efficiency of the screw if $\mu = 0{\cdot}2$.

13. In a coupling for railway coaches the pitch of the left- and right-handed screw is $\frac{1}{8}$ in. A force of 50 lb. is applied at a radius of 16 in. to rotate the nut on the screw. Calculate the force drawing the coaches together if the efficiency of the screw is $33\frac{1}{3}$ per cent.

14. The mean diameter of a $1\frac{1}{4}$ in. bolt is 1·16 in., the angle of the thread is 55° and there are 7 threads per inch. If the coefficient of friction is 0·25, calculate the axial tightening-up force available with this bolt if a force of 45 lb. is being applied at the end of a spanner 18 in. long.

15. A rope fastening a ship to the dockside is coiled with seven complete turns round a bollard. If the pull on the rope from the vessel is 36,000 lb., find the pull necessary in the free end of the rope to prevent slipping. $\mu = 0{\cdot}2$.

16. The safe tension in a single-ply belt is 100 lb. per inch width, and the belt is transmitting 20 H.P. at 300 r.p.m. on a 24-in. diameter pulley. If the angle of lap is 270° and $\mu = 0{\cdot}3$, find the necessary width of belt.

17. An electrically-driven capstan at the top of an incline is used to draw loaded trucks up an incline of 10° with a coefficient of friction of 0·15. The maximum weight of the trucks is 3 tons ; the cable is passed round the capstan a number of times and a pull of 30 lb. is applied at the free end. Calculate the number of turns which must be made

round the capstan if μ for capstan and cable is 0·3 and the separate turns are not allowed to touch.

18. Ten ropes 2 in. in diameter are employed to drive a pulley 10 ft. in mean diameter running at 90 r.p.m. The angle of lap is 210°, the angle of the grooves 60° and $\mu = 0{\cdot}28$. The maximum pull in any one rope must not exceed 3500 lb. Calculate the maximum horse-power which can be transmitted.

19. The angle of contact in the case of a steel band-brake on a cylindrical brake drum (Fig. 105) is 240°. Find the value of T_1 and the necessary pull P at the end of the brake lever to hold a load L of 2 tons if $\mu = 0{\cdot}35$.

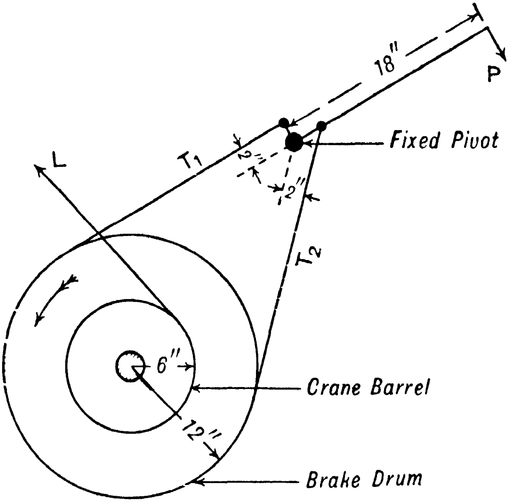


FIG. 105.

20. Find the horse-power absorbed in friction in the case of a vertical shaft supported by a collar bearing if the outside and inside radii are 2 in. and 3 in. respectively. The shaft speed is 180 r.p.m., the vertical thrust is 800 lb. and $\mu = 0{\cdot}15$.

21. A plate clutch having two friction surfaces is to transmit 60 H.P. at 2400 r.p.m. The internal diameter of the friction surfaces is 7 in. and the external $10\frac{1}{2}$ in. Calculate (*a*) the torque to be transmitted, (*b*) the necessary axial pressure between the surfaces, assuming the intensity of pressure to be constant. $\mu = 0{\cdot}3$. (U.L.C.I.)

CHAPTER VII

SIMPLE BRACED FRAMES UNDER NON-UNIFORM LOADING. RECIPROCAL FIGURES. METHOD OF SECTIONS. FORCES ACTING AT A POINT BUT NOT IN ONE PLANE

FRAMED or braced structures are widely employed because of the comparative ease with which a structure of the requisite strength may be built up with economy in weight of material.

A few examples of simple pin-jointed braced frames are shown in Fig. 106. It will be seen that a firm type of structure is obtained naturally when the members of the frame are arranged to form a triangle or a number of triangles, as in the case of 1 and 3. In 2 the number of members or links is deficient, and this particular linkage forms a mechanism and can only be converted to a structure by providing one or more stiff joints at the angular points.

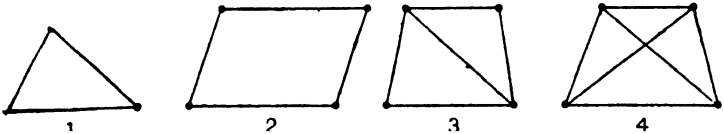


FIG. 106.

A simple way of determining whether a structure is firm or otherwise can be derived from the triangular make-up. If n is the number of nodes or joints and N the number of members, then :

the structure is firm if $N = 2n - 3$
the structure is underfirm if $N < 2n - 3$
the structure is overfirm if $N > 2n - 3$

In 4 there are more links or members than are necessary from the simply firm point of view, and the arrangement is said to be redundant or overfirm. Such redundancy does occur occasionally in practice where additional bracing is required, but good workmanship is essential in the construction to prevent initial stresses being set up between the various parts before any external loads are applied. The work of this chapter will refer to simply firm braced structures, as the methods employed for finding the forces in the members of redundant frames are beyond the scope of this book.

Forces in simple braced roof trusses and built-up girders. In drawing force diagrams for roof trusses and built-up girders, it is essential to remember that two kinds of forces have to be considered : (*a*) the *external forces*, that is, the forces due to external loads and reactions of the supports which must together be in equilibrium among themselves ; and (*b*) the *internal forces* induced into the various members of the structure by the action of the external forces.

With the external forces, the conditions of equilibrium of forces acting on a body set out in Chapter II must apply, the body in this case being a " braced structure ".

In the calculations for design it is generally assumed that :

(1) the structure is made up of members or bars pin-jointed together with frictionless pins, so that the internal forces act along the axes of the bars and are concurrent at the axis of the pins ;

(2) the external loads are concentrated at the joints, and in cases where the loads are not so given, the total of isolated loads and distributed loads on a bar must be split proportionally between the two ends.

By making use of these two assumptions, the calculated forces in the internal members usually exceed those actually acting, because in practice the joints are of a more rigid riveted type. Hence any error due to the assumption of pin-joints should be on the " safe side ".

Ties and Struts. The members of a structure are classified as **ties or struts** according to the nature of the forces in them.

A $\left\{\begin{matrix}\text{tie}\\ \text{strut}\end{matrix}\right\}$ is a member of a braced structure which is resisting $\left\{\begin{matrix}\text{tension}\\ \text{compression}\end{matrix}\right\}$ and is indicated in the space diagram by arrows pointing $\left\{\begin{matrix}\text{away from}\\ \text{towards}\end{matrix}\right\}$ the joints at the extremity of the member.

In order to find the magnitude of the forces acting in the various ties and struts comprising a braced structure, the external forces and reactions are first determined in such a way that these forces act only at the nodes or joints, and all the unknown forces are found by using the principles of equilibrium. Since it is assumed that all the internal forces act along the axes of the bars, and all the external forces and reactions have been calculated, it is possible to draw a vector force diagram for each joint or node. Generally, the first vector diagram is drawn for one of the supports, because there are usually sufficient known forces at this joint. The results obtained from the first diagram should give sufficient information to enable one to draw the vector diagram for one of the adjacent joints ; this process can be repeated until all the internal forces have been found.

In practice, the various vector diagrams are combined into what is known as a **reciprocal figure** as this gives a neater solution, especially for involved framed structures. When drawing a vector diagram it is customary to take the external and internal forces acting at each joint in proper order, starting with a known force and then proceeding to the next adjacent to it, and so on until the reciprocal figure is complete.

Example 1. *Fig. 107 shows a loaded girder for an overhead travelling crane. Draw the reciprocal figure for the given loading and tabulate the forces in the various members of the girder.*

The various steps in obtaining a solution may be as follows :

(*a*) Draw the space diagram to a suitable scale in such a position that room is allowed on the paper for the vector diagram.

(*b*) Letter the spaces in the space diagram according to Bow's notation (see p. 34) and Vol. I, p. 77.

(*c*) Calculate the unknown reactions. These reactions must be vertical to balance the vertical loads. As the bays are equal and 9 ft. long, take moments about the left-hand support,

$$W_2 \times 4 = 16 \times 3 + 12 \times 2, \text{ or } W_2 = 18 \text{ tons.}$$

Hence, $$W_1 = 16 + 12 - W_2 = 10 \text{ tons.}$$

(*d*) Now draw to a suitable scale the vector diagram *abcd* for the external forces and reactions, in which $ab = 12$ tons and $bc = 16$ tons, taking the vectors and letters in the clockwise order, that is, *ab*, *bc*, *cd*, *da*. In this case the vector diagram becomes a straight line called the

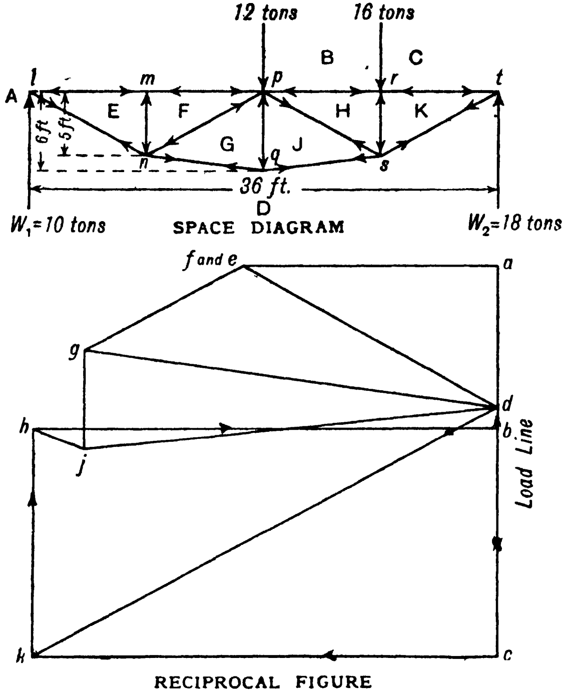


FIG. 107.

load line. Arrows are not inserted in this case as they would obviously cause confusion.

(*e*) Starting at the node *l*, draw the vector diagram *dae* to represent the forces acting at this node in the clockwise order *da*, *ae*, *ed* ; *ae* being parallel to AE and *ed* parallel to ED. The direction of the force *DA* is upwards, so that *ae* and *ed* in the diagram *dae* indicate that the forces acting at the node *l* are as shown in the space diagram. That is, AE is a strut and ED a tie. These stress indications are usually inserted on the space diagram at each stage as the reciprocal figure is completed. The directions of the forces for each vector polygon or triangle can be found by following round the polygon, starting with a force of which the direction is known.

Proceeding now to the adjacent node *m*, it will be seen that there can be no force in EF, because there is no load at *m* and the force in AE must balance the force in AF.

Proceeding next to node *n*, the vector triangle *defgd* is drawn and from it the nature of the forces acting at *n* can be indicated on the space diagram as before. Notice that as there is no force in EF, then *e* and *f* form one point.

In the same way, *gfabhjg* represents the vector polygon for node *p*, *dgjd* for node *q*, *djhkd* for node *s*, *khbck* for node *r*, and *dkcd* for node *t*. The reciprocal figure is now completed.

(*f*) Measure the lengths of the various vectors in the reciprocal figure to scale and tabulate the magnitudes of the forces and their natures as below.

AE	ED	EF	FA	FG	GD	GJ	JH	HB	JD	HK	KC	KD
17·5	20	0	17·5	13	29	7·5	4	32·5	29	16	32·5	37
S	*T*	—	*S*	*S*	*T*	*S*	*T*	*S*	*T*	*S*	*S*	*T*

S = Strut. *T* = Tie.

Notice that arrows are not inserted on the vectors in the reciprocal figure in order to avoid confusion.

Example 2. *A roof truss is shown in Fig.* 108 *with ordinary loading and a wind load of* 2000 *lb. normal to the left-hand side of the truss. This wind load of* 1000 *lb. on each bay is divided proportionally at the joints on this side of the truss. Determine the reactions of the supports and draw the reciprocal figure. Tabulate the forces acting in each member. The truss is supposed to be fixed at q and supported on rollers at s.*

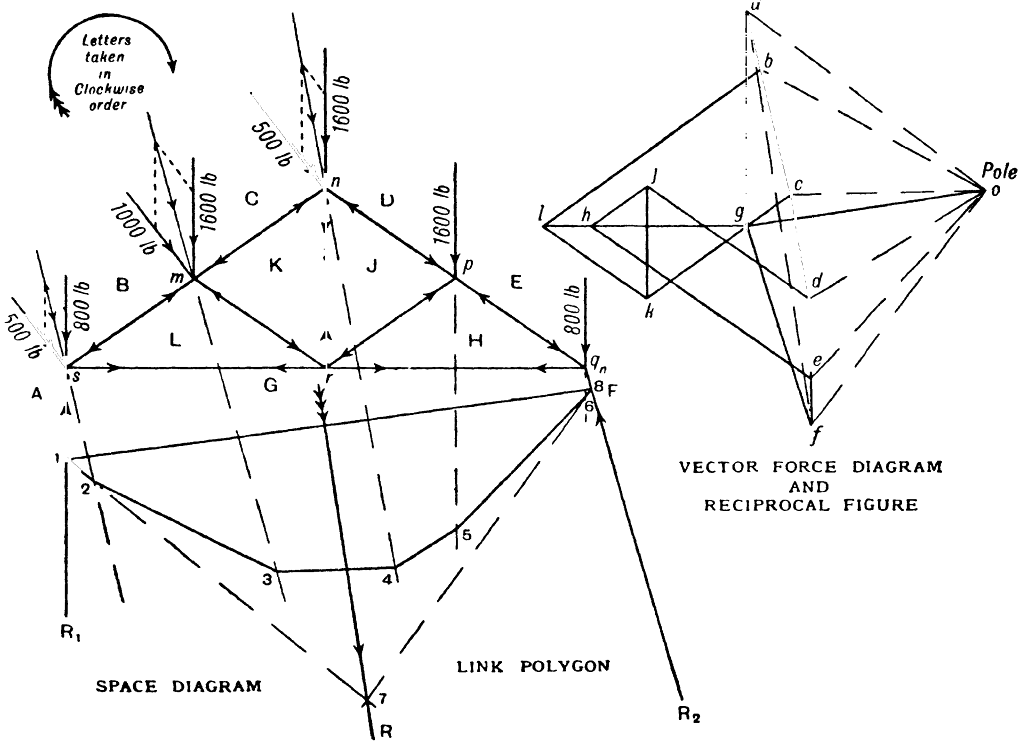
The steps in this purely graphical solution may be as under :

(*a*) Choose a suitable scale and draw the space diagram.

(*b*) Choose a suitable scale and find the resultant of the wind and dead loads acting at each of the joints S, *m* and *n*. The two forces acting at each of these joints can be combined to form one resultant force by representing the dead load and wind load by the two adjacent sides of a parallelogram and finding its diagonal according to the method of the parallelogram of forces.

(*c*) Name the various forces by lettering the spaces according to Bow's notation.

(*d*) *Find the value of the reactions R_1 and R_2.* Draw the vector force diagram *abcdef* for the external forces *AB, BC, CD, DE, EF*. The clos-



SPACE DIAGRAM

LINK POLYGON

VECTOR FORCE DIAGRAM AND RECIPROCAL FIGURE

Fig 108.

ing line *af* gives the magnitude and direction of the resultant. Choose a pole *o* and join *o* to the vertices *a*, *b*, *c*, *d*, *e*, *f*. Complete the link polygon 2, 3, 4, 5, 6, 7, in which 2, 3 is parallel to *ob*, 3, 4 parallel to *oc*, and so on. Draw lines from 2 and 6, respectively parallel to *oa* and *of*, to meet at 7, and the resultant *af* or R must pass through 7. To find the direction of the resultant at *q*, produce R and R_1 until they meet and join *q* to the same point of intersection. This latter line gives the direction of the reaction R_2, because the three external forces R, R_1 and R_2 acting on the roof truss must be in equilibrium and their lines of action meet in one point. In the force diagram draw from *f* a line parallel to R_2 and from *a* a line parallel to R_1, meeting the former line at *g*. Then *ga* represents R_1 in magnitude and direction, and *fg* similarly represents R_2.

As a check on the construction, produce 7, 6 to 8 on R_2 and 7, 2 to 1 on R_1 and join 1 to 8. Then 1, 8 should be parallel to *og*.

(*e*) The reciprocal figure to determine the magnitudes of the internal forces can now be drawn as in Example 1. Thus *gablg* represents the vector diagram for the node *s*, *lbchl* for node *m*, and so on. In addition, the arrows can be inserted on the members in the space diagram to indicate the direction of the forces at the joints and thus indicate which members are ties and which struts. Scale measurement of the reciprocal figure will give the magnitudes of the forces in the members of the truss.

Member -	BL	LG	CK	KL	KJ	DJ	JH	HG	EH
Force, lb. -	5040	3760	3380	2380	2160	3720	1320	2880	5040
Nature - -	*S*	*T*	*S*	*S*	*T*	*S*	*S*	*T*	*S*

Reaction $R_1 = AG = 4140$ lb. Reaction $R_2 = GF = 4040$ lb.

Wind loads. The following National Physical Laboratory formula gives the wind pressure in lb. per sq. ft. on a vertical wall in terms of the velocity of the wind :

$$P_v = \text{pressure in lb./ft.}^2 = 0\cdot0027 \times (\text{Velocity of wind in m.p.h.})^2.$$

A formula due to Hutton gives the normal pressure P_θ in lb. per sq. ft. on the windward side of a roof inclined at an angle θ degrees to the horizontal thus :

$$P_\theta = P_v \sin\theta^{1\cdot84\cos\theta-1}.$$

This formula makes no allowance for the suction effect on the leeward side, which is often considerable.

Often three separate reciprocal figures are drawn for any given roof truss for (1) wind load on the right-hand side, (2) wind load on the left-hand side, and (3) for the dead loads due to the weight of the structure and any loads suspended from it. Each reciprocal figure is then of a simple character. The results obtained from the three figures can be combined to give the maximum loads in each member. Usually in obtaining the forces in the members of a roof truss due to wind loads, only one end is assumed fixed to its support as in Example 2, p. 186, while the other is assumed to rest on rollers and be free to move (that is, with the reaction at the point taken as vertical). In every case the resultant wind load and the two reactions form three forces in equilibrium, and therefore their lines of action meet in one point, a fact enabling the direction of the reaction at the fixed end to be determined.

The Bailey Bridge. This bridge, made famous by its wide employment during the Second World War, is named after Sir Donald Bailey, of the Ministry of Supply, who took the leading part in its design and development. It is constructed from a set of interchangeable parts and in its simplest and lightest form can carry a load of 20 tons. The bridge is built up from sections 10 ft. long, each section consisting of seventeen standard parts while the foundations are constructed from nine other parts. Fig. 109 shows in outline the shape and size of a basic section which, it should be noted, is not of the simple frame type. Without pontoons it is possible to cross a gap up to 240 ft., and a much wider range if midstream supports or pontoons are available. The spans are erected in succession on rollers on the bank and pushed forward by the constructors.

The 11-ft. wide carriage-way of the near half of the bridge is completed first, because the weight of this helps to effect a balance as the bridge is pushed forward to touch down on the prepared foundation on the other side of the gap.

By doubling or trebling the simple basic construction, that is, by fixing sections like those in Fig. 109 side by side, it is possible to build a bridge to carry the heaviest tanks. Joints between the sections are secured by two-inch pins.

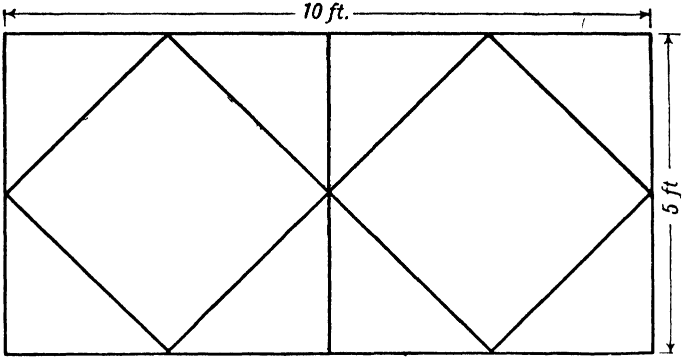


Fig. 109.

It is interesting to note that in Italy the Royal Engineers bridged the 300-ft. gap across the River Trigni with a Bailey Bridge in 36 hours. The Somerfeld portable track, so widely used because of its flexibility and strength for roads and runways on muddy or marshy ground, is also used on these bridges.

The " Butterley " Standard Bridge. This bridge is designed for the rapid erection of temporary and permanent bridges and it employs only five standardised units made on jigs to a high standard of accuracy. A bridge of 270 ft. span was built across a British waterway during the Second World War in 120 hours by less than a dozen men. All the parts are easily handled with light appliances whether for a light footbridge or for a heavy single-span bridge of 150 ft., and no riveting is done at any stage of the production or erection. The welding necessary in the construction of standardised units is of a high degree of accuracy. Each girder of a completed bridge is of the Warren equilateral type, each bar being 10 ft. long. The three-span bridge with two supporting piers is constructed from each bank and barges are used to support the ends moving out across the river until a pier is reached. Centre spans are constructed by cantilevering from each pier until the two halves meet over midstream.

Suspension bridges. In this type of bridge, the load is taken by steel cables which pass over rollers on vertical piers at the ends of the bridge and are then anchored down on the land side. Vertical hangers placed at regular intervals transmit the load from the road or railway to the cable above.

To determine the tension in the cables, it is usual to consider the equilibrium of that half of the cable between the centre of the bridge and the rollers on the top of the pier. The horizontal tension H in the cable at the centre of the bridge, the inclined pull T in the cable at the top of a pier and the resultant load W on the portion of the cable constitute three forces which must be in equilibrium. To find H, moments are taken about the rollers at the top of the pier, and then a vector triangle will give the magnitude of T. The resultant thrust on the piers will depend upon the tensions and the relative inclinations of the cable on the two sides and will equal the resultant of those tensions.

Method of sections. When it is necessary to find the force in only one or two members of a framed structure, or when it is necessary to check a particular result, the method of sections can be employed.

Imagine a framed structure divided into two parts by a line XY cutting across the bars A, B and C (Fig. 110) and that the right-hand

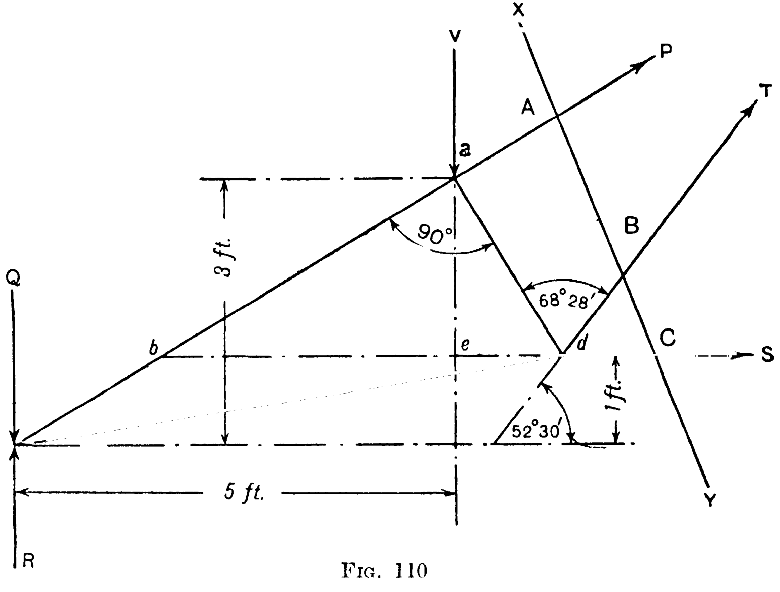


FIG. 110

side is removed. Now suppose that the internal forces in A, B and C be balanced respectively by the external forces P, T and S. Obviously, the portion of the structure will remain in equilibrium under the action of the external forces P, T, S, V, Q and R, of which P, T and S are unknown. If the moments of all these external forces are taken about d, the intersection of the line of action of forces T and S, the latter will have no moment and the moment of P will balance the resultant moment of all the remaining external forces about d. Hence the magnitude of P can be found, and it is then possible to find T by taking moments about the point of intersection of P and S. S can be found by taking moments about any point not in the line of T. Alternatively, a force polygon may be drawn to determine T and S after P has been found.

The direction of the unknown force can be determined by noticing the sign of the moment of the unknown force necessary for equilibrium, as illustrated in the following example.

Example 3. *In Fig.* 110, $V = 1000$ *lb.*, $Q = 500$ *lb.*, $W = 500$ *lb.*, $R = 2500$ *lb. Calculate the value of* P, S *and* T *given* $ad = 2{\cdot}3$ *ft.*, $de = 1{\cdot}18$ *ft. where* a *is the point of action of force* V.

(a) Taking moments about d with clockwise moments as positive :

$$P \times 2{\cdot}3 - 1000 \times 1{\cdot}18 + 2500 \times 6{\cdot}18 - 500 \times 6{\cdot}18 = 0,$$

or $$P \times 2{\cdot}3 = 1000 \times 1{\cdot}18 - 2000 \times 6{\cdot}18, \text{ or } P = -\ 4861 \text{ lb.}$$

This result indicates that P has an anticlockwise moment about d, and thus P acts towards a, and A is a strut.

(b) Taking moments about b, the point of intersection of P and S, and remembering $be = \frac{2}{3}$ of $5 = 3\frac{1}{3}$ ft.

$$T \times bd \sin 52^\circ\ 30' + (R - Q) \times 1\tfrac{2}{3} + V \times 3\tfrac{1}{3} = 0.$$

$$T \times 4{\cdot}51 \sin 52^\circ\ 30' = -\ 2000 \times 1\tfrac{2}{3} - 1000 \times 3\tfrac{1}{3}, \text{ or } T = -\ 1863 \text{ lb.}$$

T has therefore an anticlockwise moment about b, and thus B is a tie.

(c) Taking moments about a

$$S \times 2 - T \times 2{\cdot}3 \sin 68^\circ\ 28' + (R - Q) \times 5 = 0\ ;$$

$$S \times 2 = 1863 \times 2{\cdot}3 \sin\ 68^\circ\ 28' - 10{,}000, \text{ or } S = -\ 3007 \text{ lb.}$$

S has thus an anticlockwise moment about a, and thus C is a tie.

Forces acting at a point but not in the same plane. The common three-legged support for a tackle used for lifting heavy objects where no crane is available is a common example of forces acting at

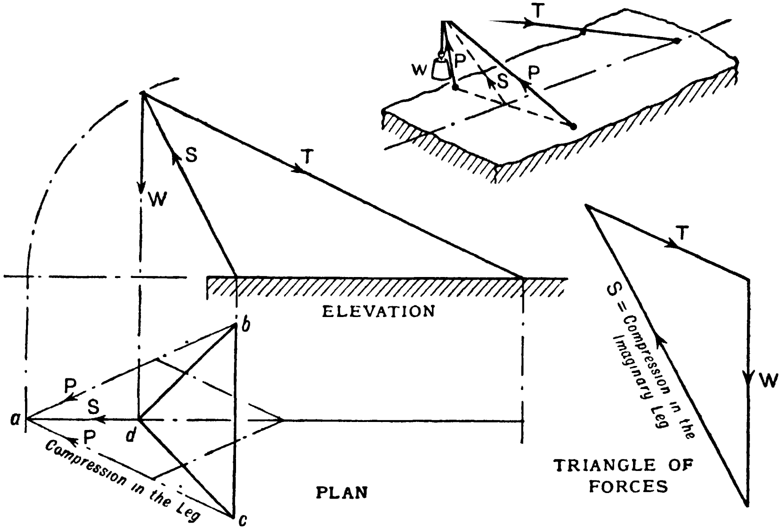


Fig. 111.

one joint but not in the same plane. Another well-known example is provided by the giant sheer-legs used in shipyards for lifting very heavy loads.

In the latter case, the forces in the two inclined and back stay can be found for a given configuration, provided an elevation and plan of the sheer-legs can be drawn as in Fig. 111. The tension in the back stay and the load are forces in one plane, and if an imaginary force *S* is included which is *both in the plane of the legs and in the plane of T and W*, a vector triangle of forces can be drawn according to the principles of Chapter II, from which the magnitude of *T* and *S* can be determined. If *ab* and *ac* represent the true length of the legs to scale, the force *S* can be resolved in these directions and thus the compressive force *P* in each of them determined by scale measurement.

Expt. 5. Objects. *To verify the methods used to find the forces acting in a loaded inverted tripod.*

Apparatus. Drawing board and paper, cords, weights, pulleys and spirit level.

METHOD OF PROCEDURE. Four forces PA_1, PB_1, PC_1, and PW (Fig. 112) are arranged to act at a point P, the forces in three cases being due to weights passing over pulleys and in the fourth to a freely hanging load, the cord passing through a small hole in the drawing board. The latter is placed horizontally by using a spirit level and, by means of a scale screwed to a block of hard wood of true rectangular section throughout, it is possible to choose three points A_1, B_1 and C_1, all at the same height above the board, and to plot their plans A, B and C on the paper on the board below. Arrange to have P in the plane of the board and join P to A, B and C. Produce AP to G in the line BC. The drawing of this plan will facilitate the remainder of the work.

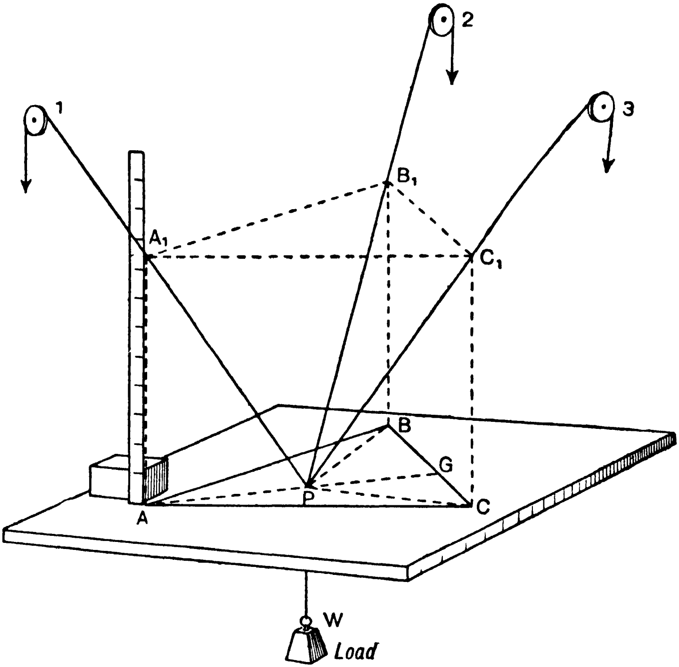


FIG. 112.

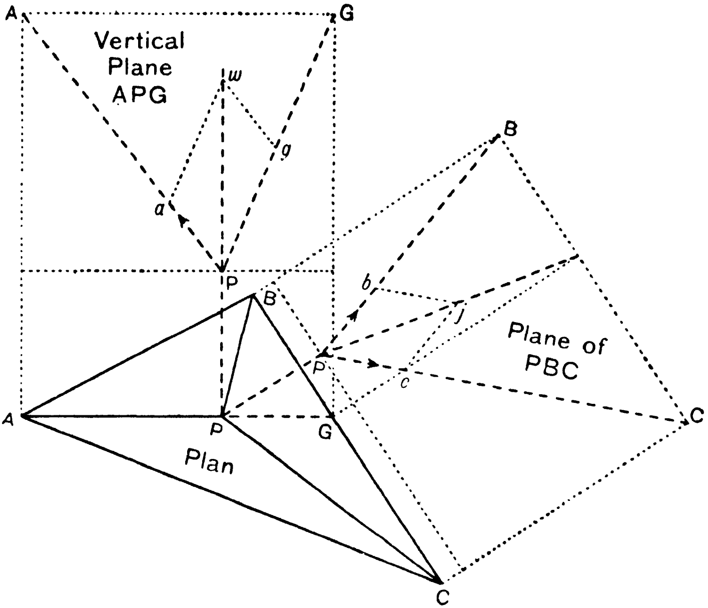


FIG. 113.

In Fig. 113 the plan ABC is reproduced together with the true lengths of the strings in the planes APG and PBC. In the plane APG, *Pw* represents the load *W* to scale. Resolve this load in the directions of the string PA and the imaginary string PG, both in the planes of APG and PBC. Transfer *Pg* to *Pj* in the plane of PBC and resolve this imaginary force in the true directions of the strings P*b* and P*c*.

OBSERVATIONS AND DERIVED RESULTS. Measure P*a*, P*b* and P*c* to scale, and the forces represented should agree with the weights at the ends of the strings respectively passing over pulleys 1, 2 and 3. It is also instructive to remember that if the forces *PA*, *PB* and *PC* are split into their horizontal and vertical components, then the three horizontal components will form a closed vector triangle of forces, while the sum of the vertical components will be equal to the load *W*.

In the case, when the strings A_1P, B_1P, C_1P of Fig. 112 are equal in length and make equal angles with the horizontal and with each other, so that the triangle ABC of Fig. 113 is equilateral, then the tension in each string must be the same. If the strings are inclined at an angle θ to the vertical, the vertical components in each of the three strings must be $\frac{1}{3}W$ and, consequently, the tension in each string will be $\frac{1}{3}W \sec \theta$. The horizontal component of the tension in each string will be $\frac{1}{3}W \tan \theta$ and these horizontal components will balance each other.

When the arrangement shown in Fig. 112 is inverted so that the three strings or ties are replaced by three struts to support a vertical load from the vertex in such a way that the struts form the edges of a pyramid with a triangle for base, similar methods to those already given can be employed to find the compressive forces in the struts.

Example. *Three poles of equal length are arranged as a tripod with their upper ends lashed together and their lower ends fixed on the ground. These lower ends are equally spaced and form the vertices of an equilateral triangle. If the poles are each inclined at 50° to the vertical and a load of 15 cwt. is hung from the upper junction of the poles, find the compressive force in each pole. Assume that the lines of action of the forces acting along the struts and the load meet in one point.*

As the poles are placed symmetrically and are equally inclined to the horizontal, the compressive force in each pole will be the same and equal to

$$\frac{1}{3}W \sec \theta = \frac{1}{3} \times 15 \times 1{\cdot}5557 \text{ cwt.}$$
$$= \mathbf{7{\cdot}8 \text{ cwt.}}$$

To complete this chapter, a summary of the graphical conditions of equilibrium suitable for problems on coplanar forces as found in the theory of structures is given below. Equilibrium is assured if either of the three conditions is satisfied.

SUMMARY OF GRAPHICAL CONDITIONS OF EQUILIBRIUM

(1) One funicular polygon and the force polygon closes.

(2) Two different funicular polygons close when the same pole is used for both.

(3) A funicular polygon closes for each of three poles, provided these poles are not in the same straight line.

EXERCISES ON CHAPTER VII

1. Find the reactions at A and B for the loaded Warren girder shown in Fig. 114. Find the forces in the members, stating whether tensile or compressive.

2. Determine the forces in the members of the truss shown in Fig. 115, and distinguish between the members in compression and those in tension. (U.L.C.I.)

3. The figure (Fig. 116) shows a loaded girder of an overhead travelling crane. Draw the force diagram and give the magnitudes of the forces in members A, B and C, stating whether they are in tension or compression. (U.L.C.I.)

4. Find the forces in the members of the Warren girder when loaded as in Fig. 117. Make a table showing the magnitude of the forces in each member and the nature of those forces.

5. Figs. 118, 119 and 120 show loaded frames. In each case draw the reciprocal figure, and tabulate the magnitudes of the reactions and the forces in the members and the nature of those forces.

6. The frame shown in Fig. 121 is loaded between the joints B and C. Calculate the equivalent load at B and C, the resultant of the two external loads and the magnitudes and natures of the forces in the members.

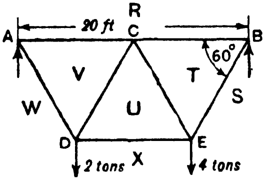


FIG. 114.

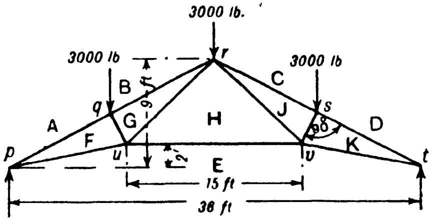


FIG. 115.

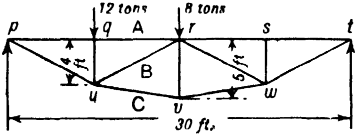


FIG. 116.

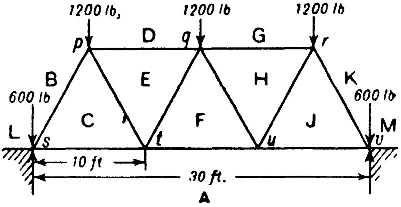


FIG. 117.

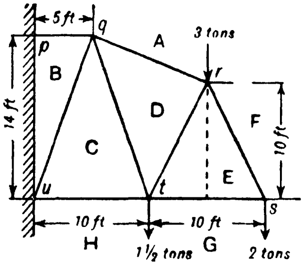


FIG. 118.

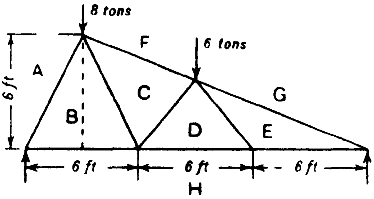


FIG. 119.

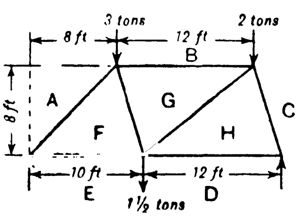


FIG. 120.

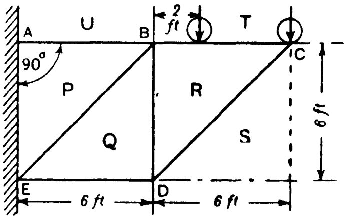


FIG. 121.

7. Obtain the values of the forces in the members of the loaded frames shown in Figs. 122 and 123.

8. The reaction at B in the case of the loaded frame shown in Fig. 124 is vertical. Find the value of the reactions at A and B and the forces in each member of the structure by means of a reciprocal figure.

9. Fig. 125 shows a roof truss with a resultant wind load of 4000 lb. on the left-hand side. Determine the forces at each support and in the members of the structure, stating clearly which are ties and which struts. Also draw a reciprocal figure for the same wind load on the right-hand side, and tabulate the results.

10. In Fig. 126 the roof truss is subjected to combined wind and dead loads. Find the magnitudes and directions of the reactions at the supports and draw the reciprocal figure. From the latter find the value of the forces in each member of the structure.

11. Use the method of sections to find the magnitude and nature of the forces in the bars A, B and C of the Warren girder shown in Fig. 127.

12. A roof principal (Fig. 128) is loaded with vertical loads as shown and a horizontal load of 2 tons. It is pinned at B. Find the forces in the members DC, ED and FG. (A.M.I.Mech.E.)

13. Use the " method of sections " to find the forces in the members AB, AC, AD and CD of the truss shown in Fig. 129. (U.L.C.I.)

14. The circle O in Fig. 130 represents the plan of a vertical flagstaff; the radiating lines 120° apart represent the plans of guy ropes attached to the staff at a height of 16 ft. above the ground. It is known that the tension in the guy rope AB is 60 lb. Find the tension in each of the other ropes, and the compressive force acting along the flagstaff.

15. Two legs of a tripod are each 13 ft. long, the third leg being 12 ft. 6 in. long. If the feet of the legs lie on the corners of an equilateral triangle of side 10 ft. lying in a horizontal plane, find the forces in each leg when a weight of 5 tons is suspended from the apex of the tripod. (U.L.C.I.)

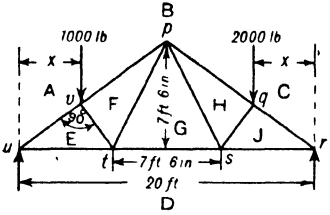


FIG. 122.

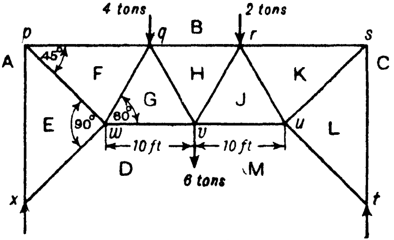


FIG. 123.

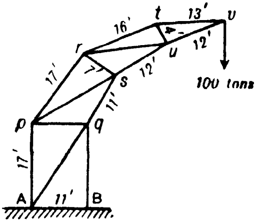


FIG 124.

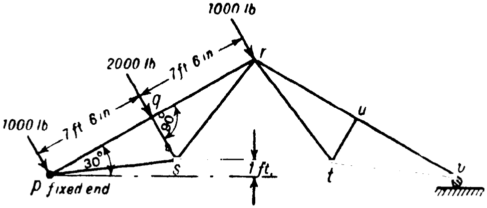


FIG. 125.

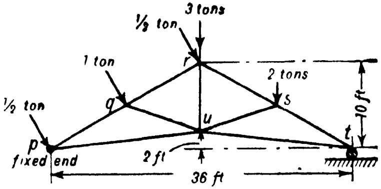


FIG. 126.

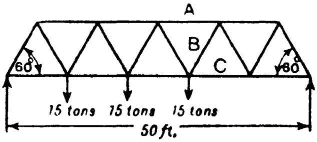


FIG. 127.

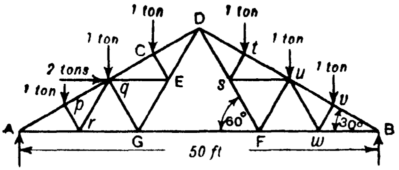


FIG. 128.

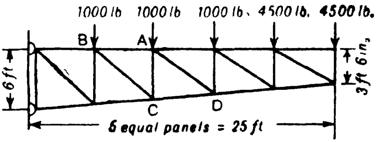
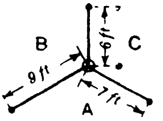


FIG. 129.



Angles at A, B and C = 120°

FIG. 130.

CHAPTER VIII

BEAMS AND CANTILEVERS. MOMENT OF INERTIA OF BEAM SECTIONS. SIMPLE THEORY OF BENDING

Shearing and Bending of a Beam. For a *simply loaded* cantilever or beam in equilibrium under the action of external forces, the latter are all in one plane and perpendicular to the beam. Fig. 131 (*a*) shows a cantilever under the action of the resultant force P of all the forces to the right of the section XX. This resultant force tends to turn the right-hand portion of the cantilever about a horizontal axis at XX and also to slide this part vertically across the section XX. For, if two equal and opposite forces S and T, each equal in magnitude to P, be applied at XX, the equilibrium of the cantilever will not be affected and P and T will form a couple Px giving a **bending moment** (**B.M.**) about XX, and S will tend to cause a sliding down at XX or a **shearing force** (**S.F.**) at XX. Also, in the simply loaded beam, Fig. 131 (*b*), the external forces balance and each of the two parts on either side of a section XX may be considered as a cantilever.

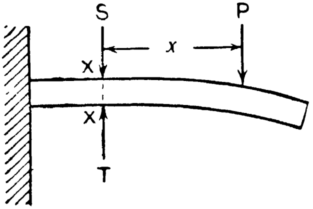


FIG. 131 (*a*).

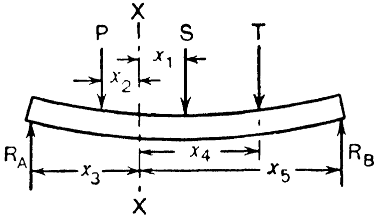


FIG. 131 (*b*).

In this case, for equilibrium,

$$R_A + R_B = P + S + T,$$

and, by taking moments about the right hand support to find R_A,

$$R_A(x_3 + x_5) = P(x_2 + x_5) + S(x_5 - x_1) + T(x_5 - x_4).$$

Hence the following definitions of shearing force and bending moment:

The shearing force at a section of a cantilever or beam is equal to the algebraic sum of all the forces on *one* side of the section.

The bending moment at a section of a cantilever or beam is equal to the algebraic sum of the moments of all the forces on *one* side of the section.

To calculate the shearing force or bending moment at any section of a beam, first calculate the reactions of the supports. The shearing force is then the resultant of all the external forces to *one* side of the section, and the bending moment is the resultant moment of all the external forces on *one* side of the section. When calculating the bending moment and shearing force in the case of a uniformly distributed load, which may include the weight of the beam itself, that part of the load on one side of the section may be considered as a resultant load concentrated at the centre of gravity of the distributed load.

Shearing force is generally said to be positive when it causes the portion of the beam on the left-hand side of a section to rise; and negative when the portion on the right-hand side of a section rises. Bending moment is often taken as positive when the upper surface of the beam is made convex; and negative when the upper surface is made concave. This means that anticlockwise moments on the left-hand side of a section are considered positive; and on the right-hand side of a section clockwise moments are considered positive. This is but a convention, and it is only necessary for a student to maintain a clear conception of signs in order to arrive at correct values of S.F. and B.M.

In Fig. 131 (*b*) the bending moment at XX is $Px_2 - R_Ax_3$, or alternatively $Sx_1 + Tx_4 - R_Bx_5$.

Inclined loads. When a beam is subjected to an inclined load or series of inclined loads, these loads must be split into their components at right angles, one normal to the beam and the other along the beam. In Fig. 132 the force P acting on the beam at an angle θ is split into its components P_V and P_H, the latter causing a horizontal thrust in the portion of the beam AC. For this beam the equilibrium conditions are:

$$R_V + R_B = P_V, \quad R_H = P_H, \quad R_Vl = P_Vx_2;$$

and the bending moment at the section XX is :

$$R_V x_1 \quad \text{or} \quad -R_B(l-x_1)+P_V(l-x_1-x_2).$$

At XX the shearing force is R_V and the thrust is P_H.

Shearing force, thrust and bending moment diagrams. When the shearing force, thrust and bending moment are calculated at a sufficient number of sections and the results plotted on a straight-line base representing the length of the beam, and the points joined by smooth curves or straight lines, as the case may be, the curves obtained are called respectively **shearing force, thrust** and **bending moment diagrams.**

As is usual in graphical work, positive values are plotted above the base, and negative below. These diagrams for a simple case of inclined loads are shown in Fig. 132. It should be noted that the laboratory method of applying loads to a beam by means of a knife-edge application is seldom carried out in practice, except in weighing and testing machines. Usually the isolated load is spread over a short length of beam, and the effect on the three types

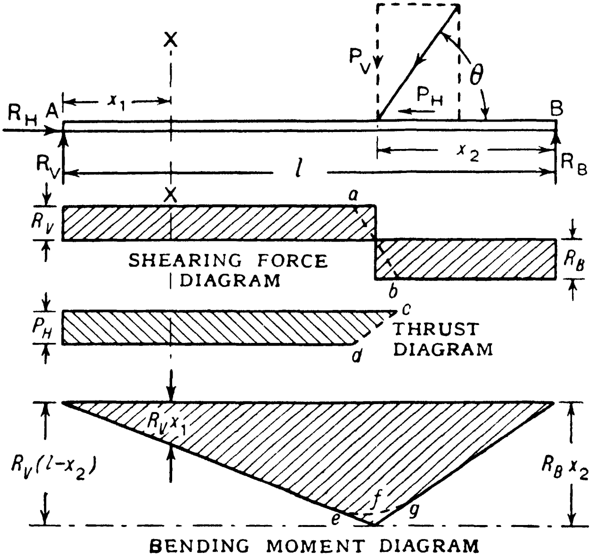
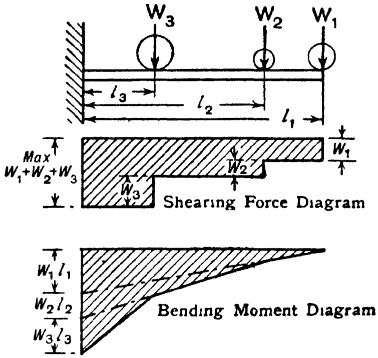
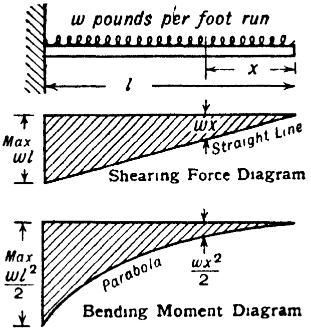


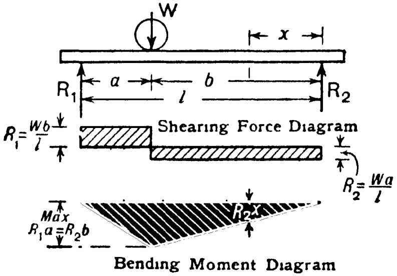
FIG. 132.



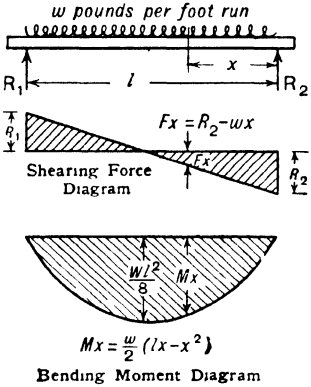
(1) CANTILEVER
(SIMPLE LOADS)



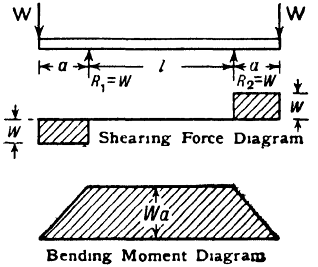
(2) CANTILEVER
(UNIFORM LOAD)



(3) BEAM
(ISOLATED LOAD)



(4) BEAM
(UNIFORM LOAD)



(5) BEAM
(UNIFORM B M BETWEEN SUPPORTS)

FIG. 133.
Standard cases of Bending Moment and Shearing Force.

of diagram is to cause a more gradual alteration in the shearing force, thrust and bending moment as indicated respectively in Fig. 132 by the dotted lines *ab, cd, efg*.

Standard cases. Some of the more important standard cases of shear and bending moment diagrams are shown in Fig. 133. When the loads are both concentrated and distributed, the shearing force and bending moment can either be calculated directly for any given section of the beam or calculated separately for each type of loading and the results added algebraically. For a large number of isolated loads such as are often experienced in practice, the graphical method of determining shearing force and bending moment (as shown, p. 238, Vol. I) is very convenient.

Example. *A horizontal beam XY is loaded as shown in Fig.* 134 *and is supported at P and Q. Obtain the reactions of the supports, and draw*

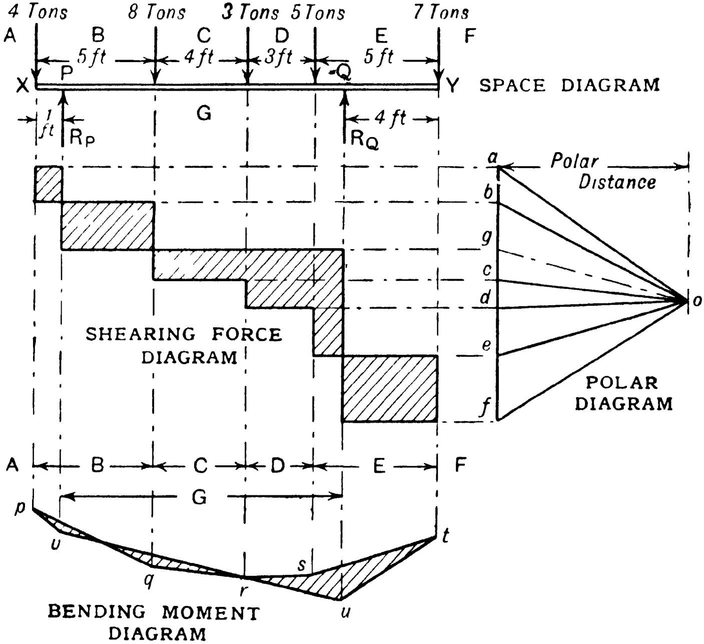


FIG. 134.

the shearing force and bending moment diagrams by means of a graphical solution.

Draw to scale the space diagram and letter it according to Bow's notation. Then draw the load line *abcdef*, and at an integral and convenient distance from this line mark a pole *o*. Join *oa*, *ob*, *oc*, *od*, *oe* and *of*.

To obtain the B.M. diagram, start at any point *p* on the vertical line through the first load and draw the lines *pq*, *qr*, *rs*, *st*, *tu*, *pv* respectively in the spaces *B*, *C*, *D*, *E*, *F* and *A* and respectively parallel to *ob*, *oc*, *od*, *oe*, *of*, *oa*.

Finally, close the diagram by joining *vu*, and in the polar diagram draw from *o* a line parallel to *vu*, meeting the load line *abcdef* in *g*. Then the *vertical* height of the B.M. diagram at any given point is the B.M. at this position on the beam measured to the scale :

B.M. scale in tons ft. = space diagram scale × load scale × polar distance.

For example, if the space scale is 1 in. = 2 ft., the load scale 1 in. = 5 tons, and the polar distance P is 2 in., then the B.M. scale is 1 in. $= 2 \times 5 \times 2$ or 20 tons ft.

The S.F. diagram is obtained by projecting horizontally from the points *abcdefg* in the load line to the projections of the spaces A, B, C, D, E, F and G respectively. The scale of this diagram is obviously the same as the load scale. By measurement from diagram,

$$\text{Reaction of beam } ga = R_P = 8\tfrac{3}{4} \textbf{ tons.}$$
$$fg = R_Q = 18\tfrac{1}{4} \textbf{ tons,}$$

Maximum B.M. occurs at Q and is equal to 28 tons ft.

It should be noticed that when the ends of the beam overhang the supports, the maximum B.M. is considerably reduced, and at the two places where the B.M. changes sign its value is zero. The above results can easily be checked by calculation.

Mathematical aspect of relationship between S.F. and B.M. diagrams. Referring to Fig. 135 and taking moments about B,

$$R_A l = wl \,.\, l/2 + W_1 x_1 + W_2 x_2.$$
$$\therefore R_A = wl/2 + W_1 x_1/l + W_2 x_2/l.$$

Now, consider any section XX between the loads W_1 and W_2 ; then,

$$\text{shearing force at } XX = -wx - W_1 + R_A$$
$$= -wx - W_1 + wl/2 + W_1 x_1/l + W_2 x_2/l. \;.....(1)$$

This expression is only true between the isolated loads, because it

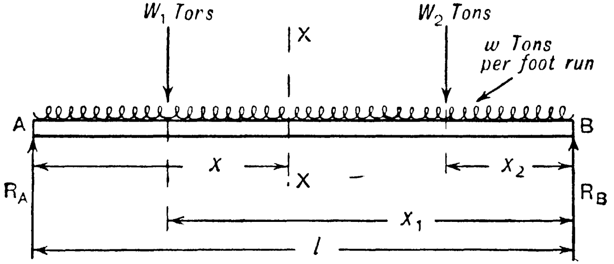


FIG. 135.

is most important to remember that *the presence of isolated loads causes a break in the continuity of the* S.F. *and* B.M. *diagrams.* If the B.M. at XX is M, then

$$M = - W_1(x - l + x_1) - wx^2/2 + wlx/2 + W_1x_1x/l + W_2x_2x/l.$$

Differentiating, $dM/dx = - W_1 - wx + wl/2 + W_1x_1/l + W_2x_2/l.$ This is the same as the expression (1), indicating that the rate of change of B.M. is equal to the S.F. The maximum value of the B M. at any section is given by $dM/dx = 0$, that is, when $F = 0$.

Also, since $dM/dx = F$ or $dM = F \, . \, dx$, and considering the S.F. diagram between any two sections, say at x_4 and x_5 not marked in Fig. 135, over which the S.F. is continuous, and taking signs into account, then

$$M = \int_{x=x_4}^{x=x_5} F \, dx \, ;$$

or the area of the S.F. diagram represents the change in value of the B.M. between those two sections.

Stresses in beams. In **ordinary bending,** the loads and reactions on a beam cause it to become curved, so that some longitudinal layers or fibres of the beam are stretched and others compressed. In between, there must be a surface called the **neutral surface** where there is no strain or stress due to bending. The intersecting line where the neutral surface meets a transverse section of the beam (Fig. 136) is called the **neutral axis** of the section. The deformations due to the external forces set up resisting tensile and compressive forces in the material of the beam. Thus, at any section of the beam the resulting moment, or **Moment of Resistance,** due to these resisting tensile and compressive forces will act in the opposite sense and balance the external B.M. on the beam at that section.

In **simple bending,** the effect of shearing is ignored, and in Fig. 136 a portion of a beam is assumed to be under the action of pure couples, that is, subjected to a constant bending moment.

In the study of the theory of simple bending, the following assumptions for homogeneous materials are made and, although not perfectly true, are nearly so within the elastic limit of the material :

(1) the original plane transverse sections of a beam remain plane after bending ;

(2) the material follows Hooke's Law, that is, the strain is proportional to the stress producing it ; and the modulus of elasticity (E) is the same in tension and compression and of equal value in all directions ;

(3) the limit of elasticity or proportionality of the material is not exceeded.

Consider two transverse sections PQ and KL close together on a beam (Fig. 136 (*a*)) ; suppose the beam is subjected to the action of two equal and opposite couples at its ends. The planes will be inclined to one another after bending as shown in Fig. 136 (*b*), and will intersect at O, which will be the centre of curvature of radius R for that portion of the beam between the sections. A very thin layer MN parallel to the axis of the beam and distant y from the neutral axis EF will be altered in length to M_1N_1. The strain in this layer by bending will be

$$\frac{M_1N_1 - MN}{MN} = \frac{M_1N_1 - EF}{EF} = \frac{M_1N_1}{EF} - 1 ;$$

but, by similarity of sectors of angle θ,

$$\frac{M_1N_1}{EF} = \frac{R+y}{R},$$

or $$e = \text{strain} = \frac{M_1N_1 - EF}{EF} = \frac{(R+y)\theta - R\theta}{R\theta} = \frac{y}{R}.$$

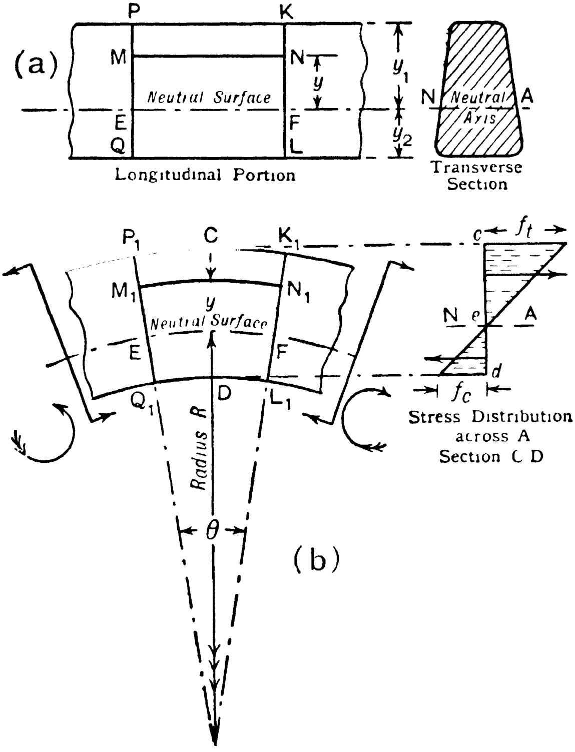


FIG. 136.

Suppose the longitudinal stress producing this strain be p; then, if the elastic limit of the material has not been exceeded,

$$p = Ee = \frac{Ey}{R}, \quad \text{or} \quad \frac{p}{y} = \frac{E}{R}. \quad \text{........................(1)}$$

Hence, p is proportional to y, the distance from the neutral axis.

The result (1) obtained in the last paragraph allows calculations to be made for the smallest radius of curvature to which materials can be bent when the modulus of elasticity, the elastic limit and the depth of the section of the material are known. If the radius of a drum is very large compared with that of the strip, wire or piping wound upon it, then the maximum stress induced in the material may be kept within the elastic limit.

The success of " Pluto ", in which a pipe line was laid across the English Channel to supply petrol to the Allied Armies entering Europe in 1944, was largely due to the use of the Hais pipe, specially designed for flexibility and strength. This piping was wound upon large drums 40 ft. in diameter, and afterwards the drums were towed across the Channel, the piping being unwound, and laid upon the sea-bed.

The Hais pipe is of composite construction, of 3·08 in. bore, the innermost layer consisting of extruded lead nearly 0·2 in. thick, which is wrapped and armoured. Steel strip and galvanised steel wires separated from each other and from the lead by bitumen-impregnated cotton tape and by paper tape treated with petroleum make the pipe extremely strong and flexible. The outside of the pipe is covered with gas-tarred jute yarn, which gives the pipe an external diameter of 4·49 in. By filling the pipe with water under pressure, it can be laid without fear of external crushing. On test, an internal pressure of 3500 lb. per sq. in. was necessary to burst or fracture the pipe. The cable was constructed in lengths of 30 miles, while one drum was able to take 70 miles wound upon it. The simple theory just evolved for bending of pipes of homogeneous construction will obviously not be applicable for Hais piping of composite construction and subjected during the laying to variable tension and internal and external pressure, but similar strength, flexibility and comparative lightness could not be attained with simple piping.

The neutral axis. The equilibrium of the portion of the beam to the left of section *CD* (Fig. 136, *b*) requires that the internal forces acting across the section must form a resisting couple equal to the external couple acting, and the total tensile forces acting must balance the total compressive forces.

Therefore, taken over the whole section,

$$\Sigma p\,\delta a = 0,$$

and since $p = Ey/R$, then $\frac{E}{R}\Sigma y\,\delta a$ must be zero.

But $\Sigma y \, \delta a$ represents the whole moment of area of the section about the neutral axis. Hence, this expression can only be zero if the axis passes through the centroid of the section.

Example 1. *A beam of symmetrical section,* 10 *in. deep, is bent so that the maximum skin or outer layer stress due to bending is* $7\frac{1}{2}$ *tons per square inch. If* $E = 13{,}500$ *tons per sq. in., find the radius of curvature.*

Since $p = Ey/R$, then $R = Ey/p$, where $y = 5$ in.

$$\therefore \; R = 13{,}500 \times 5/7\tfrac{1}{2} = \textbf{9000 in. or 750 ft.}$$

Example 2. *A piece of spring steel wire is bent round a drum* 4 *ft. in diameter. If* $E = 13{,}500$ *tons per sq. in., the diameter of the wire is* $\frac{1}{8}$ *in. and the elastic limit has not been reached, calculate the maximum bending stress induced in the wire.*

In this case, $R = 2$ ft. and $y = \frac{1}{16}$ in.

As $p = Ey/R$, p will be a maximum when y is a maximum.

$$\therefore \; p = \frac{13{,}500 \text{ tons per sq. in.} \times \frac{1}{16} \text{ in.}}{24 \text{ in.}} = \textbf{35·2 tons per sq. in.}$$

Now consider a transverse section of any beam at any point, Fig. 137. The neutral axis will always pass through the centroid of the section. Consider a thin layer GH of thickness δy distant y from NA and over which the stress is p.

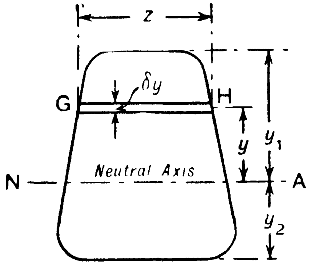


FIG. 137.

Then, moment of resistance of this layer about NA

$$= \text{stress} \times \text{area of layer} \times \text{arm of force} = p \, . \, z \, . \, \delta y \, . \, y.$$

Total moment of resistance over the section

$$= M = \int_{y=y_2}^{y=y_1} pzy \, dy = \frac{E}{R} \int_{y=y_2}^{y=y_1} zy^2 \, . \, dy = \frac{EI}{R} \, . \quad \ldots\ldots\ldots\ldots\ldots(2)$$

The quantity to be integrated is called the **moment of inertia** or second moment of the section about the neutral axis, and is denoted by the letter I. The moment of inertia of an area about an axis is thus the sum of the products of the areas of its elements and the squares of their distances from the axis.

Hence, since for equilibrium the moment of resistance equals the external B.M. on the beam, and combining (1) and (2), we have:

$$\frac{M}{I}=\frac{E}{R}=\frac{p}{y}$$

as the formula for simple bending in a concise and easily usable form.

It should be noted that the maximum stress will occur on the outermost layers farthest from the neutral axis, that is, for the values y_1 and y_2.

The quantities I/y_1 and I/y_2 are called the **moduli of the section** and denoted by **Z**, and for symmetrical transverse sections the two moduli become equal.

Unsymmetrical sections often occur in practice when angles, channels and tees and built-up sections are employed. Also with cast-iron sections it is customary to shape them so that the tension flange or portion is much greater in area, so that the neutral axis comes nearer to the surface subjected to maximum tensile stress. This is because cast iron is weak in tension compared with its compressive strength.

Moment of inertia. The moment of inertia or second moment of section about an axis depends not only on the area but also on the disposition of that area about that axis. Expressions for I for different sections are readily found by the methods of the integral calculus.

Example 3. *Find the value of I for a rectangular section of width b and depth d about a symmetrical axis parallel to b.*

Consider a strip of width b and thickness δy distant y from the symmetrical axis.

Moment of inertia of strip about the axis $= b\,\delta y\,.\,y^2$.

$$I \text{ for whole section} = \int_{-d/2}^{d/2} by^2\,dy = \Big[by^3/3\Big]_{-d/2}^{d/2} = bd^3/12.$$

N.B. $Z = I/\tfrac{1}{2}d = bd^2/6$.

The following two theorems are very useful for determining the moments of inertia of plane figures or solids about different parallel axes.

Parallel axis theorem. The moment of inertia of a $\left\{\begin{array}{l}\text{plane figure}\\ \text{solid}\end{array}\right\}$ about any axis exceeds that about a parallel axis through its centre of gravity by the product of the $\left\{\begin{array}{l}\text{area of the plane figure}\\ \text{mass of solid}\end{array}\right\}$ and the square of the distance between these two axes.

Thus, if I_G is the moment of inertia of an area A about an axis passing through the centre of gravity, I_X the moment of inertia of the same area about a parallel axis XX, d the distance between the axes, then

$$I_X = I_G + Ad^2.$$

Polar moment of inertia. The sum of the moments of inertia of an area about two mutually perpendicular axes XX and YY in its plane is equal to the polar moment of inertia about an axis through O, the intersection of the other two axes, and perpendicular to the plane of the area.

Thus $$I_X + I_Y = I_0.$$

Example 4. *Obtain the value of* (*a*) *the polar moment of inertia of a circular section about an axis through its centre and perpendicular to its plane. Also, find the moment of inertia and modulus of section of* (*b*) *a circle of radius R about a diameter,* (*c*) *an annulus about a diameter.*

(*a*) Consider a concentric ring of thickness δr at r from the centre O.

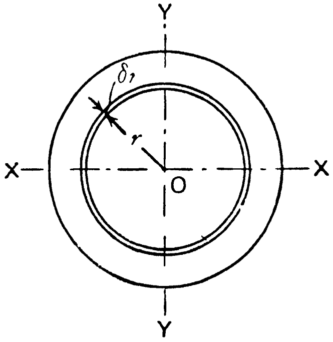


FIG. 138.

Moment of inertia of ring about $O = 2\pi r\,\delta r\,.\,r^2$.

$$I_0 = \int_0^{d/2} 2\pi r^3\,\delta r = \left[\frac{\pi r^4}{2}\right]_0^{d/2} = \frac{\pi d^4}{32}.$$

(*b*) Since, from the polar moment theorem, $I_0 = I_X + I_Y$, then, in this case, $I_X = I_Y$ and $I_X = \frac{1}{2}I_0 = \pi d^4/64$ and

$$Z = I_X/\tfrac{1}{2}d = \pi d^3/32.$$

(*c*) For a hollow section of outside diameter D_1 and inner diameter D_2, then

$$I_X = \pi D_1^4/64 - \pi D_2^4/64 = \pi\,(D_1^4 - D_2^4)/64,$$
$$Z = I_X \div (D_1/2) = \pi\,(D_1^4 - D_2^4)/32D_1.$$

N.B. The modulus in the case of the annulus is *not* the difference of the Z's of the inner and outer areas, but the difference of the I's divided by the full *depth* of the section.

Deflection of a beam under uniform bending moment. Between the supports A and B the beam is subjected to a uniform bending moment. Hence, this part of the beam will be deflected to form the arc of a circle. Referring to Fig. 139 and using the property of intersecting chords of a circle,

$$2R\,.\,\Delta = (L/2) \times (L/2)$$

or

$$R = L^2/8\Delta.$$

But

$$\frac{E}{R} = \frac{M}{I}, \quad \therefore\ \frac{E}{R} = \frac{8E\Delta}{L^2} = \frac{M}{I};$$

$$\therefore\ \Delta = \frac{ML^2}{8EI} = \frac{WxL^2}{8EI},$$

since $M = Wx$ for every section of the beam between the supports.

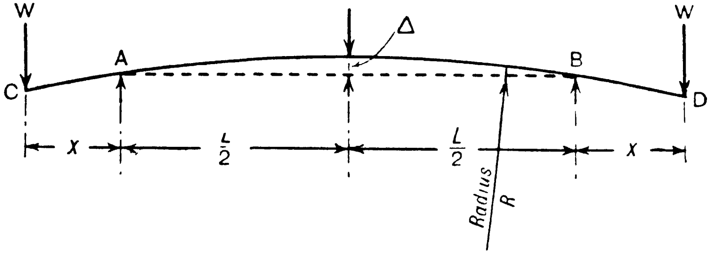


FIG. 139.

Expt. To verify the formula $\Delta = ML^2/8EI$.

Apparatus. Deflection bench, knife edges and supports or hangers at A, B, C and D (Fig. 139), weights, dial test indicator for measuring the central deflection. Series of beams of different cross-sections and of nearly as possible the same material and manufacture.

Method of procedure. Arrange a series of experiments to verify that:

(1) $M/\Delta =$ a constant, by keeping L constant, and E and I constant by using the same beam, varying M and measuring the corresponding Δ.

(2) $I\Delta =$ constant, by keeping M and L constant and using different-sized beams of the same manufacture and heat treatment, varying I and measuring Δ.

(3) $L^2/\Delta =$ a constant, by keeping M constant, using the same beam, varying L and measuring Δ.

The good results which can be achieved in the above series of experiments should give the student confidence in the use of the formulae derived in this chapter. These formulae, which summarise the theory of simple bending, even with the assumptions and approximations involved, enable the engineer to calculate strains and stresses and to design beams with a margin of safety and a fair degree of economy in the use of material.

Modulus of rupture. This is a stress calculated from the formula:

stress × Z = bending moment at fracture,

where the bending moment is that causing fracture. This stress is obviously not the true stress, but the figure obtained is used for comparative purposes between beams of the same and different materials.

The test for modulus of rupture is frequently employed in the case of cast irons, timber and plastic materials.

Expt. To find the modulus of rupture of three similar specimens of cast iron of circular section.

Apparatus. A hydraulic testing machine with fittings for applying a central load to a short beam supported on knife-edges near its ends.

Method of procedure. By means of the testing machine apply a central load to each specimen in turn, and record in each case the load W lb. necessary to produce fracture. Use a span L of 12 in. in each case, and if R is the mean radius of a specimen, the maximum bending moment M on the beam will equal $WL/4$, and the moment of inertia I of a cross-section will be $\pi R^4/4$.

Then $f=\text{modulus of rupture}=\dfrac{My}{I}=\dfrac{WLR}{4I}=\dfrac{WL}{\pi R^3}$.

OBSERVATIONS.

Specimen	W	R	πR^3	WL	f lb./in.^{2}
1	1200 lb.	0·455	0·296	14,400	48,600
2	900 lb.	0·418	0·229	10,800	47,100
3	865 lb.	0·407	0·212	10,380	48,900

DERIVED RESULTS. The mean value of the modulus of rupture is 48,200 lb. per sq. in. See p. 247, Expt. 30 of Vol. I for modulus of rupture of wooden beams.

Example 5. *Find the position of the neutral axis and the moment of inertia about the axis NA of the section shown in Fig.* 140.

To find the position of the centroid of the section, take the moments of area about the 8 in. base line.

Total area $= 6\times 1+6\times 1+8\times 2=28$ in.^{2}

$28\bar{y}=6\times 1\times 8\tfrac{1}{2}+6\times 1\times 5+8\times 2\times 1,$

$28\bar{y}=97$ or $\bar{y}=3{\cdot}46$ in.

To find the value of I about NA, find the values of I for each rectangle about its own axis, and then use the parallel axes theorem.

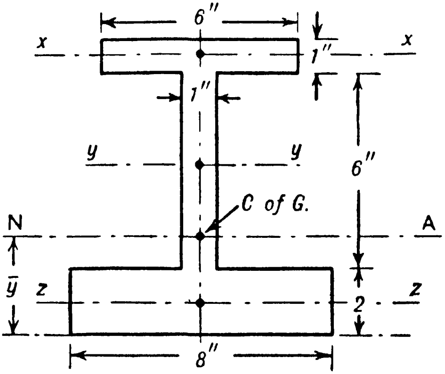


FIG. 140.

	Area (in.)^2	Distance of C.G. from NA. (in.)	I of each rectangle about its own axis (in.)^4
Top flange	6	$8\cdot5 - 3\cdot46 = 5\cdot04$	$6 \times 1^3/12 = 0\cdot5$
Web	6	$5 - 3\cdot46 = 1\cdot54$	$1 \times 6^3/12 = 18$
Bottom flange	16	$3\cdot46 - 1 = 2\cdot46$	$8 \times 2^3/12 = 5\frac{1}{3}$

$$\begin{aligned} I_{NA} &= (0\cdot5 + 6 \times 5\cdot04^2) + (18 + 6 \times 1\cdot54^2) + (5\tfrac{1}{3} + 16 \times 2\cdot46^2) \\ &= 152\cdot91 \qquad + 32\cdot23 \qquad + 102\cdot17 \\ &= \mathbf{287\cdot3\ in.^4\ units.} \end{aligned}$$

Example 6. *A beam which has a symmetrical section and a moment of inertia of* 2500 *in.*^{4} *units and a depth of* 20 *in. is to take a central load of* 45 *tons. If the stress is not to exceed* 6 *tons per sq. in., find the greatest permissible span if the beam is assumed to be simply supported.*

Let L ft. = span. Then maximum B.M. at centre is given by $WL/4$, where W is the central load.

$$M = \frac{WL}{4} = \frac{45 \times L \times 12}{4} = 135L \text{ tons in.}$$

$$I = 2500 \text{ (in.)}^4 \text{ units}, \quad y = 10 \text{ in.}, \quad p = 6 \text{ tons in.}^2$$

Since $\qquad M = pI/y,$

then $\qquad 135L = 6 \times 2500/10, \quad$ or $\quad L = \mathbf{11\cdot1\ ft.}$

Example 7. *A floor is supported by joists* 3 *in. wide and* 9 *in. deep and the joists have a span of* $10\frac{1}{2}$ *ft. at* 12-*in. centres. If the greatest stress due to bending is not to exceed* 1200 *lb. per sq. in., find the maximum load the floor can carry in lb. per sq. ft.*

Each joist carries a floor area $10\frac{1}{2}$ ft. × 1 ft. and a load of $10\frac{1}{2}\,w$ lb. if the load is w lb. per sq. ft.

$\therefore\ M = (10\frac{1}{2}w \times 10\frac{1}{2})/8$ lb. ft., since $M = WL/8$.

Also $M = pI/y, \quad \therefore\ 110\frac{1}{4}w = (8 \times 1200 \times 3 \times 9^3)/(12 \times 4\frac{1}{2} \times 12),$

or $\qquad w = \mathbf{294\ lb.\ per\ sq.\ ft.}$

Example 8. *A water main is* 5 *ft. in external diameter, thickness of metal* $1\frac{1}{2}$ *in. The span or unsupported length is* 9 *ft. Consider the main as a simply supported beam, and calculate the maximum stress due to bending if the main is full of water.*

$$\begin{aligned} I \text{ for metal section} &= \pi(R^4 - r^4)/4 \\ &= \pi(30^4 - 28\tfrac{1}{2}{}^4)/4 = 118{,}000 \text{ in.}^4 \text{ units.} \end{aligned}$$

$$\begin{aligned} \text{Distributed load} &= \text{wt. of main} + \text{wt. of water} \\ &= \pi\,.\,(2\tfrac{3}{8})^2\,.\,9\,.\,62\cdot5 + \pi(30^2 - 28\cdot5^2) \times 9 \times 480/144 \\ &= 18{,}240 \text{ lb.} \end{aligned}$$

$$M = WL/8 = (18{,}240 \times 108)/8 = 246{,}240 \text{ lb. in.}$$

Also $y = 30$ in. and $p = My/I$;

$$\therefore\ p = 246{,}240 \times 30/118{,}000$$
$$= 62{\cdot}6 \text{ lb. per sq. in.}$$

Elementary principles of reinforced concrete design. It is not possible, within the scope of this book, to deal at great length with design in reinforced concrete, but an example to illustrate the general principles may be taken with advantage. The general assumptions as regards the conditions of bending are :

(*a*) the bending stress on a section is proportional to the distance from the neutral axis ;

(*b*) the reinforcement takes the whole of the tension and the concrete takes the compression ; and

(*c*) the total compressive moment of the concrete about the neutral axis is equal to the total tensile moment of the reinforcement about the neutral axis.

Consider the beam section shown in Fig. 141. Let the maximum compressive stress in the concrete be f_c and the tensile stress in the reinforcement f_t. The average stress over the area $b \times y$ is $\frac{1}{2}f_c$.

Then for the concrete, the total force on the section above NA

$$= \tfrac{1}{2}f_c by.$$

Total force on reinforcement of area a sq. in. $= f_t a$.

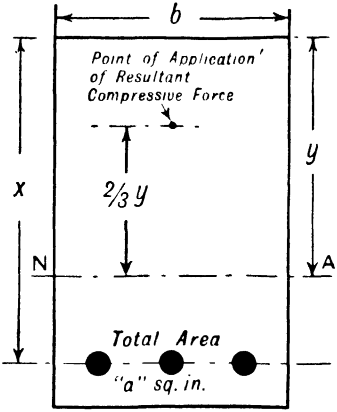


FIG. 141.

Since the total compressive force = total tensile force,

then $\frac{1}{2}f_c by = f_t a$ or $y = 2f_t a/bf_c$. ....(1)

Also, total compressive moment = total tensile moment;

$$\therefore \ \tfrac{1}{2}f_c by \,.\, \tfrac{2}{3}y = f_t a(x-y),$$

$$\text{or} \quad a = \frac{f_c by^2}{3f_t(x-y)}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

The expressions (1) and (2) enable y and a to be found.

Example 9. *Find the area of reinforcement required in a beam if* $b = 5$ *in. and* $x = 11$ *in., if the allowable stresses are* 0·2 *ton per sq. in. for concrete and* 5 *tons per sq. in. for the reinforcement.*

From (1), $y = 2f_t a/f_c b = 2\,.\,5\,.\,a/(0{\cdot}2 \times 5)$

$= 10a.$

From (2), $a = \dfrac{f_c by^2}{3f_t(x-y)} = \dfrac{0{\cdot}2 \times 5 \times y^2}{3 \times 5(11-y)} = \dfrac{y^2}{165 - 15y}$;

$$\therefore \ a = \frac{(10a)^2}{165 - 15(10a)} = \frac{100a^2}{165 - 150a},$$

whence $a = 0{\cdot}66$ sq. in.

With three rods, their diameters will be $d = \sqrt{\dfrac{0{\cdot}66 \times 4}{3\pi}} = 0{\cdot}53$ in.

Therefore, **three rods each $\frac{9}{16}$ in. in diameter are suitable.**

Note. The moment of resistance for a reinforced concrete section will be either $\frac{1}{2}f_c by\,.\,(x - \frac{1}{3}y)$, or $f_t a(x - \frac{1}{3}y)$.

EXERCISES ON CHAPTER VIII

1. A horizontal beam of 20 ft. span supported at its end is subjected to isolated loads of 10, 20 and 30 tons respectively at 6, 8 and 16 ft. from one support and acting normal to the beam. Draw the shearing force and bending moment diagrams for the beam and find the maximum bending moment.

2. Isolated vertical loads of 4, 5 and 6 tons are carried by a horizontal beam of 18 ft. span supported at its ends. The loads act respectively at 3 ft., 9 ft. and 16 ft. from one support. Draw the shearing force and bending moment diagrams for the beam and find the maximum bending moment.

3. A cantilever 14 ft. long carries three loads of 3, 4 and 5 tons at distances of $2\frac{1}{2}$, $7\frac{1}{2}$ and 14 ft. from the fixed end. If the cantilever also carries a uniform load of 0·5 ton per ft. run, draw the B.M. and S.F. diagrams. What is the B.M. and S.F. at a section 5 ft. from the support?

4. A beam spans 28 ft. and overhangs the right-hand end by 4 ft. A concentrated load of 2 tons is carried on the overhanging end, a concentrated load of 6 tons at 8 ft. from the left-hand support, and a uniformly distributed load of 0·5 ton per ft. run over a 10-ft. length starting from the left-hand support. Employ a graphical method to draw the S.F. and B.M. diagrams, and find the B.M. and S.F. at the 6 ton load.

5. An overhanging beam is supported and loaded as shown in Fig. 142. Calculate the reactions of the supports, the shearing forces and bending moments at each load, and draw the S.F. and B.M. diagrams. State the maximum B.M. and where it occurs. Also mark the "points of inflexion" or zero B.M.

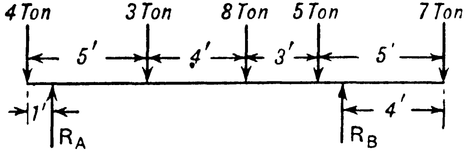


FIG. 142.

6. Draw to scale the B.M. and S.F. diagrams for a bridge of 50-ft. span supported at the ends and loaded with a 2–8–0 type locomotive, as shown in Fig. 143. State the maximum B.M. and mark its position. Neglect the weight of the bridge.

7. In Question 6 and Fig. 143 calculate:

(a) the reactions of the supports,

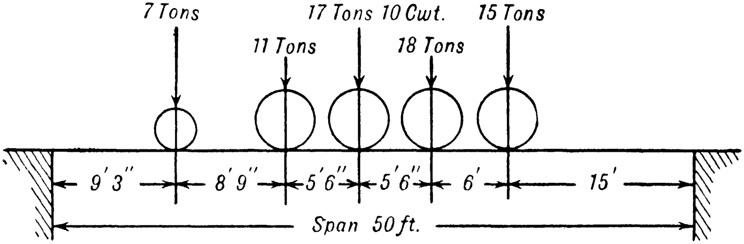


FIG. 143.

(*b*) the position of the centre of gravity of the engine measured back from the centre of the bogie axle,

(*c*) the bending moment at the section of the bridge immediately below the centre of gravity of the engine.

Neglect the weight of the bridge.

8. Find the greatest intensity of stress arising from a B.M. of 90 inch tons acting on a beam of symmetrical section 8 in. deep if I equals 75 inch^4 units.

9. Calculate the moment of resistance for a beam 12 in. deep if I for the section is 245 (inch)^4 units and the maximum skin stress due to bending is $7\frac{1}{2}$ tons per sq. in.

10. Find the positions of the centroids of the sections in Fig. 144 and the moments of inertia of the sections about axes through the centroid parallel to the flanges.

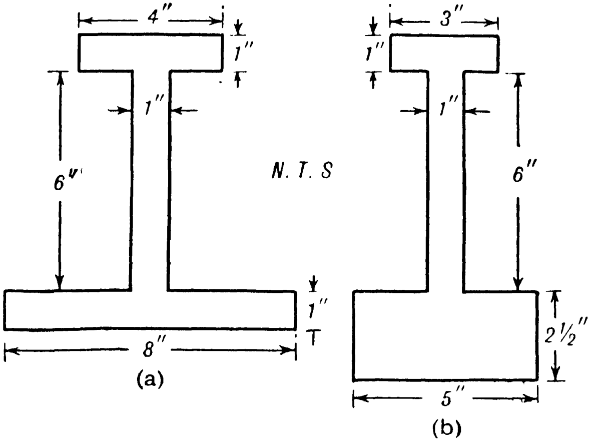


FIG. 144.

11. A beam 8 in. wide and 10 in. deep is of rectangular section. If the stress due to bending is not to exceed 7 tons per sq. in., calculate the maximum bending moment the beam can withstand.

12. To what radius may a beam of symmetrical section 10 in. deep be bent without producing a skin stress greater than 6 tons per sq. in. Take E as 13,500 tons per sq. in. What would be the moment of resistance if the second moment of the section is 211 (inch)^4 units?

13. A wooden beam 12 in. deep and 6 in. wide with a span of 15 ft. and simply supported at its ends carries a central load of 3 tons. Find

the greatest intensity of stress due to bending and the approximate value of the radius of curvature at mid-span. What uniform loading per foot run would produce the same maximum intensity of bending stress? $E = 900{,}000$ lb. per sq. in.

14. To what radius may a beam of symmetrical section 12 in. deep be bent without producing a skin stress greater than 7 tons per sq. in., if $E = 13{,}500$ tons per sq. in.? What would the moment of resistance be if I for the section is 251 (inch)^4 units?

15. What is the greatest central load that a simply supported beam having a span of 3 ft. and a rectangular section $1\frac{1}{2}$ in. wide and 2 in. deep may carry if the greatest stress due to bending is not to exceed 7 tons per sq. in.?

16. What distributed load may be carried by a simply supported beam of 20 ft. span if the depth and moment of inertia of the section are respectively 12 in. and 375 (inch)^4 units? The maximum permissible stress is $7\frac{1}{2}$ tons per sq. in. What isolated central load can be carried to produce the same maximum stress due to bending?

17. A steel wire of circular section is bent around a drum 6 ft. in diameter. If the elastic limit for the steel is 14 tons per sq. in., and the Modulus of Elasticity is 14,000 tons per sq. in., find the maximum diameter of the wire in order that there shall be no permanent deformation when it is removed from the drum. Also find :

(*a*) The moment of inertia of the section of the wire about its neutral axis.

(*b*) The maximum bending moment on the wire.

18. A floor has to carry a load of $1\frac{1}{4}$ cwt. per sq. ft. The joists are 12 in. deep and 4 in. wide and have a span of 20 ft. How far apart may the centre lines be placed if the stress due to bending is not to exceed 1100 lb. per sq. in.? Assume the joists are simply supported at the ends.

19. A wooden floor is supported by wooden joists 10 in. deep and 4 in. wide, and they are placed 15 in. apart, centre to centre. The clear span is 20 ft. If the maximum stress in the wood due to bending is not to exceed 1200 lb. per sq. in., determine the uniformly distributed load per sq. ft. this floor can safely carry.

20. A beam of rectangular section 6 in. deep and 4 in. broad resists a B.M. of 120 tons in. Show how the intensity of direct stress varies throughout the depth of the beam. Divide the part which is in tension into three strips each 1 in. deep and 4 in. wide, and estimate for each strip (1) the total tension in tons across the section, and (2) the moment in tons inches of its pull about the neutral axis. (A.M.I.Mech.E.)

21. A rectangular beam 9 in. deep and $3\frac{1}{2}$ in. wide is required to support a load of 200 lb. per foot run. If the maximum intensity of

bending stress is not to exceed 1000 lb. per sq. in., find the greatest span the beam can have.

22. A steel beam 36 in. long is symmetrically and horizontally supported on knife-edges placed 30 in. apart. The centre of the beam receives an upward deflection of 0·24 in. when loads of 42 lb. are applied at each of the extreme ends. What deflection would be caused if loads of 14 lb. had been applied at the ends? Also calculate the deflection in the case of the 42 lb. loads if both I for the beam section and E for the material of the beam were doubled.

23. In an experiment to determine the approximate modulus of elasticity of brass, a bar 3 ft. 8 in. long and of square section $\frac{9}{16}$ in. × $\frac{9}{16}$ in. was symmetrically supported on knife-edges placed horizontally and exactly 24 in. apart. Loads of $3\frac{1}{2}$ lb. were applied at each end of the beam, and the deflection at the centre was found to be 0·017 in. Calculate the modulus for this specimen.

24. A similar experiment to that described in Question 23 was carried out on a steel beam. In this case the beam was 0·75 in. wide and 0·375 in. deep and the deflection 0·035 in. under the same conditions. Calculate the value of E for this specimen of steel.

25. A beam of span 20 ft. supported horizontally at its ends carries a uniformly distributed load of 1 ton per foot run and a vertical central load of 5 tons. Calculate :

(*a*) the maximum bending moment acting in tons inches ;

(*b*) the modulus of the section of the beam required if the stress due to bending is not to exceed 5 tons per sq. in. ;

(*c*) prove the formula used in finding the maximum bending moment in a beam supported at each end and loaded with a uniformly distributed load.

26. An engineer desires to bend a spring steel strip of selected homogeneous material $\frac{1}{16}$ in. in thickness, 1 in. wide and 8 ft. in length to a perfectly circular arc of 5 ft. diameter. Use your knowledge of the theory of bending, and illustrate by sketches how this may be done by inducing a constant bending moment. What bending moment will be required, and what maximum stress will be set up if $E=13{,}500$ tons per sq. in.?

27. Sketch the B.M. and S.F. diagrams, inserting the principal values, for an axle of a railway wagon, gauge 4 ft. $8\frac{1}{2}$ in., loads 6 ft. apart, symmetrically placed and $2\frac{1}{2}$ tons each. If the axle is 5 in. in diameter, calculate the maximum stress in it due to bending.

28. A horizontal beam AB of 9 ft. span and simply supported at its ends carries loads of 5, 4 and 2 tons at respectively 2 ft., 5 ft. and 8 ft. from A. Draw the S.F. and B.M. diagrams for the beam, and state the maximum values for each diagram. $E=13{,}500$ tons per sq. in. Also :

(*a*) find the equivalent uniform load per foot run on this beam that would produce this maximum bending moment ;

(*b*) find the radius of curvature of the beam at the section where the maximum B.M. acts if $I = 150$ in.^{4} units ;

(*c*) calculate the necessary modulus of section if the stress due to bending is not to exceed 6 tons per sq. in.

29. A girder AB, 12 ft. long, of which the section has a moment of inertia of 374 (in.)^4 units, is 5 in. wide and 10 in. deep. The girder is freely supported horizontally at A and C, C being 10 ft. from A. Loads are applied vertically of magnitudes 4, 3 and 5 tons respectively 2 ft., 7 ft. and 12 ft. from A. Draw the B.M. and S.F. diagrams for this beam, and find (*a*) the maximum B.M. and S.F. to which the girder is subjected ; and (*b*) the greatest intensity of tensile and compressive stress in the beam due to bending.

30. A rolled girder 10 in. deep has a span of 16 ft. and is freely supported at each end. If it carries a uniformly distributed load of 4000 lb. per foot run on half its length from one support to the centre of the span, draw the B.M. and S.F. diagrams, and calculate the greatest intensity of bending stress in the girder, I being 345 (inch)^4 units about the neutral axis of the beam section. (A.M.I.Mech.E.)

31. A steel beam carrying a single load midway between its end supports is thereby stressed to a maximum intensity of 10 tons per sq. in. Find the greatest intensity of stress under the action of the same central load in a second beam in all respects geometrically similar to the first but having all its linear dimensions 20 per cent. greater. (A.M.I.Mech.E.)

32. A bar 5 ft. long of rectangular section 1 in. wide by 3 inches deep is subjected to a pull of 30 tons, the axis of the pull being parallel to the axis of the bar, but distant one-tenth of an inch from it. Determine (*a*) the intensity of direct stress ; (*b*) the B.M. to which the bar is subjected ; and (*c*) the extreme or maximum tensile and compressive stress intensities.

33. A reinforced concrete beam 6 in. wide and depth 13½ in. is reinforced with four bars placed symmetrically 12 in. below the top of the beam. If the permissible stresses are 0·2 ton per sq. in. for the concrete and 5 tons per sq. in. for the reinforcement, calculate the diameter of the bars and the moment of resistance of the section.

CHAPTER IX

LINEAR AND ANGULAR MOTION, FORCE AND TORQUE. VELOCITY AND ACCELERATION. MOMENTUM. WORK, POWER AND ENERGY. FLYWHEELS

Linear and Angular Motion

Motion in one plane may be classified as (1) linear, or (2) angular, sometimes called rotational, or (3) a combination of the two. Frequently, problems have to be solved in which motion is simply translational, or simply rotational, or a combination of the two. In addition, the motion is often such that the acceleration is uniform, or may be considered as such. The equations of motion with uniform acceleration for both linear and angular motion are derived below, and as they are widely used they should be memorized by the student.

Equations of motion with uniform acceleration. The equations for the two fundamental types of motion are developed side by side for the purposes of comparison, commencing with the usual symbols for displacement, velocity and acceleration, and obtaining the equations by integration and elimination.

Translational motion	Rotational motion
Displacement, s	θ
Velocity, $v=\frac{ds}{dt}$.	$\omega=\frac{d\theta}{dt}$.
Acceleration,	
$\frac{dv}{dt}=\frac{d^2s}{dt^2}=$ linear acceleration $=a$.	$\frac{d\omega}{dt}=\frac{d^2\theta}{dt^2}=$ angular acceleration $=\alpha$.
Integrating, $\frac{ds}{dt}=v=at+c_1$,	$\frac{d\theta}{dt}=\omega_2=\alpha t+c_1$,
or $\mathbf{v}=\mathbf{u}+\mathbf{a}t$, ...(1)	or $\boldsymbol{\omega}_2=\boldsymbol{\omega}_1+\boldsymbol{\alpha}t$, ...(1)
if $v=u$, when $t=0$.	if $\omega_2=\omega_1$, when $t=0$.

Integrating again,	
$s = ut + \frac{1}{2}at^2 + c_2$	$\theta = \omega_1 t + \frac{1}{2}\alpha t^2 + c_2$
$\mathbf{s = ut + \frac{1}{2}at^2}$, ...(2)	$\boldsymbol{\theta = \omega_1 t + \frac{1}{2}\alpha t^2}$, ...(2)
if $s = 0$, when $t = 0$.	if $\theta = 0$, when $t = 0$.
Also, s = average velocity × time	
$\mathbf{s} = \left(\frac{\mathbf{u + v}}{2}\right)\mathbf{t}$. ....(3)	$\boldsymbol{\theta} = \left(\frac{\boldsymbol{\omega_2 + \omega_1}}{2}\right)\mathbf{t}$. ......(3)
By eliminating t from (2) and (3)	
$\mathbf{v^2 = u^2 + 2as}$. ....(4)	$\boldsymbol{\omega_2^2 = \omega_1^2 + 2\alpha\theta}$. ....(4)

Examples to illustrate the use of the equations (1) to (4) are given below. It should also be remembered that the relationships *between* linear and angular velocity are given by

$$v = \omega r \text{ or } \omega = v/r\,; \quad \text{and } a = \alpha r \text{ or } \alpha = a/r.$$

Example 1. *A car starting from rest with a constant acceleration attains a speed of* 45 *m.p.h. in* 600 *yd. Find the acceleration and the time taken.*

$u = 0$; $s = 1800$ ft. ; $v = 45$ m.p.h. $= 66$ ft. per sec.,
since 60 m.p.h. = 88 ft. per sec.

Using (4) above for linear motion : $v^2 = u^2 + 2as$;

$$a = (v^2 - u^2)/2s = 66^2/3600 = 1{\cdot}21 \text{ ft. per sec. per sec.}$$

Example 2. *A point on the rim of a flywheel* 8 *ft. in diameter has a velocity of* 15 *ft. per sec., and this is uniformly accelerated to* 21 *ft. per sec. in* 3 *minutes. Find the initial and final angular velocity of the flywheel and the total number of revolutions made. Also find the angular acceleration.*

$$\text{Initial velocity} = \omega_1 = \frac{v_1}{r} = \frac{15 \text{ f./s.}}{4 \text{ ft.}} = 3{\cdot}75^c \text{ per sec.}$$

$$\text{Final velocity} = \omega_2 = \frac{v_2}{r} = \frac{21 \text{ f./s.}}{4 \text{ ft.}} = 5{\cdot}25^c \text{ per sec.}$$

Using (3) above for angular motion,

$$\begin{aligned} \theta &= \text{No. of radians turned through} = (\omega_2 + \omega_1)t/2 \\ &= (5{\cdot}25 + 3{\cdot}75)\,180/2 \qquad = 810 \text{ radians} \\ &= 810/2\pi \text{ or } 128{\cdot}9 \text{ r.p.m.} \\ \alpha &= \text{angular acceleration} = (\omega_2 - \omega_1)/t \text{ from (1) above} \\ &= (5{\cdot}25 - 3{\cdot}75)/180 \qquad = 1{\cdot}5/180 \\ &\qquad = 0{\cdot}0083^c \text{ per sec. per sec.} \end{aligned}$$

Example 3. *At a certain instant a rotor has a velocity of* 5 *radians per sec. and is uniformly accelerating at the rate of* 3 *radians per sec. What will be its velocity after it has made* 1200 *more revolutions?*

$$1200 \text{ revs.} = 2400\pi \text{ radians} = \theta \; ; \quad \omega_1 = 5^c \text{ per sec.}$$

$$\alpha = 3^c \text{ per sec. per sec.}$$

Using (4) above : $\omega_2{}^2 = \omega_1{}^2 + 2\alpha\theta = 5^2 + 2 \times 3 \times 2400\pi,$

$$\omega_2 = 212{\cdot}7^c \text{ per sec. or } 33{\cdot}86 \text{ r.p.s.}$$

Space, velocity and acceleration diagrams. When a space-against-time diagram is plotted for a locomotive, vehicle or a machine part, the distances s covered are usually plotted as ordinates and times t as abscissæ. The slope of this diagram at any given time obviously is ds/dt, or the velocity at that time. If the slopes are obtained at various times and plotted as ordinates at these times, then a velocity-against-time graph will be produced. The slope of the graph of velocity against time, that is, dv/dt, obviously gives the acceleration at the time considered ; and therefore by finding the slopes at various times it is possible to draw an acceleration-against-time graph. If the mass of the locomotive or machine part is constant, then since force is proportional to acceleration, the acceleration-against-time graph will also represent the force-against-time graph to a different scale. In this case it is only the force necessary to produce the acceleration which is represented. Often, however, with big accelerations and decelerations (or retardations) met with in modern engineering problems, these forces are of considerable magnitude and the acceleration-against-time graph is very important.

The *area* under the velocity-time and acceleration-time graphs has also an important significance. Thus the area of the velocity-time graph between any two ordinates at t_1 and t_2 represents the distance covered in that time interval, because the area represents the average velocity multiplied by the time.

Similarly, the area under the acceleration-time graph between any two ordinates t_1 and t_2 represents the total change of velocity, because

$$\text{Acceleration} = \frac{\text{velocity change}}{\text{time change}} \quad \text{and}$$

$$\text{Acceleration} \times \text{time} = \frac{\text{velocity change}}{\text{time change}} \times \text{time change}$$
$$= \text{velocity change.}$$

Example. *In the speed-time curve for an electric train the time scale was* 1 *in.* = 20 *sec. and the speed scale* 1 *in.* = 10 *m.p.h. After* 30 *sec. from rest, the tangent to the curve was at* 42° *to the horizontal, and the area under the curve on a base of* 180 *sec. was* 26·4 *sq. in. Find the acceleration after* 30 *sec., the distance covered by the train in* 3 *minutes and the average speed.*

$\tan 42^\circ = 0{\cdot}9004 =$ apparent slope of curve after 30 sec.

1 in. = 10 m.p.h. = $14\frac{2}{3}$ ft. per sec.

$$\text{Real slope} = \text{acceleration} = \frac{0{\cdot}9004 \times 14\frac{2}{3} \text{ ft./sec.}}{20 \text{ sec.}}$$
$$= 0{\cdot}6603 \text{ ft./sec./sec.}$$

1 sq. in. of the diagram represents $\frac{14\frac{2}{3} \text{ ft.}}{\text{sec.}} \times 20 \text{ sec.} = 293\frac{1}{3}$ ft.

26·4 sq. in. represents $293\frac{1}{3} \times 26{\cdot}4$ or 7744 ft.

Average speed = 7744/180 or 43 ft./sec. = 29·3 m.p.h.

Torque and the laws of motion. At this stage it is appropriate to define torque and to restate Newton's Laws of Motion (Vol. I, p. 261), and to study them with particular relation to rotary motion.

Torque is the turning moment acting on a body which produces or tends to produce angular motion and, in rotary motion, corresponds to force in linear motion. Torque is measured in pounds feet or pounds inches.

First or inertia law. A body continues in a state of rest or of

$$\left\{\begin{matrix} \text{uniform motion in a straight line} \\ \text{or} \\ \text{constant angular velocity about an axis} \end{matrix}\right\}$$

unless compelled by external $\left\{\begin{matrix} \text{force} \\ \text{or} \\ \text{torque} \end{matrix}\right\}$ to change that state.

Second law. The rate of change of $\left\{\begin{matrix} \text{linear} \\ \text{or} \\ \text{angular} \end{matrix}\right\}$ momentum of a body is proportional to the $\left\{\begin{matrix}\text{force}\\ \text{or}\\ \text{torque}\end{matrix}\right\}$ acting, and takes place $\left\{\begin{matrix}\text{in the direction of the straight line in which the force acts}\\ \text{or}\\ \text{about the axis of the torque.}\end{matrix}\right\}$

It should be noticed that the first law is a particular case of the second law, when the rate of change of momentum, whether angular or linear, is zero.

Dealing with the second law and linear motion, if W/g is the mass of a body and v its instantaneous velocity, the expression for its momentum (see also p. 19) will be Wv/g. Hence, the rate of change of momentum will be $\frac{d}{dt}\left(\frac{Wv}{g}\right)$; and if the mass is constant, this expression becomes $\frac{W}{g}\cdot\frac{dv}{dt}$, or $\frac{Wa}{g}$ if a is the acceleration at the instant.

Thus, $\mathbf{F} = \textbf{force acting} = \mathbf{Wa/g}$. ....(5)

Third law. To every $\left\{\begin{matrix}\text{force}\\ \text{or}\\ \text{torque}\end{matrix}\right\}$ there is an equal and opposite $\left\{\begin{matrix}\text{force}\\ \text{or}\\ \text{torque}\end{matrix}\right\}$ generally called the $\left\{\begin{matrix}\text{reaction}\\ \text{or}\\ \text{torque reaction.}\end{matrix}\right\}$

Example. *A train weighing* 450 *tons is drawn up an incline of* 1 *in* 48 *at a speed of* 12 *m.p.h. by a locomotive of* 90 *tons weight. What horse-power is required if the frictional resistance of the engine and train is* 10 *lb. per ton? What speed could be attained by this train on the level with the same frictional resistance and utilising the same horse-power?*

(U.E.I.)

Total weight of train $= 450 + 90$ or 540 tons.
Tractive force due to incline $= 540 \times 1/48$ tons or 25,200 lb.
Tractive force due to friction $= 10 \times 540$ lb.
Total tractive effort $= 30{,}600$ lb.
Distance traversed per min. $= 12 \times 5280/60$ or 1056 ft.
∴ horse-power needed $= 30{,}600 \times 1056/33{,}000$
$= 979{\cdot}2$.

On the level, the tractive effort necessary = 5400 lb.

∴ the locomotive can now travel 30,600 × 1056/5400 ft. per min.

or, **5984 ft. per min., or 68 m.p.h.**

Torque and angular acceleration. Imagine a body (Fig. 145) turning at a given instant with angular acceleration α radians per second per second about the axis O. Suppose the body to be divided into a large number of small particles A, B, ... of mass W_1/g, W_2/g, ... at distances R_1, R_2, ... from O and acted on by forces F_1, F_2, ... respectively at right angles to R_1, R_2, ...

The instantaneous linear acceleration of A, B, ... will be αR_1, αR_2, ... respectively. It is required to find the torque T necessary to give the body this acceleration. The total forces required on the particles will be

$$F_1 + F_2 + \ldots = (W_1 \alpha R_1 + W_2 \alpha R_2 + \ldots)/g.$$

The combined torque T necessary will be

$$\begin{aligned} T = F_1 R_1 + F_2 R_2 + \ldots &= (W_1 \alpha R_1^2 + W_2 \alpha R_2^2 + \ldots)/g \\ &= \alpha (W_1 R_1^2 + W_2 R_2^2 + \ldots)/g. \end{aligned}$$

The quantity above in brackets is called the **moment of inertia** of the

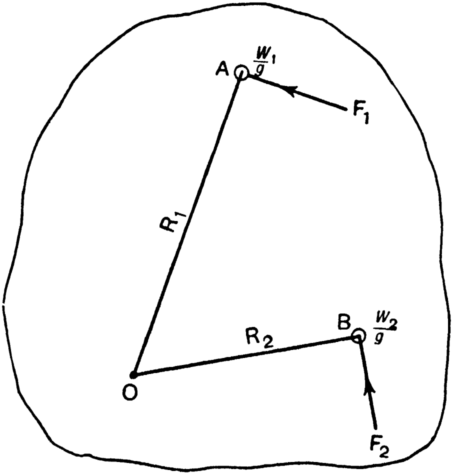


FIG. 145.

body about the axis O, and is denoted by the letter **I** ; or **I/g** in engineer's units. Hence,

$$\mathbf{T}=\mathbf{I}\alpha/\mathbf{g}. \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(6)$$

It should be noticed that the symbol I for moment of inertia takes the place of mass in the corresponding formula for linear motion (p. 228). As stated in Newton's First Law, a body which possesses inertia cannot change its own condition of rest or motion ; mass and moment of inertia are really measures of inertia for linear or angular motion respectively.

Radius of gyration. The radius of gyration k of a body about a given axis is that radius at which, if an equal mass were concentrated, it would have the same moment of inertia. Thus, if the mass of the body is W/g, the moment of inertia will be Wk^2/g.

Example. *A flywheel starting from rest attains a speed of* 3600 *r.p.m. after* 2 *minutes. The flywheel weighs* 56 *lb. and its radius of gyration is* 6 *in. Calculate the angular acceleration to which the flywheel has been subjected and the torque producing that acceleration.*

3600 r.p.m. $=120\,\pi$ radians per second.

$$\alpha=\text{angular acceleration}=\frac{120\pi}{120}=\pi \textbf{ radians per sec. per sec.}$$

$$I/g=\text{moment of inertia of flywheel}$$
$$=Wk^2/g=56\times(\tfrac{1}{2})^2/g \text{ engineer's units.}$$
$$T=\text{torque}=I\alpha/g=56\times\pi/4g=1{\cdot}366 \textbf{ lb. ft.}$$

Impulse. If a force or torque acts for a definite time, usually short, the effect is known respectively as an **impulse** or **angular impulse.** Should these effects of forces or torques over a short time act on a rigid body of constant mass, the impulse is measured by the total change of momentum.

Thus, $$\text{impulse}=\int F\,dt$$
$$=\text{total change of momentum ;}$$
$$=\text{total change of velocity}\times W/g.$$

Similarly, $$\text{angular impulse}=\int T\,dt$$
$$=\text{total change of angular momentum}$$
$$=\text{total change of angular velocity}\times I/g.$$

With a constant force or torque these expressions reduce to

$$Ft = W(v_2 - v_1)/g, \quad \dots\dots\dots\dots\dots\dots\dots(7)$$

where v_2 and v_1 are the final and initial velocities for linear motion; and

$$Tt = Wk^2(\omega_2 - \omega_1)/g, \quad \dots\dots\dots\dots\dots\dots\dots(8)$$

where ω_2 and ω_1 are the final and initial velocities for angular motion.

The statements (7) and (8) represent the second law of motion with reference to total changes of momentum, whereas expressions (5) and (6) refer to rate of change of momentum. Expression (8) if divided by t reduces to (6), because $\omega_2 - \omega_1 = \alpha t$.

Example. *A fixed plate receives a stream of water at a velocity of* 300 *ft. per second, delivered from a nozzle* 1 *in. in diameter and perpendicular to the plate. Calculate the impulse on the plate.*

$$\text{Mass of water per second} = \pi \times 1^2 \times 300 \times 62{\cdot}5/4 \times 144$$
$$= 4{\cdot}082 \text{ lb.}$$
$$\text{Impulse} = \text{total change of velocity} \times W/g = 300 \times 4{\cdot}082/g$$
$$= 38 \text{ lb. sec.}$$

Third law and the conservation of momentum. The Third Law may also be stated as " Forces (or torques) always occur in pairs ". Many examples of this law may be quoted; for example, the gudgeon pin exerts a force on the connecting rod and the connecting rod reacts with equal force on the gudgeon pin; the torque exerted by the engine on the crank shaft is resisted by the torque reaction set up by machines driven from the crank shaft.

According to the Third Law, action and reaction are equal and opposite; and this applies to both the forces and impulses acting. Therefore, if one body gains momentum, the other body concerned in the mutual action will lose exactly the same amount; and the total or resultant change of momentum is zero. This principle of the **conservation of momentum** is true for any isolated system of bodies and is also true for any and every direction of motion. The truth of the principle is not affected by forces of friction, because if a frictional force acts on a body against its direction of motion this force is a means of transferring momentum from the body to the earth. For example, the explosive charge in a gun gives momentum to the shell and an equal and opposite amount to the gun. The momentum of the recoiling gun is transmitted to the earth and so is that of the shell on impact, so that the resultant momentum given to the earth is also zero.

Example. *A gun weighing* 42 *tons fires a shell weighing* 1540 *lb. and gives it a muzzle velocity of* 2400 *ft. per sec. Find the initial speed of recoil of the gun.*

If the recoil is controlled by a time average force of 500 *tons, find how long the recoil takes.*

$$\text{Momentum of gun} = \text{momentum of shell.}$$
$$42 \times 2240 \times v = 1540 \times 2400.$$
$$v = 1540 \times 2400/(42 \times 2240)$$
$$= \mathbf{39{\cdot}3\ ft./sec.}$$
$$\text{Time for recoil} = 1540 \times 2400/(500 \times 2240)$$
$$= \mathbf{3{\cdot}3\ sec.}$$

Time average of a force or a twisting moment. Another important type of diagram is that depicting the time average of a force or twisting moment. As implied, the force or twisting moment is plotted as an ordinate on a time base. Because

$$\text{force} \times \text{time} = \text{mass} \times \text{velocity} = \text{momentum},$$

the area under the force-time diagram will represent momentum. The area of the diagram divided by the appropriate time interval will therefore give the time average of the force. It should be noted that the *time average* is quite different from the *space average* of a force.

Motion of connected bodies. Many engineering problems occur in connection with lifts and hoists in buildings and ships, and winding gear in the case of mines and quarries, where heavy loads are raised and lowered by ropes or cables which are employed to control the accelerating and decelerating or retarding forces often with the aid of balance weights. Newton's Laws can be used to solve these problems, and two important cases are given below.

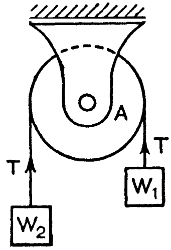


FIG. 146.

Suppose a frictionless pulley free to turn about axis A supports two loads W_1 and W_2 with the aid of an inextensible cord of negligible weight, as shown in Fig. 146. If W_1 is larger than W_2, motion will occur in a clockwise direction. W_1 will descend and W_2 will rise with an acceleration a. Suppose the tension in the cord is T, then considering the equilibrium of W_1, the accelerating force is $W_1 - T$, and therefore, by statement (5),

$$W_1 - T = W_1 a/g.$$

Similarly, W_2 will rise due to an accelerating force $T - W_2$, and

$$T - W_2 = W_2 a/g.$$

Adding,
$$W_1 - W_2 = (W_1 + W_2) a/g$$

or
$$a = \frac{(W_1 - W_2) g}{W_1 + W_2}. \tag{9}$$

Substituting this value of a in the first expression,

$$T = 2W_1 W_2/(W_1 + W_2). \tag{10}$$

It should be noticed that the tension in the cord is not necessarily the same as would be the case if the loads were at rest.

If the acceleration a above had been required, the result could have been obtained more quickly by considering the accelerating force of the system as $W_1 - W_2$ and the loads moved as $W_1 + W_2$. Thus

$$W_1 - W_2 = (W_1 + W_2) a/g, \text{ or } a = (W_1 - W_2) g/(W_1 + W_2).$$

In this case the tension T is ignored or considered as an internal force which does not affect the momentum of the system.

Now consider the case when W_2 reposes on an incline at θ to the horizontal and the coefficient of friction between the load and the incline is μ and the acceleration is a. In this case the force and

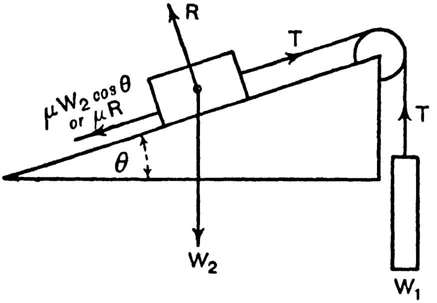


FIG. 147.

acceleration equations for W_1 and W_2 are respectively

$$W_1 - T = W_1 a/g,$$
$$T - W_2 \sin\theta - \mu R = W_2 a/g.$$

Adding to eliminate T, $W_1 - W_2 \sin\theta - \mu R = (W_1 + W_2)\,a/g$.

But $R = W_2 \cos\theta$, and therefore

$$W_1 - W_2 (\sin\theta + \mu\cos\theta) = (W_1 + W_2)\,a/g,$$

or $$a = \{W_1 - W_2 (\sin\theta + \mu\cos\theta)\}\,g/(W_1 + W_2). \quad \text{..........(11)}$$

Substituting this expression for a,

$$T = W_1 W_2 (1 + \sin\theta + \mu\cos\theta)/(W_1 + W_2). \quad \text{...........(12)}$$

If the incline is smooth, then $\mu = 0$ in (11) and (12); and if W_2 moves over a horizontal surface, then $\theta = 0$.

Notice that if the pulley A had appreciable inertia and bearing friction and if the rope were elastic, the expressions (11) and (12) would no longer be accurate.

Summary of the relations between linear and angular motion. The comparison between linear and angular motion can now be carried a stage further :

LINEAR	ANGULAR
Force = Mass × Acceleration $= \frac{Wa}{g} = \frac{W}{g} \cdot \frac{d^2s}{dt^2}$.	Torque = Moment of Inertia × Acceleration $= \frac{I\alpha}{g} = \frac{Wk^2\alpha}{g} = \frac{Wk^2}{g} \cdot \frac{d^2\theta}{dt^2}$.
Momentum = Mass × velocity $= \frac{W}{g} \cdot \frac{ds}{dt}$.	Moment of Momentum = Moment of Inertia × velocity $= \frac{Wk^2}{g} \cdot \frac{d\theta}{dt}$.
Impulse = Change of momentum. $Ft = W(v_2 - v_1)/g$.	Angular impulse = change of angular momentum. $Tt = Wk^2(\omega_2 - \omega_1)/g$.
Work = Force × distance in direction of force $= Fs$.	Work = Torque × angle turned through in radians $= T\theta$.
Kinetic Energy $= \frac{Wv^2}{2g} = \frac{W}{2g}\left(\frac{ds}{dt}\right)^2$.	Kinetic Energy $= \frac{I\omega^2}{2g} = \frac{I}{2g}\left(\frac{d\theta}{dt}\right)^2$.
Work done = Change of K. Energy. $Fs = W(v_2^2 - v_1^2)/2g$.	Work done = Change of K. Energy. $T\theta = I(\omega_2^2 - \omega_1^2)/2g$.

WORK, POWER AND ENERGY

Energy. Suppose a force F acts on a body of mass W/g over a small distance δs and increases its velocity from v_1 to v_2 with a mean acceleration a. Then the work done will be $F\ \delta s$. However, $F = Wa/g$ and $a\ \delta s = (v_2^2 - v_1^2)/2$ from (4). Hence,

$$F\ \delta s = Wa\ \delta s/g = W(v_2^2 - v_1^2)/2g. \quad\ldots\ldots\ldots\ldots\ldots\ldots(13)$$

Thus the work done is equal to the change of kinetic energy, which in this case increases from $Wv_1^2/2g$ to $Wv_2^2/2g$ (see p. 281, Vol. I).

Similarly, if a torque T acting through an angle $\delta\theta$ on a body of moment of inertia I/g increases the velocity from ω_1 to ω_2 with a mean acceleration α, it will do an amount of work equal to $T\ \delta\theta$.

Then, $$T\ \delta\theta = I\alpha\ \delta\theta/g = I(\omega_2^2 - \omega_1^2)/2g. \quad\ldots\ldots\ldots\ldots\ldots\ldots(14)$$

In this case the work done by the torque increases the kinetic energy from $I\omega_1^2/2g$ to $I\omega_2^2/2g$.

The relations (13) and (14) obviously remain true if the force F and torque T are variable quantities, provided F and T represent the mean values with respect to the distance δs and angle $\delta\theta$ respectively.

In the general case, (13) and (14) become respectively :

$$\text{Work done} = \Sigma F\ \delta s = W(v_2^2 - v_1^2)/2g$$

and $$\text{Work done} = \Sigma T\ \delta\theta = I(\omega_2^2 - \omega_1^2)/2g.$$

Example. *A flywheel is required to store* 6000 *ft. lb. of energy while its speed is gradually increased from* 199 *to* 201 *r.p.m. What should it weigh if its radius of gyration is* 2 *ft.? Calculate its kinetic energy at* 200 *r.p.m.*

$$\omega_2^2 = \left(\frac{201 \times 2\pi}{60}\right)^2, \quad \omega_1^2 = \left(\frac{199 \times 2\pi}{60}\right)^2, \quad I = Wk^2/g.$$

Using (13), work stored = change of kinetic energy.

$$\begin{aligned} 6000 &= Wk^2(\omega_2^2 - \omega_1^2)/2g. \\ W &= 12{,}000g/k^2(\omega_2^2 - \omega_1^2) \\ &= 12{,}000 \times 3600g/(4 \times 800 \times 4\pi^2) \\ &= 11{,}500 \text{ lb.} \end{aligned}$$

$$\begin{aligned} \text{K.E. at 200 r.p.m.} &= \frac{11{,}500 \times 4}{2g}\left(\frac{200 \times 2\pi}{60}\right)^2 \\ &= 315{,}000 \text{ ft. lb.} \end{aligned}$$

Space average of a force or twisting moment. When force is plotted as an ordinate on a distance base, or when a torque is plotted as an ordinate on a base of radian measure, the area under the graph

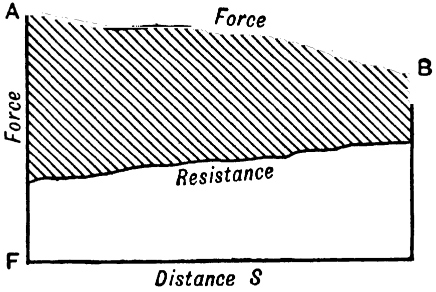


FIG. 148.

obviously represents work done. If the force acting on a machine is greater than the resistance set up, then the difference between the force and resistance is available to increase the kinetic energy of the machine. Thus the shaded area on the diagram below the line AB represents the gain of kinetic energy. If the resistance exceeds the applied force or effort, the area between the graphs of effort and resistance would be considered negative.

Probably the most common example of a space-force graph is the ordinary indicator diagram for a steam engine, where the ordinates represent to some scale the force acting on the piston at any given piston position.

Rolling bodies. When a body rolls on a level track or upon an incline, it possesses two types of kinetic energy, translational and rotational. Its total energy consists of the instantaneous energy of rotation about an axis through its centre of gravity, together with the translational energy considered as due to the entire mass of the body concentrated at the centre of gravity. Thus, if ω is the angular velocity of the body about its centre of gravity, v the velocity of the centre of gravity, and r the distance of the centre of gravity from the track, then the total kinetic energy is

$$\text{K.E.} = Wv^2/2g + Wk^2\omega^2/2g,$$

where W/g is the mass of the body, and k its radius of gyration about the axis of rolling.

$$\begin{aligned}\text{K.E.} &= Wv^2/2g + Wk^2v^2/2gr^2\\ &= (1 + k^2/r^2)\, Wv^2/2g.\end{aligned}$$

This expression shows that the inertia of the rolling body is increased by $(1 + k^2/r^2)$ times as compared with that of the same body if it were sliding along the track at the same speed without angular motion. In the case of a thin hoop, where k approaches the value of r, the kinetic energy is practically doubled when rolling is compared with sliding. The total energy of a train is the sum of the rotational energy of all wheels plus the translational energy of the train itself.

Flywheels. The turning moment in the crank shaft of a single-cylinder engine varies considerably throughout a revolution, being zero at the inner and outer dead-points and having maximum values in between. Diagrams of turning moment can be constructed with turning moments as ordinates and crank angle displacement as abscissæ, or they can be constructed on a polar basis in which the ordinates are measured radially outwards from the crank pin circle. In the case of two or more cylinders, the turning moment diagrams for each cylinder can be combined to make one diagram ; but in no case is the turning moment quite uniform even with careful arrangement of the crank positions for the cylinders. The area of the diagram represents the work done by the engine, and if the speed of the engine is the same at the end as it was at the beginning of the revolution, the work done by the engine is equal to the resistance overcome. In this case, if the resistance diagram is superimposed on the turning moment diagram, the areas of the two diagrams will be equal.

By studying these diagrams it is easy to find the points of maximum and minimum speed during a revolution. For example, in Fig. 149 the curve CDJKLMNB represents the turning moment diagram, and ADEFGHB the uniform resisting torque diagram. Obviously the maximum speed will occur at E and the minimum at H. The greatest fluctuation of energy will be the algebraic sum of the shaded areas between E and H, FLG being considered positive, and EFK and GMH as negative. Thus, the maximum fluctuation of energy is the difference between the kinetic energy of the moving parts measured at the greatest and least speeds during the revolution. The coefficient of fluctuation of energy is the ratio of the maximum fluctuation of energy to the total work done during the revolution.

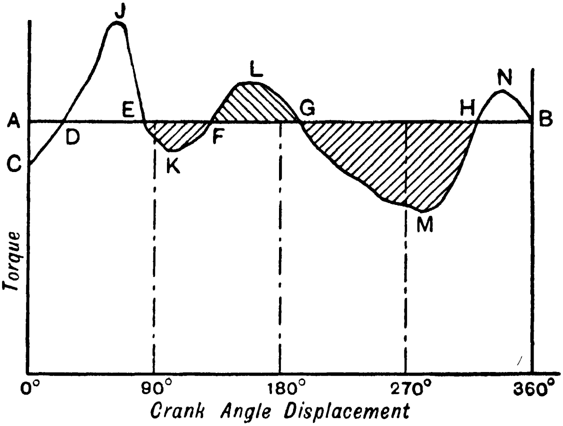


FIG. 149.

To reduce the variation of speed of an engine during each cycle, a flywheel is fitted which alternatively stores and gives out energy. As the average energy of the flywheel is generally much greater than the other moving parts of the engine, the fluctuations of speed are usually small compared with the average speed.

Flywheel design. Flywheel design is a complex subject, but an elementary consideration can be given here. Let W/g be the required mass of the flywheel, ω the mean velocity, ω_2 the maximum and ω_1 the minimum velocity during a revolution, k the radius of gyration, I/g the moment of inertia in engineer's units, K the maximum fluctuation of energy and E the mean energy at the mean velocity.

Then $K = I(\omega_2^2 - \omega_1^2)/2g = I(\omega_2 + \omega_1)(\omega_2 - \omega_1)/2g$.

But $\omega_2 + \omega_1 = 2\omega$ and $E = I\omega^2/2g$.

$$\therefore\ K = I\omega(\omega_2 - \omega_1)/g = 2E(\omega_2 - \omega_1)/\omega$$

$$= 2E \times \text{fractional fluctuation of velocity} = 2EN.$$

Example. *A compound steam engine develops 450 I.H.P. at 90 r.p.m., and it is found from the turning moment diagram that the maximum fluctuation of energy is 20 per cent. of the mean energy per revolution. If the fluctuation of speed is not to exceed 1/150 and the radius of gyration of the wheel is 7½ ft., determine the necessary weight of the flywheel.*

$$E = \text{Work developed per revolution} = \frac{450 \times 33{,}000}{90} \text{ ft. lb.}$$
$$= 165{,}000 \text{ ft. lb.}$$
$$K = 20 \text{ per cent. of } E = 33{,}000 \text{ ft. lb.}$$
$$(\omega_2 - \omega_1)/\omega = N = 1/150,$$
$$\omega = 90 \times 2\pi/60 = 3\pi^c \text{ per sec.}$$
$$\therefore \; E = I\omega^2/2g = Wk^2\omega^2/2g = 33{,}000/2N.$$
$$W = \frac{33000g}{k^2\omega^2 N} = \frac{33{,}000 \times 32{\cdot}2 \times 150}{(7\tfrac{1}{2})^2 \times 9\pi^2 \times 2240} \text{ tons}$$
$$= \mathbf{14{\cdot}24 \text{ tons.}}$$

Hoop stress in flywheels. A rough approximation to the hoop stress induced into the flanges of flywheels can be found by assuming the stress in the rim is due to the necessity of supplying an inward centripetal force to produce angular motion in the particles of the rim.

If w is the density of the material, and a the cross-sectional area of the rim, then the weight of rim subtending an angle $\delta\theta$ at the centre of the wheel will be $wa r\,\delta\theta$, where r is the average radius of

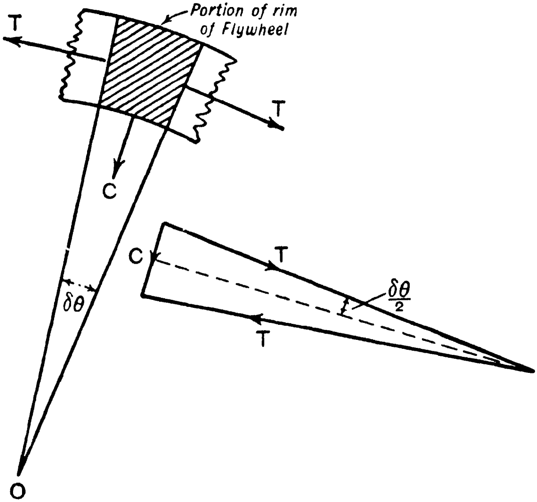


FIG. 150.

the rim. The two tensions T must supply the centripetal force of magnitude $war\ \delta\theta\ v^2/gr$, where v is the mean velocity of the rim at radius r.

Resolving the two forces in the direction of r and equating :

$$C = 2T \sin \delta\theta/2 = war\ \delta\theta\ v^2/gr,$$

or $T = wav^2/g$, since $\delta\theta = \sin \delta\theta$ for small angles.

But $T = fa$, where f is the hoop stress.

Hence $f = wv^2/g$.

Example. *Find the hoop stress in a flywheel if the peripheral velocity of the rim is* 70 *ft. per sec. and the density of the material is* 490 *lb. per cu. ft.*

$$f = wv^2/g = 490 \times 70 \times 70/32{\cdot}2 \text{ lb. per sq. ft.}$$
$$= \mathbf{517{\cdot}7 \text{ lb. per sq. in.}}$$

EXERCISES ON CHAPTER IX

1. State the Laws of Motion.

In an experiment to verify the second law, a trolley was drawn along a plane by the action of a falling weight attached to the trolley by a light string. The times taken were for the trolley to move through 5 ft. from rest for various values of the falling weight.

Falling weight (lb.)	0·2	0·3	0·4	0·5	0·6	0·7	0·8
Time (sec.)	6·3	5·1	4·4	3·9	3·6	3·3	3·1

Show how you would use these figures to verify the second law of motion. If the trolley weighs 24 lb., what is the value of "g"? (U.E.I.)

2. A train is timed between two stopping stations, 3 miles apart, to cover the distance in 6 minutes. It can be uniformly accelerated to maximum speed at 2 ft. per sec.^{2}, and can be uniformly retarded from maximum speed at 6 ft. per sec.^{2} What is the maximum speed and the distance travelled at this speed?

3. A plane is inclined at an angle θ to the horizontal. Neglecting friction, what should be the acceleration of a body sliding down the plane?

A car, weighing, with passengers, 2000 lb., runs down a hill in neutral. The hill has a gradient of 1 in 24, the 24 being measured along the road surface. The car traverses the whole length of the hill, that is, 0·5 mile in 1·5 minutes. At the top of the hill the car was already travelling at 10 m.p.h. Calculate : (*a*) the uniform acceleration ; (*b*) the mean retardation due to friction ; (*c*) the mean frictional resistance against which the car is travelling. (U.E.I.)

4. A locomotive draws a train weighing 290 tons up a gradient of 1 in 200 with uniform acceleration such that a speed of 30 m.p.h. is attained in 4 minutes from rest. If the frictional resistances are 12 lb. per ton and the air resistance varies as the square of the speed and is 43 lb. at 12 m.p.h., find the H.P. required at the speed of 30 m.p.h. (U.E.I.)

5. In Question 4, if the total load on the coupled driving axles is 140 tons, find the least coefficient of friction necessary at the speed stated.

6. A nozzle 3 in. in diameter delivers a stream of water at a velocity of 210 ft. per sec. perpendicular to a fixed plate. What is the impulse of the water on the plate?

7. (*a*) In a velocity-time curve state what the slope of the curve at any point represents, also what is represented by the area between the velocity-time curve, the time axis and the ordinates at the two given times.

(*b*) An electric train can pick up its running speed of 40 miles an hour by uniform acceleration in 20 sec. starting from rest. The application of the brakes produces a uniform retardation of 5 ft. per sec. per sec. Find the time taken by the train under these conditions for a journey starting from one station and stopping at the next station one mile farther on. (U.E.I.)

8. A horizontal shaft, 4 in. in diameter, resting in bearings at its ends, transmits power by belts to various machines. The tensions in the belts cause a horizontal force of 1500 lb., and a vertical downward force of 500 lb. The weight of the shaft and pulleys is 10 cwt. If $\mu = 0{\cdot}07$, find the horse-power absorbed in friction at 100 r.p.m.

9. A disc flywheel weighs 200 lb. and is 24 inches in diameter. Find the time the wheel will take to reach a speed of 1000 r.p.m. neglecting friction. The torque acting is 40 lb. ft.

10. A flywheel weighs 1·6 tons and has a radius of gyration of 2·5 ft. It is running at 180 r.p.m., and its speed is reduced to 150 r.p.m. in 30 sec. Find (*a*) the average angular retardation, and (*b*) the torque resisting motion.

11. A flywheel weighing $4\frac{1}{2}$ tons has a radius of gyration of 4·5 ft. Determine the moment of inertia of the flywheel in engineer's units, and the mean effective torque which may be applied to the flywheel in order to increase its speed by 150 r.p.m. in 45 sec.

12. A steel flywheel cylindrical in shape has a diameter of 12 in. and a thickness of 2 in. Calculate its moment of inertia about the axis of rotation if 1 cubic inch of steel weighs 0·29 lb. The axis of rotation coincides with the axis of the cylinder.

If the speed of this flywheel increases uniformly from 200 to 300 r.p.m. in 20 sec., find the angular acceleration. Also find the torque producing the angular acceleration.

13. A flywheel increases its speed from 240 r.p.m. to 360 r.p.m. in 30 sec. and it is to store an extra 6000 ft. lb. of energy in doing so. If the radius of gyration of the wheel is 1 ft., what should the weight be?

14. The coefficient of friction between a shaft and its bearing is 0·04. The shaft is 8 in. in diameter and carries a flywheel which weighs 10 tons. Find the work done during each revolution of the shaft when the motion is uniform.

If the shaft is rotating at 270 r.p.m., find how many revolutions it will make before coming to rest if the only retarding force is friction. Radius of gyration of flywheel 3 ft. Neglect the inertia of the shaft.

15. The mass and radius of gyration of a flywheel are respectively 10 tons and 6 ft. If it is revolving at 120 r.p.m., find how long it would take and how many revolutions it would make in coming to rest against a retarding torque of 1000 lb. ft. What is its angular momentum at 120 r.p.m.?

16. A drum 2 ft. in diameter and having a radius of gyration of 9 in. and weighing 600 lb. is employed to raise a weight of 6 cwt. with an upward acceleration of ½ ft. per sec. per sec. Find the torque necessary on the drum.

17. Find the maximum torque on the winding drum shaft for a pit cage, given the following data:

Weight of pit cage and load = 6 tons.
Depth of shaft = 400 ft.
Diameter of winding drum = 21 ft.
Radius of gyration of drum = 7 ft. 6 in.
Weight of winding drum = 17½ tons.
Starting acceleration of pit cage = 1·2 ft. per sec. per sec.
Weight of cable per foot = 3 lb.

18. A shell weighing 1200 lb. is projected at a velocity of 1800 ft./sec. from a gun weighing 36 tons. Find the initial velocity of recoil of the gun. Also find the space average of the force necessary to stop the recoil of the gun in 30 in.

19. Derive the formula you use for determining the kinetic energy of a moving body.

Find the kinetic energy of a motor car, which weighs 1 ton, when it is travelling at 35 m.p.h. If it is running down a long incline of 1 in 50 at the above speed and power be suddenly cut off, how far will it travel before coming to rest, the resistance to motion being 110 lb.? (U.L.C.I.)

20. A four-wheeled truck has a total weight of 15 tons. The weight of each axle, with its wheels, is ½ ton, while the radius of gyration of each wheel is 12 in. and the diameter at the tread is 30 in. What is the total kinetic energy of the truck when running at 15 m.p.h.? If the brakes on each wheel are pressed to the rim with a force of 1200 lb. and the coefficient of friction between the rubbing surface is 0·2, find how far the truck will travel up an incline of 1 in 840 before coming to rest.

21. A 60-pounder shell moving at 1000 ft. per sec. breaks up and a fragment weighing $\frac{1}{3}$ of the weight of the shell is brought to rest. Find the momentum of the remainder of the shell. What energy was expended in the break up?

22. A bullet weighing one ounce travelling horizontally at 2000 ft./sec., becomes embedded in a stationary block of mass 8 lb. which is free to move horizontally. Find the velocity of the block after impact, and the ratio of the original kinetic energy of the bullet to the kinetic energy of the block and embedded bullet.

23. Establish the formula connecting the moment of inertia, angular acceleration and torque producing it in the case of a rotating body.

A wheel weighs 1 ton and it has a radius of gyration of 3 ft.; its speed is increased from 90 to 120 r.p.m. in 30 sec. Determine:

(*a*) the average angular acceleration in radians per sec. per sec.;
(*b*) the torque required to produce this change;
(*c*) the change in the kinetic energy of the wheel. (U.E.I.)

24. Show that, when a body is rotating about a fixed axis under the action of a constant torque, the work done is the product of the torque and the angle turned through in radians.

The rotor of a turbine weighs 20 tons and has a radius of gyration of 5 ft. The speed of the rotor is increased from 190 to 215 r.p.m. in 5 sec. Determine (1) the torque necessary to produce the change, and (2) the change in kinetic energy of the rotor.

25. A flywheel is subjected to a torque of 600 lb. ft. Find the time the flywheel will take to reach a speed of 300 r.p.m. if friction is neglected and the radius of gyration and the weight of the shaft and wheel are respectively 2·2 ft. and 3 tons. Find the energy of rotation of the above wheel at 300 r.p.m. and the number of revolutions it would make in coming to rest under a friction torque of 40 lb. ft.

26. A disc flywheel, 6 ft. in diameter and weighing 2 tons, is keyed to a machine shaft. Determine the torque necessary to reduce the speed of the shaft from 150 r.p.m. to 90 r.p.m. in 5 sec. How many ft. lb. of energy would the wheel give out during the 5 sec.? (U.L.C.I.)

27. A flywheel supported on a horizontal axle 2 in. in diameter is pulled round by a cord on the axle carrying a weight. It is found that a weight of 4 lb. will just overcome the friction. A further weight of 16 lb. (making 20 lb. in all) is applied, and after 2 sec. from rest, the weight has gone 12 ft. Find the moment of inertia of the wheel.

28. Determine the moment of inertia of a flywheel and its shaft from the observations given below. The flywheel is supported in bearings with its shaft horizontal and it is rotated by means of a stout thread wound round the shaft. This thread carries a weight of 0·21 lb. at its free end, and of this weight 0·01 lb. is necessary to overcome the friction of the bearing. The weight falls 30 in. in 28·6 sec. Radius of shaft 1 in. Weight of flywheel and its shaft 25 lb.

29. A projectile stores two million ft. lb. of kinetic energy at 3000 ft. per sec. How much kinetic energy is lost if its speed drops to 500 ft. per sec.? Find its kinetic energy of rotation if its speed of rotation about its axis of flight is 24,000 r.p.m. and the radius of gyration about the same axis is 1 in.

30. Explain what the letters represent in the formula $Wv^2/2g$ which is used for determining the kinetic energy of the moving body.

At a certain instant a body weighing 20 lb. has stored in it 360 ft. lb. of kinetic energy. Determine the speed with which it is moving and also the reduction of speed that takes place when its kinetic energy is reduced to 30 ft. lb. (U.E.I.)

31. A portion of a mechanism is displaced through 40° from its position of equilibrium and about a point O. The centre of gravity of the mechanism is 16 in. below O. If the weight is 100 lb., find (*a*) the restoring torque; (*b*) the kinetic energy of rotation after release and passing its position of equilibrium.

32. The axle of a disc 12 in. in diameter rests on two inclined parallel rails, the disc being between the rails. The inclination of the rails is 1 in. in 6 ft. length of rail. Find the velocity of the disc after rolling 8 ft. along the rails from rest:

(*a*) when the axle is $\frac{1}{2}$ in. diameter;
(*b*) when the axle is 3 in. diameter.

The radius of gyration of a disc is the radius divided by $\sqrt{2}$. Neglect the moment of inertia of the axle.

33. Find the weight of flywheel required for a steam engine in which it is desired to limit the cyclical variation of speed to 0·6 per cent on either side of the mean. The ratio of excess energy during any one stroke to the total work done per stroke is 0·28. The engine is designed to indicate 480 H.P. at 110 r.p.m., and the radius of gyration of the flywheel is 6·4 ft.

34. A certain rolling mill requires 400 H.P. during the 10 seconds that the billet is passing through the mills. An electric motor whose maximum output is 340 H.P., drives the mill, and a flywheel keeps the speed between 80 and 72 r.p.m. during the operation. Find:

(*a*) the moment of inertia of the flywheel;
(*b*) the weight of the flywheel, in tons, if its radius of gyration is 9·5 ft. (U.E.I.)

35. In a single-cylinder double-acting engine the lengths of the crank and connecting rod are 10 in. and 45 in. respectively, and the diameter of the cylinder is 18 in. Assuming the net steam pressure to be constant throughout the stroke and to be equal to 35 lb. per sq. in., draw on a base representing crank angles a diagram of the torque or turning moment acting on the crank shaft for a revolution, and from it find the value of the maximum torque. (U.E.I.)

CHAPTER X

TORSION. HORSE-POWER TRANSMITTED BY A SHAFT. DETERMINATION OF SHAFT DIAMETERS. TORSION METERS, COUPLINGS, CLOSE-COILED SPRINGS

Torsion. When a shaft (Fig. 151) is subjected to two equal and opposite twisting couples in parallel planes normal to the shaft axis and at the ends of the shaft, it is said to be subjected to simple

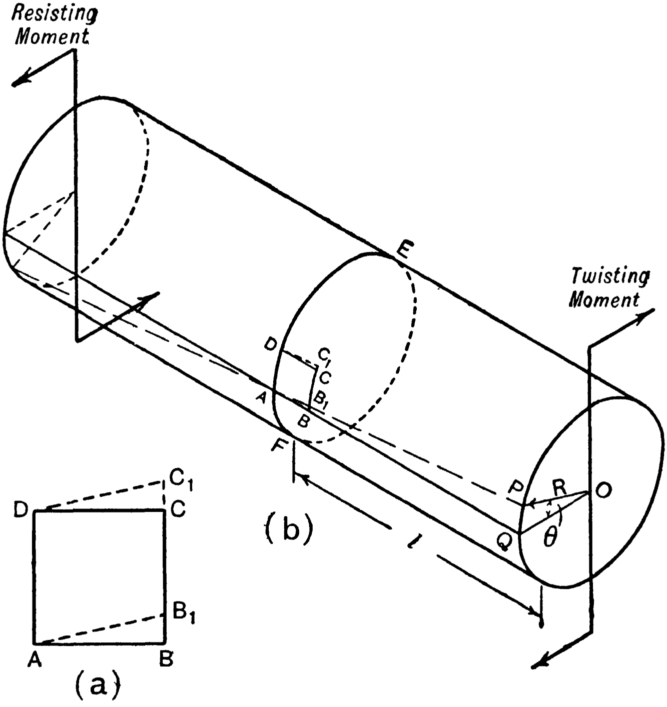


FIG. 151.

torsion. The couples will cause deformation or twisting, which will set up resisting shear stresses in the shaft in plane sections normal to the shaft axis; and at any given section the resisting moment on the area of the section due to these shear stresses will balance and oppose the twisting couples or torque. The stress at any point on the cross-section is due to pure shear, and the strain is such that all sections perpendicular to the axis except ADFE, the neutral plane of torsion, make a small rotatory movement relative to adjacent sections.

Consider a very thin square layer ABCD, where AD lies on the cross-section not subjected to stress. When the twisting couples are applied, this square becomes AB_1C_1D, and the shear strain is BB_1/AB. On a length l of shaft, the shear strain will become PQ/l for all parts of the curved surface or, since $PQ = R\theta$, where R is the shaft radius and θ the angle *in radians* which PQ subtends at the centre O, then the strain will be $R\theta/l$.

If f is the shear stress at the surface at radius R, then from the relationship,

modulus of rigidity, $$C = \frac{\text{shear stress}}{\text{shear strain}},$$

$$C = f \div R\theta/l = fl/R\theta,$$

or $$\frac{f}{R} = \frac{C\theta}{l} \text{ . ...(1)}$$

Assuming that radial lines remain radial after strain (and this is very nearly true), it can be easily understood that the shear stress increases in proportion to the radius, being zero along the shaft axis. The quantities θ, C and l are the same for any given cross-section subjected to a torque; hence the stress q at any point on the cross-section will be proportional to the radial distance r of that point from the shaft centre. Thus, in general,

$$\frac{q}{r} = \frac{f}{R} = \frac{C\theta}{l} \text{ . ...(2)}$$

Note that the *torque angle* is measured in *radians*.

Torsional moment of resistance. If the shear stress at radius r is q, this shear stress may be considered as acting on an elemental ring (Fig. 152) of thickness δr at a radius r from centre O of the section. If the section be considered as made up of a large number of such elemental rings each of thickness δr, the total moment of resistance for the section can be obtained. The moment of resistance of the elemental ring shown in Fig. 152 is area × stress × radius, or $2\pi r\ \delta r\ q \times r$, and since, from (2), $q = fr/R$, the moment of resistance is

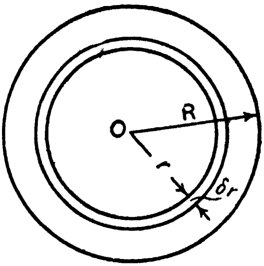


FIG. 152

$$2\pi r^3 f\ \delta r/R.$$

The total moment of resistance over the section is

$$\int_{r=0}^{r=R} 2\pi r^3 f\ \delta r/R = \left[\frac{\pi r^4 f}{2R}\right]_{r=0}^{r=R} = \frac{\pi R^4 f}{2R} = \frac{f}{R} J,$$

where $J = \pi R^4/2$, and is called the **polar moment of inertia** of the section about the centre O.

Thus, since the resisting moment of the shaft will balance the applied torque T,

$$T = f\,.\,J/R. \qquad \text{(3)}$$

Combining (2) and (3),

$$\frac{T}{J} = \frac{f}{R} = \frac{q}{r} = \frac{C\theta}{l}. \qquad \text{(4)}$$

In the case of hollow shafts, J, the polar moment of inertia, is $\pi(R^4 - r^4)/2$, where R is the outer radius and r the inner radius.

Obviously, with a hollow shaft and the same weight of material per unit length, the outside radius can be much greater than with a solid shaft; hence the polar moment of inertia is greater and the hollow shaft consequently is stronger than the solid one of the same weight per unit length.

At the greater radius of the hollow shaft, the material is doing more work than at the centre of a solid shaft where the material is only lightly stressed.

Example 1. *What diameter of shaft would be necessary to transmit a torque of* 100,000 *lb. ft. if the shear stress on a cross-section is not to exceed* 8000 *lb. per sq. in.?*

$T = 100{,}000$ lb. ft. $= 1{,}200{,}000$ lb. in. $f = 8000$ lb./in.^{2}

Using (3) above, $T = Jf/R = \pi R^3 f/2$,

or $R^3 = 2T/\pi f = 2{,}400{,}000/8000\pi$,

$R = 4{\cdot}57$ in. or Dia. $= 9{\cdot}14$ in., or say, $9\frac{1}{4}$ in.

Example 2. *In the above example calculate the length of shaft (in diameters) if the angle of twist on the length considered is* 1°. *The modulus of rigidity is* 12,000,000 *lb./in.*².

Using (1), $C\theta/l = f/R$, where $\theta = 1°$ or 0·0175 radian.

$$\text{No. of diameters} = \frac{l}{2R} = \frac{C\theta}{2f} = \frac{12{,}000{,}000 \times 0{\cdot}0175}{2 \times 8000}$$
$$= 13{\cdot}12.$$

Horse-power transmitted by a shaft. The horse-power is given by work done per minute divided by 33,000, and

$$\text{work done} = \text{torque} \times \text{radians turned through}$$
$$= T \times 2\pi N/33{,}000,$$

where N is revolutions of shaft per minute, and T is torque transmitted in lb. ft.

In this case, T is the mean torque ; but in practice this varies, and the maximum and minimum values may be considerably above or below the mean torque. Since the maximum shear stress depends on the maximum torque transmitted, this maximum must be known, or allowed for, in the designing of shafts for a given average or mean horse-power.

Example 1. *Calculate the greatest intensity of shear stress due to torsion in a shaft* 4 *in. in diameter transmitting* 80 *H.P. at* 60 *r.p.m. if the maximum torque exceeds the mean by* $33\frac{1}{3}$ *per cent. Also calculate the maximum angle of twist in degrees per foot of length if* $C = 12{,}000{,}000$ *lb./in.*²

$J = \pi R^4/2 \quad = \pi \times 2^4/2 = 8\pi$ in.⁴ units

Horse-power $= T \times 2\pi N/33{,}000$.

∴ Mean torque $= 33{,}000 \times \text{H.P.}/2\pi N$

$= 33{,}000 \times 80/(2\pi \times 60) = 22{,}000/\pi$ lb. ft.

Maximum torque = mean torque × $133\frac{1}{3}/100 = 352{,}000/\pi$ lb. in.

Using the relationship $T/J = f/R$,

$$f = \frac{TR}{J} = \frac{352{,}000 \times 2}{\pi \times 8\pi} = 8915 \text{ lb. in.}^2$$

To find θ, use $C\theta/l = T/J$, where $l = 12$ in.

Then, $\theta = Tl/CJ = 352{,}000 \times 12/(12{,}000{,}000 \times 8\pi^2)$ radians

$= 0{\cdot}26°$.

Example 2. *Design a shaft to transmit* 100 *H.P. at* 180 *r.p.m. if the angle of twist is not to exceed* 1° *in* 5 *ft. length of shaft.* $C = 12{,}000{,}000$ *lb. per sq. in.*

Using $C\theta/l = T/J$, $\qquad J = Tl/C\theta.$

$l = 60$ in. $\qquad J = \dfrac{\pi R^4}{2} = \dfrac{3{,}300{,}000 \times 12 \times 60}{2\pi \,.\, 180 \times 12{,}000{,}000 \times 0{\cdot}0175}$

$T = \text{H.P.} \times 33{,}000/2\pi N.$ $\qquad R = 1{\cdot}589$ in.

$= 33{,}000 \times 100 \times 12/(2\pi \,.\, 180)$ lb. in.

$\theta = 1°$ or 0·0175 radian,

and diameter $= 3\frac{1}{8}$ **in. or** $3\frac{1}{4}$ **in.**

The design in this case is from the *rigidity* point of view.

Example 3. *Calculate the necessary size of a solid shaft to transmit* 100 *H.P. at* 180 *r.p.m. if the maximum stress due to torsional shear is not to exceed* 10,000 *lb. per sq. in. Take the maximum torque as being* 40 *per cent. in excess of the mean.*

As above, $\qquad T = 3{,}300{,}000 \times 12/(2\pi \times 180)$ lb. in.

Maximum torque $= 1{\cdot}4 \times 3{,}300{,}000 \times 12/(2\pi \times 180)$ lb. in.

$= 15{,}400/\pi$ lb. in.

Using $T/J = f/R$, $\qquad J = TR/f.$

$$\frac{\pi R^4}{2} = \frac{TR}{f}, \text{ or } R^3 = \frac{2T}{\pi f},$$

$$R^3 = \frac{2 \times 15{,}400}{\pi^2 \times 10{,}000}, \text{ or } R = 1{\cdot}462 \text{ in.,}$$

or, diameter $= 2{\cdot}924$, say **3 in. shaft.**

In this example, the design is from the point of view of *maximum stress.* It may be necessary in practice to check a design for both rigidity and maximum stress.

Example 4. *In an experiment to determine the modulus of rigidity of mild steel bar* 102·5 *in. long and* 0·497 *in. in diameter, the following corresponding values of torque and angle of twist were recorded :*

Angle of twist, degrees,	0	3	7	10	14	17·5	21
Torque, in. lb. - -	0	42·8	85·7	128·6	168·5	214·4	257·3

Plot a graph of torque against angle of twist, and use the graph to determine the modulus of rigidity of the sample of mild steel.

$J \quad \pi R^4/2 = \pi d^4/32 = \pi(0{\cdot}497)^4/32 = 0{\cdot}005994$ in.^{4} units. $\quad l = 102{\cdot}5$ in.

Mean value of slope T/θ from graph

$= 220$ lb. in./18° $= 220$ lb. in./0·3142 radian

$= 700{\cdot}1$ lb. in.

But $C\theta/l = T/J$, or $C = Tl/J\theta$

$$= 700{\cdot}1 \times 102{\cdot}5 \text{ lb. in.}^2/(0{\cdot}005994 \text{ in.}^4)$$
$$= \mathbf{11{,}980{,}000 \text{ lb. per sq. in.}}$$

Example 5. *Determine the number of bolts necessary in the flanged coupling for the solid shaft shown in Fig. 153. In this example take the permissible shear stress in the shaft and bolts to be f lb. per sq. in. and f/2 lb. per sq. in. respectively.*

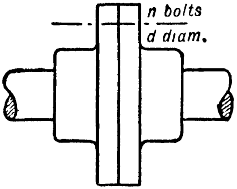


FIG. 153.

The desirable number of bolts will be such that the combined resisting moment of the bolts at the bolt circle will equal the moment of resistance of the shaft itself.

Let n = no. of bolts required, and d the diameter of the bolts.

$$\text{Moment of resistance of bolts} = n\frac{\pi d^4}{2}\,\frac{f}{2} \times \frac{D}{2} = \pi n d^2 Df/16.$$

$$\text{Moment of resistance of shaft} = \text{torque on shaft} = T$$
$$= Jf/R \text{ from (3) (p. 247)}$$
$$= \pi R^3 f/2.$$

Thus, $\pi n d^2 Df/16 = \pi R^3 f/2$

and $\mathbf{n = 8R^3/d^2D}$.

Example 6. *Compare the weights of hollow and solid shafts of the same material and length to transmit the same horse-power at the same speed and at the same maximum stress. The internal diameter of the hollow shaft is one half of the external diameter.*

What is the horse-power transmitted by a hollow shaft, 12 in. external diameter and 8 in. internal diameter, at a speed of 210 r.p.m. with a maximum stress of 4 tons per sq. in.? (U.E.I.)

Since the shafts are to transmit the same horse-power at the same speed, each shaft must receive the same torque.

Let r and R be the outside radii of the solid and hollow shaft respectively.

Solid shaft	*Hollow shaft*
$J = \pi r^4/2.$	$J = \pi(R^4 - R^4/16)/2 = 15\pi R^4/32.$
$\frac{T}{f} = \frac{J}{r} = \frac{\pi r^3}{2}.$	$\frac{T}{f} = \frac{J}{R} = \frac{15\pi R^3}{32}$

Since the torque and maximum stress are the same for both shafts, then T/f is the same for each, hence

$$\frac{\pi r^3}{2} = \frac{15\pi R^3}{32} \quad\text{or}\quad \frac{r^3}{R^3} = \frac{15}{16} \quad\text{or}\quad \frac{r}{R} = \mathbf{0{\cdot}9788}.$$

Ratio of weights will be as $r^2 : \frac{3}{4}R^2 = 1{\cdot}277 : 1$.

$$\begin{aligned} T &= \pi f(12^4 - 8^4)/16 \times 2 \times 6 \text{ tons in.} \\ &= 1040\pi/3 \text{ tons in.} \\ \text{Horse-power} &= 2\pi NT/(33{,}000 \times 12) \\ &= \frac{2\pi^2 \times 210 \times 1040 \times 2240}{3 \times 33{,}000 \times 12} = 8129. \end{aligned}$$

Close-coiled helical springs (cylindrical spiral springs). A spring in the form of a closely-coiled helix under axial load can be regarded as being subjected to torsion only; because the shear stress due to the axial load is generally very small compared with the shear stress due to the torsion of the wire forming the helical spring.

A closely-coiled spring is shown diagrammatically in Fig. 154*a*, in which d is the diameter of the wire, R the mean radius of the coils and W the axial load producing a deflection Δ. If n is the number of coils and l the length of wire, then

$$l = 2\pi Rn \text{ very nearly.}$$

The expression for deflection can best be obtained by considering any half coil of the spring, which will be practically horizontal in a close-coiled spring. Imagine arms fixed to the ends of the half coil as EF and GH (Fig. 154 *b*), with equal and opposite vertical forces W acting at F and G. The half coil will be subjected to a torque WR, and if the angle of twist is ϕ radians the distance between F and G, or the deflection, will be $R\phi$. As the number of coils is increased, the total deflection will be due to the cumulative effect of each half coil. Thus for n coils the total deflection will be $2Rn\phi$ or Δ.

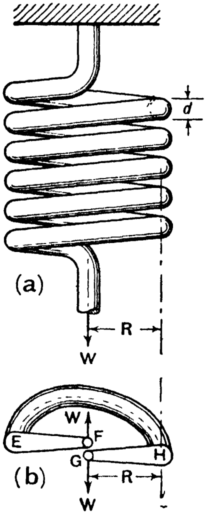


FIG. 154.

The whole wire has to withstand a torque T of WR lb. in., and this torque will produce a total angle of twist at the free end of the wire equal to $2n\phi$ or θ radians, where

$$C\theta/l = T/J\,; \quad \text{or} \quad \theta = Tl/CJ.$$

Thus, $$\Delta = R\theta = RTl/CJ\ ;$$

but $$T = WR,\ J = \pi d^4/32,\ l = 2\pi Rn.$$

$$\therefore\ \Delta = \frac{64nWR^3}{Cd^4}\ . \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5)$$

Example. *Find the deflection of a closely-coiled helical spring made of* $\frac{1}{4}$ *in. diameter steel wire if there are* 20 *coils of mean diameter* 3 *in. and the axial load is* 25 *lb. Take* C = *modulus of rigidity* = 12,000,000 *lb. per sq. in. Also find the stress in the material due to this axial load.*

$$\Delta = 64WR^3n/Cd^4 = 64 \times 25 \times (1\tfrac{1}{2})^3 \times 20/(\tfrac{1}{4})^4 \,.\, 12 \,.\, 10^6$$
$$= \mathbf{2{\cdot}3\ in.}$$

$$f/\tfrac{1}{2}d = T/J \quad \text{or} \quad f = Td/2J = WRd/2J$$
$$= 16WR/\pi d^3$$
$$= 16 \times 25 \times 1\tfrac{1}{2}/\pi(\tfrac{1}{4})^3$$
$$= \mathbf{12{,}220\ lb./in.^2}$$

EXERCISES ON CHAPTER X

1. Derive an expression connecting the torque applied to a round shaft and the maximum shear stress induced. A shaft 8 in. in diameter transmits 1400 horse-power at a speed of 160 r.p.m. Determine the maximum stress induced in the shaft. (U.E.I.)

2. A shaft 8 in. in diameter revolves at 160 r.p.m. Find the horse-power transmitted by the shaft when it has an angle of twist of 1 degree for each length of 20 diameters. The modulus of rigidity of the material is 12 million lb. per sq. in. (U.E.I.)

3. A shaft 5 in. in diameter and 50 ft. long transmits 120 H.P. at 150 r.p.m. Find the total angle of twist on this length and the maximum shear stress in the shaft.

4. If the shear stress in a 2 in. diameter shaft is 3000 lb. per sq. in., find the horse-power transmitted at 60 r.p.m.

5. In a shaft running at 90 r.p.m. the maximum twisting moment exceeds the mean by 50 per cent. If the shaft is 4 in. in diameter and is transmitting 70 H.P., find the maximum intensity of torsional shear stress. What is the greatest twist in degrees per foot of length if the modulus of rigidity of the shaft is 12,500,000 lb. per sq. in.?

6. Obtain the expression for the polar moment of inertia of a circular section.

A solid steel shaft is required to transmit 250 H.P. at 250 r.p.m. The maximum twisting moment in each revolution is 25 per cent greater than the mean. If the maximum intensity of shearing stress is not to exceed 10,000 lb. per sq. in., find the diameter of the shaft.

7. A shaft 2 in. in diameter can safely transmit a torque of 1600 lb. ft. What diameter of shaft made of the same material would be required to transmit 60 H.P. at 240 r.p.m.

8. What diameter of shaft is necessary to transmit 4000 H.P. at 220 r.p.m. if the angle of twist is not to exceed 1 degree on a length of 20 diameters? $C = 12{,}000{,}000$ lb. per sq. in.

9. A shaft 2 in. in diameter is rotating at 150 r.p.m. The shaft is found to twist through an angle of $4\frac{1}{4}$ degrees on a length of 45 ft. Calculate :

(*a*) the torque which the shaft is transmitting in lb. ft. ;
(*b*) the horse-power which the shaft is transmitting.

C = modulus of transverse elasticity = 12,000,000 lb. per sq. in.

10. A solid circular-section propeller shaft for a ship is 10 in. in diameter and makes 35 r.p.m. The angle of twist over a length of 24 in. is measured by a torsion meter and found to be 0·2 degree. If the modulus of rigidity is 5500 tons per sq. in., find :

(*a*) the maximum shear stress on the shaft ;
(*b*) the moment of turning or torque on the shaft ;
(*c*) the horse-power being transmitted by the shaft.

11. In a hollow shaft the outside diameter is $1\frac{1}{2}$ times the inside diameter. Obtain these diameters when the shaft transmits 9000 H.P. at 240 r.p.m. if the shear stress is not to exceed 10,000 lb. per sq. in.

12. Determine the shaft speed in r.p.m. to transmit 200 H.P. if the shaft diameter is $3\frac{1}{2}$ in. and the shear stress is not to exceed 10,000 lb. per sq. in.

13. Calculate the maximum torque and the mean horse-power being transmitted in the case of a hollow shaft of which the outer diameter is 8 in. and the inner 5 in., if the shear stress is not to exceed 8000 lb. per sq. in. and the shaft speed is 300 r.p.m. Assume the maximum torque to be 25 per cent more than the mean.

14. Determine the diameter of a solid shaft which transmits 10,000 kW. at 3000 r.p.m. if the twist is not to exceed 1 degree in 20 diameters and the shear stress is not to exceed 7500 lb. per sq. in. Also find the mean torque on the shaft. $C = 12{,}000{,}000$ lb. per sq. in.

15. During a torsion test on a steel specimen of diameter 0·22 in. and length 30 in. the following observations were made :

Torque, lb. in. -	0	0·6	1·2	1·8	2·4	3	3·6	4·2	4·8	5·4
Angle of twist -	0	0·37	0·75	1·12	1·5	1·95	2·3	2·7	3·05	3·5

Find the mean ratio of torque to angle of twist in radians and the value of the modulus of rigidity for the material.

16. In a torsion test on a steel rod 0·219 in. in diameter, the following results were obtained. The angle of twist was measured on a length of 30 in.

Angle of twist in degrees -	0	0·8	1·6	2·4	3·2	4·0	4·75	5·6	6·4	7·2	8·05
Torque applied in lb. in. -	0	1·2	2·4	3·6	4·8	6·0	7·2	8·4	9·6	10·8	12

Plot a graph of torque *v.* angle of twist and determine the modulus of rigidity of the material.

17. The following observations were made during a torsion test on a rod of silver steel:

Diameter of specimen = 0·218 in.

Distance between gauge points of strain indicator = 30·3 in.

Torque in lb. in. - -	0	1·2	2·4	3·6	4·8	6·0	7·2	8·4	9·6	10·8	12
Angle of twist in degrees -	0	0·75	1·5	2·25	3·1	3·9	4·65	5·4	6·1	7	7·7

Plot a graph, determine the slope, and find the value of the modulus of rigidity, C, of the material.

18. In an experiment to determine the modulus of rigidity of mild steel, a bar 102·5 in. long and 0·497 in. in diameter gave the following corresponding values of torque and angle of twist.

Angle in degrees -	0	3	7	10	14	17·5	21
Torque, in lb. in. -	0	42·875	$85\frac{3}{1}$	$128\frac{5}{8}$	$168\frac{1}{2}$	$214\frac{3}{8}$	$257\frac{1}{4}$

Plot a graph of torque against angle of twist and use it to determine the modulus of rigidity of mild steel.

19. (*a*) Describe an experiment which shows the relation between the deflection of a closely-coiled helical spring, the mean radius of its coils and the angle of twist in the material.

(*b*) If a closely-coiled helical spring is made of $\frac{1}{2}$ in. round steel and has 20 coils of mean radius 3 in., calculate the deflection of the spring under a 60 lb. load.

Modulus of rigidity of steel = 12 million lb. per sq. in.

20. Wire one quarter of an inch in diameter is used to make a closely-coiled helical spring. If the mean diameter is 6 in., what axial pull can this spring withstand if the shear stress is not to exceed 12,000 lb. per sq. in.? $C = 12$ million lb./in.^{2}

21. A spiral spring is made from round steel closely coiled in 10 turns the outside diameter of which is 5 times that of the rod. If the spring is to stretch 0·5 in. under an axial load of 300 lb., find the diameter of the rod. The case may be taken as one of pure torsion. $C = 12{,}000{,}000$ lb. per sq. in. (U.E.I.)

22. A shaft and coupling are required to transmit 30 H.P. at 120 r.p.m. There are four bolts in the coupling and the bolt circle is 8 in. diameter. Estimate the working diameters of the shaft and the bolts if the working stress in the former is 8000 lb. per sq. in. and in the latter 3500 lb. per sq. in. The shaft may be assumed to be subjected to pure torsion, and the maximum twisting moment is 1·4 times the mean twisting moment. (U.E.I.)

23. A closely-coiled helical spring has 10 coils. The spring requires a load of 320 lb. to compress it one inch. If the mean coil diameter is eight times the diameter of the steel from which the spring is made, what is the diameter of the steel? What is the maximum stress in the spring when it compresses 2 in.? Modulus of rigidity $= 12{\cdot}5 \times 10^6$ lb. per sq. in. (U.E.I.)

CHAPTER XI

CENTRIPETAL ACCELERATION. CONICAL PENDULUM. CENTRIFUGAL GOVERNOR. SIMPLE BALANCING IN ONE PLANE. SIMPLE HARMONIC MOTION. RECIPROCAL MOTION

Centripetal force. Imagine a body (Fig. 155) with velocity v to be moving in a circular path of radius r. Consider two successive positions A and B close together, so that the angle ACB is $\delta\theta$, and also suppose that the change of the *direction* of the velocity has occurred in a time δt. Then (Fig. 155*b*) shows the velocity diagram for the change, *ab* being the change in time δt.

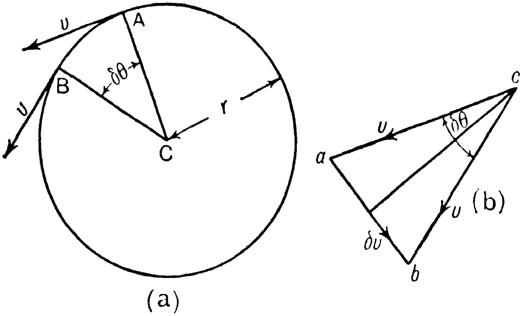


FIG. 155.

As A and B (Fig. 155*a*) are taken close together, the sector ABC and the velocity triangle *abc* tend to become similar figures, and in the limit when $\delta\theta$ becomes $d\theta$,

$$\frac{\delta v}{v} = \frac{\text{arc AB}}{r} = \frac{v\,\delta t}{r} \quad \text{or} \quad \frac{\delta v}{\delta t} = \frac{v^2}{r}.$$

But
$$\frac{\delta v}{\delta t} = \frac{\text{velocity change}}{\text{time change}} = \text{acceleration},$$

Hence the acceleration in the direction *ab* along a radius is given by v^2/r, and is called the **centripetal acceleration.**

If the angular velocity of the body about C is ω, then the centripetal acceleration may be written $\omega^2 r$ because $v = \omega r$.

Remembering that force = mass × acceleration, then

$$\text{centripetal force} = \frac{W}{g}\,\frac{v^2}{r} = \frac{W}{g}\,\omega^2 r. \qquad \ldots\ldots\ldots\ldots\ldots\ldots(1)$$

This force acts inwards towards the centre of the circle and is necessary for circular motion. **Centrifugal force** may be regarded as the reaction of the centripetal force and acts in the opposite direction, that is, outwards from the centre. These forces act at the centre of gravity of the body.

Example 1. *A train weighing* 600 *tons is rounding a horizontal curve of* 500 *ft. radius at a speed of* 30 *m.p.h. Calculate the inward or centripetal force which must be supplied by the rails to maintain the motion.*

Centrifugal force = Wv^2/gr. 30 m.p.h. = 44 f./s.

$$= 600 \text{ tons} \times \frac{44 \text{ ft.}}{\text{sec.}} \times \frac{44 \text{ ft.}}{\text{sec.}} \Big/ \frac{32{\cdot}2 \text{ ft.}}{\text{sec.}^2} \times 500 \text{ ft.}$$

$= \mathbf{72{\cdot}15\ tons.}$

In this case the rails are subjected to the reaction or outward pull of the tyres of the train wheels or the centrifugal force.

Example 2. *A boy whirls a stone weighing* 1 *lb. round in a vertical circle at the end of a cord* 2 *ft.* 6 *in. long at a speed which reaches* 60 *r.p.m. when the stone is in its lowest position. What is the tension in the cord in this position?*

Centripetal force necessary = $W\omega^2 r/g$ and $\omega = 2\pi^c/\text{sec.}$

$= 1 \times (2\pi)^2 \times 2\tfrac{1}{2}/32{\cdot}2$ or $3{\cdot}07$ lb.

Since the tension in the cord must supply the centripetal force plus the force of gravity, then

Tension in cord = 1 lb. + 3·07 lb. or **4·07 lb.**

Example 3. *If in Example* 2 *the velocity of the stone were to be the minimum possible for motion in a complete circle, find the velocity of the stone and the tension in the string at the highest and lowest points and also midway between.*

If the velocity is to be the least possible, then the cord will be just slack in the highest position and the weight just supplies the centripetal force. Tension in highest position = *zero.*

$\therefore$ if v_H is the velocity in the highest position,

$$Wv_H^2/gr = W \text{ or } v_H^2 = gr,$$

$$\text{or} \quad v_H = \sqrt{32{\cdot}2 \times 2{\cdot}5} \text{ or } 8{\cdot}97 \text{ f./s.}$$

Since no work is done or lost, if friction is neglected, then from the Principle of Conservation of Energy the total energy remains the same for all positions of the stone.

Hence the kinetic energy in the lowest position must equal the kinetic energy plus the potential energy in the highest position, or if v_L is the velocity at the lowest point,

$$Wv_L^2/2g = (Wv_H^2/2g) + 5W,$$

$$\text{or} \quad v_L^2 = v_H^3 + 10g \quad \text{or} \quad v_L = 20{\cdot}06 \text{ f./s.}$$

Tension in cord in lowest position $= W + Wv_L^2/gr$

$$= 1 + 402{\cdot}5/2\tfrac{1}{2}g \quad \text{or} \quad 6 \text{ lb. (or } 6W).$$

For the mid-position and using the same energy principle,

$$(Wv_M^2/2g) + 2\tfrac{1}{2}W = Wv_L^2/2g = (Wv_H^2/2g) + 5W,$$

$$\text{or} \quad v_M^2 = v_H^2 + 5g \quad \text{or} \quad v_M = \sqrt{7\tfrac{1}{2}g} \quad \text{or} \quad 15{\cdot}54 \text{ f./s.}$$

Tension in cord in mid-position $= Wv_M^2/gr$

$$= 7\tfrac{1}{2}g/gr = 3 \text{ lb. (or } 3W).$$

Since a centripetal force is necessary when a vehicle is rounding a curve, and since this force acts through the centre of gravity, which is almost always above the ground-level, the vehicle will tend to overturn because this necessary force can usually only be applied at ground level. If, however, the track is banked, the horizontal component of the reaction of the track supplies some, or all, of the inward force required. For example, if a car is rounding a curve of radius r and the track is banked to an angle θ to the horizontal, then the reaction R of the track must be normal and supply a horizontal component equal to Wv^2/gr in order that the car may be free from any force tending to cause a skid sideways at a speed v and mass of car W/g. If the track is insufficiently banked for the given speed, then the friction between the tyres and the road may prevent side-slip ; but this frictional force cannot prevent overturning, as the centripetal force required acts through the centre of gravity.

Determination of correct banking angle. Let Fig. 156 represent a vehicle of weight W symmetrical about a central plane through **GC** perpendicular to the plane of the paper and moving in a circle of

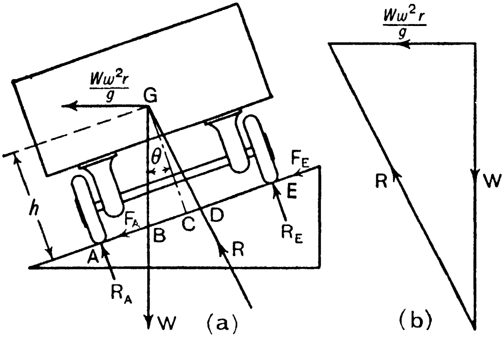


FIG. 156.

radius r, the centre of the circle being to the left of the vehicle. The latter moves in a curved track banked at an angle θ to the horizontal. In the plane of the diagram, through the centre of gravity G, the forces acting on the vehicle are gravity, W, and the resultant reaction of the track, R. The resultant of W and R must be $W\omega^2 r/g$, which is the inward centripetal force necessary for curvilinear motion at radius r and angular velocity ω radians per second.

For ideal conditions and the speed for which the banking is designed, that is, with no tendency for lateral or side-slip, or side-rail pressure in the case of a tracked vehicle, the reaction R should act along the line CG normal to AE, and the triangle of forces will then be as in Fig. 156*b*. This gives the angle between R and W as θ, and the correct banking for radius r and speed ω is given by

$$\tan\theta = W\omega^2 r/gW = \omega^2 r/g = v^2/rg. \quad\text{..................(2)}$$

In the general case, R will not act normal to AE, and as it can only act through the agency of the reactions of the wheels at A and E, it is necessary to study the problem rather more fully.

Let the normal component reactions at A and E be R_A and R_E, and the tangential components be F_A and F_E. Then the resultant of R_A, R_E, F_A and F_E will be equal to R.

Taking components of the reactions normal to the track and along the banking parallel to AE :

$$(W\omega^2 r\sin\theta/g) + W\cos\theta = R_E + R_A, \quad\text{...............(3)}$$

$$(W\omega^2 r\cos\theta/g) - W\sin\theta = F_E + F_A. \quad\text{...............(4)}$$

Also, since there is no rotation about G, the moments of the components about this point must be zero, or if $AE=d$,

$$\tfrac{1}{2}(R_E-R_A)d=(F_E+F_A)h. \quad \text{.......................(5)}$$

Eliminating (F_E+F_A) from (4) and (5),

$$R_E-R_A=2Wh\{(\omega^2 r\cos\theta/g)-\sin\theta\}/d. \quad \text{...............(6)}$$

Adding (3) and (6) and dividing by 2 will give the value of R_E; and subtracting (6) from (3) and dividing by 2 will give R_A:

$$R_E=\frac{W\omega^2 r}{g}\left\{\frac{h\cos\theta}{d}+\frac{\sin\theta}{2}\right\}+W\left\{\frac{\cos\theta}{2}-\frac{h\sin\theta}{d}\right\}, \quad \text{......(7)}$$

$$R_A=\frac{W\omega^2 r}{g}\left\{\frac{\sin\theta}{2}-\frac{h\cos\theta}{d}\right\}+W\left\{\frac{\cos\theta}{2}+\frac{h\sin\theta}{d}\right\}. \quad \text{......(8)}$$

The speed at which the vehicle will overturn outwards occurs when the wheel A leaves the ground, that is, when $R_A=0$.

Equating R_A in (8) to zero, gives the speed for overturning:

$$\omega^2=g\left\{\frac{d\cos\theta+2h\sin\theta}{r(2h\cos\theta-d\sin\theta)}\right\}, \quad \text{.......................(9)}$$

or

$$\omega^2=g\left\{\frac{(d+2h\tan\theta)}{r(2h-d\tan\theta)}\right\}, \quad \text{............................(10)}$$

which reduces to $\omega^2=gd/2rh$ if there is no banking and θ is zero.

Since R is the resultant force acting on the wheels, then GCD is the vector triangle, because GD represents R, CG represents (R_E+R_A) and DC represents (F_E+F_A).

$$\therefore\ (\mathrm{DC}/h)=(F_E+F_A)/(R_E+R_A). \quad \text{..................(11)}$$

If the wheel has no flanged tyres and is not running on rails, there will be no lateral or side-slip so long as (F_E+F_A) is within the limiting value of $\mu(R_E+R_A)$, where μ is the coefficient of limiting friction between tyres and road.

$$\therefore\ (W\omega^2 r\cos\theta/g)-W\sin\theta=\mu(W\cos\theta+W\omega^2 r\sin\theta/g),$$

or

$$\omega^2=g(\mu+\tan\theta)/r(1-\mu\tan\theta). \quad \text{................(12)}$$

This gives the speed when the vehicle is on the point of side-slip. By equating the expressions (10) and (12), the condition when the vehicle is both about to slip sideways and overturn is obtained.

The banking adopted for main roads must be such as to suit both the slow-moving, horse-driven, top-heavy hay cart and the high-speed car. What is safe for the latter may be extremely dangerous for the former and vice versa, and a compromise has to be effected. One formula commonly used to calculate the value of the super-elevation or cant at road bends is

$$\frac{h}{l}=\frac{v^2}{15r},$$

where h is the super-elevation, l the horizontal width, v the speed in miles per hour and r the radius of the bend in feet. This formula attempts to give a balance between centrifugal and gravity accelerations and gives 1·12 in. per foot for a speed of 45 m.p.h. on a bend of radius 1200 ft. (see example 1 below).

In the case of railways it is common practice to give only three-quarters or seven-eighths of the super-elevation as calculated theoretically, and to make the maximum six inches. In fact, railway engineers frequently allow a deficiency of from 2 to 3 in. below the permissible elevation without a reduction in maximum speed. They find that this can be done without excessive rail wear or discomfort or danger to passengers. It is, however, very important that the maximum value of the super-elevation should be attained gradually and that a transition curve be used between the straight line and the bend proper.

Example 1. *It is required to find the elevation of the outer rail above the inner if a safe speed of 45 m.p.h. is required and if the radius of the curve is 1200 ft. and the rail gauge is 4 ft. $8\frac{1}{2}$ in.*

From (2), p. 259, $\tan\theta=v^2/rg.$, $r=1200$ ft., $v=66$ f./s. or 45 m.p.h.

$$=66\times 66/1200\times 32{\cdot}2$$

$$=0{\cdot}1127.$$

$\therefore$ elevation $=0{\cdot}1127\times$ rail gauge approx. $=$ **6·37 in.** or, **1·35 in.** per foot.

Example 2. *A motor-car of weight 2 tons is approaching a curve of radius 100 ft. The distance between the two back wheels is $4\frac{1}{2}$ ft. and the height of the centre of gravity 4 ft. from the ground; the coefficient of friction between the wheels and road is 0·3. Find (a) the greatest speed at which the corner can be taken without skidding; (b) neglecting skidding,* *the speed at which the road would exert no reaction on the inner wheels. The road is horizontal everywhere.* (U.E.I.)

(a) As the road is horizontal, $\theta = 0$ in Fig. 156.

Expression (3) becomes $R_E + R_A = W$,

and (4) becomes $F_E + F_A = W\omega^2 r/g$.

Also $$F_E + F_A = \mu(R_E + R_A) = \mu W.$$

$$\therefore \quad (W\omega^2 r/g) = \mu W \quad \text{or} \quad \omega^2 = \mu g/r$$
$$= 0{\cdot}3 \times 32{\cdot}2/100.$$

Thus, $\omega = 0{\cdot}3109$, and $v = \omega r = 31{\cdot}09$ ft./sec or **21·2 m.p.h.**

(b) When the inner wheel lifts, $R_A = 0$ and $\theta = 0$ as before.

Expression (8) becomes $$R_A = \frac{W}{2} - \frac{W\omega^2 rh}{gd} = 0,$$

$$\text{or} \quad \omega^2 = \frac{gd}{2rh} = \frac{32{\cdot}2 \times 4{\cdot}5}{2 \times 100 \times 4}.$$

Thus $\omega = 0{\cdot}4256$ and $v = \omega r = 42{\cdot}56$ ft./sec. or **29·02 m.p.h.**

Example 3. *Obtain a solution to the previous example if the road is banked to an angle of 10° instead of being horizontal.*

(a) From (3) and (4),

$$\mu = \frac{F_E + F_A}{R_E + R_A} = \frac{(W\omega^2 r \cos\theta/g) - W \sin\theta}{(W\omega^2 r \sin\theta/g) + W\cos\theta}$$
$$= \frac{\omega^2 r - g\tan\theta}{\omega^2 r \tan\theta + g}.$$

$$\therefore \quad \omega^2 = \frac{\mu g + g\tan\theta}{r - \mu r\tan\theta} = \frac{32{\cdot}2(0{\cdot}3 + 0{\cdot}1763)}{100(1 - 0{\cdot}3 \times 0{\cdot}1763)}.$$

Thus $\omega = 0{\cdot}4024$ and $v = \omega r = 40{\cdot}24$ ft./sec. or **27·43 m.p.h.**

(b) From (10), $\omega^2 = 32{\cdot}2\{(4\frac{1}{2} + 8\tan 10^\circ)/(8 - 4\frac{1}{2}\tan 10^\circ)100\}$,

$\omega = 0{\cdot}514$ and $v = \omega r = 51{\cdot}4$ ft./sec. or **35·04 m.p.h.**

The conical pendulum. A small but heavy ball or piece of material A (Fig. 157) revolving in a circle of radius r with angular velocity ω about C and suspended by a flexible thread OA of negligible mass from a fixed position O constitutes what is known as a **conical pendulum.** If small frictional resistances are ignored, the two forces keeping the ball moving in a circular path are the tension in the thread, T, and the gravitational force, W, respectively. The resultant of T and W must act horizontally and radially inwards and its magnitude is $W\omega^2 r/g$.

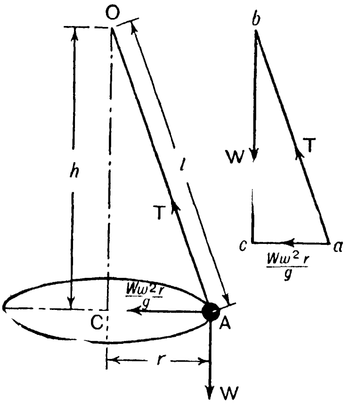


FIG. 157.

From the similarity of the vector triangle *abc* and the space triangle ACO,

$$\frac{ab}{bc}=\frac{T}{W}=\frac{l}{h}, \quad \ldots\ldots(13), \text{ and } \quad \frac{W}{W\omega^2 r/g}=\frac{bc}{ac}=\frac{h}{r}. \quad \ldots\ldots\ldots(14)$$

These equations give

$$h=g/\omega^2, \quad \ldots\ldots(15)$$

which gives h as independent of l for a given speed ω, and h as inversely proportional to the square of ω.

Thus, the height h is inversely proportional to the square of the angular velocity, and is independent of the length of the pendulum at any particular speed.

Example. *Find the angular velocity of a conical pendulum when the value of h is (a) 1 in., (b) 2 in.*

(*a*) $\omega^2=g/h=32{\cdot}2\times 12$ or $\omega=19{\cdot}66$ rad./sec. or 119·6 r.p.m.

(*b*) $\omega^2=g/h=32{\cdot}2\times 6$ or $\omega=13{\cdot}9$ rad./sec. or 84·5 r.p.m.

Governors (centrifugal type). The simplest type of centrifugal governor is constituted when a conical pendulum operates a sleeve at C (Fig. 158) through a link AC. As the sleeve moves to and fro, it operates a valve of an engine by means of a series of links. Such a governor controls engine speed and power over a much longer interval of time than that operated by a flywheel.

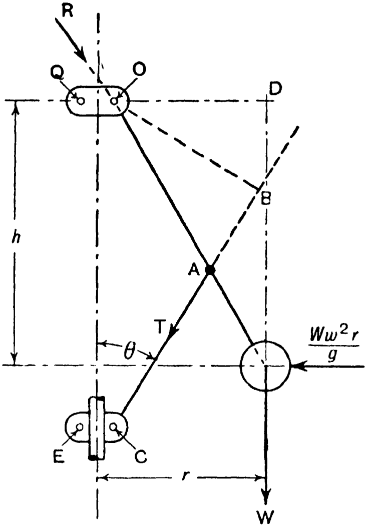


FIG. 158.

Fig. 158 shows in outline one half of such a simple governor. If the weights of arms and sleeve can be neglected and the ball considered as a heavy particle rotating at angular velocity ω at radius r and height h, then taking moments of the forces acting on the arm OA about O and noticing that the reaction of R at O will have no moment, the moments of W, T and R about O will equal that of their resultant,

$$\therefore\ W \times \mathrm{OD} + T \times \mathrm{OB} = (W\omega^2 r/g) \times h. \quad \ldots\ldots\ldots\ldots\ldots(16)$$

When ω and r are known, T can be calculated, and then by drawing a vector diagram the vector sum of R, W and T must equal $W\omega^2 r/g$. Thus, R can be found.

Example. *The spindle of a centrifugal governor as shown in Fig.* 158 *runs at* 83·9 *r.p.m. and each of the two balls weighs* 10 *lb. If the speed increases by* $12\frac{1}{2}$ *per cent without h and r changing in value, find the value of T and the total vertical upward pull on the sleeve at* C. *Neglect friction and assume* OA = AC = $4\frac{1}{2}$ *in.,* EC = 2 *in.,* $r = 4$ *in.,* $h = 5$ *in. Neglect the mass of the arms and sleeve.*

Now $T=0$ for the values of h and r at speed ω_1 or 83·9 r.p.m., and the value of T is required when h and r remain constant and ω_1 increases by $12\frac{1}{2}$ per cent to ω_2.

Using expression (16), when $T=0$, $W(r-1)=W\omega_1^2rh/g$,

$$\text{or}\quad \omega_1^2=3g/20 \quad\text{and}\quad \omega_2^2/\omega_1^2=(112\tfrac{1}{2}/100)^2.$$

$$\therefore\quad \omega_2^2=81\omega_1^2/64=243g/1280.$$

Also, as the arms are equal, $\tan\theta=(r-1)/h=0{\cdot}6$, and

$$\mathrm{OC}=2\,.\,\mathrm{OA}\,.\,\cos\theta \quad\text{and}\quad \mathrm{OB}=\mathrm{OC}\sin\theta=3{\cdot}97.$$

Using expression (16) when ω_1 becomes ω_2, h and r being constant,

$$W(r-1)+T\times 3{\cdot}97=W\omega_2^2rh/g,$$

$$30+3{\cdot}97T=200\times 243/1280,$$

$$T=2\text{ lb.}$$

Total upward pull on the sleeve at $\mathrm{C}=2T\cos\theta=4\times 5/\sqrt{34}$

$$\doteq 3{\cdot}43\text{ lb.}$$

The loaded governor. Since the height of the conical pendulum is given by $h=g/\omega^2$, the height becomes too small to be of practical use for working of a centrifugal governor at comparatively low speeds. Referring to the example on p. 263, the height is one inch for a speed of 119·6 r.p.m. In order to increase h and make the governor of practical use and available for higher speeds and power, the sleeve is loaded with a dead weight or a spring.

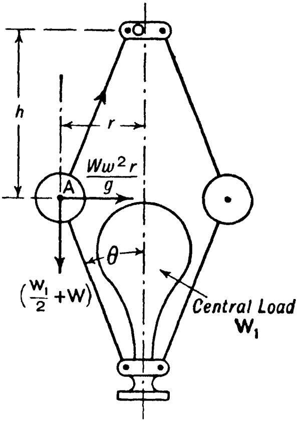


FIG. 159.

In Fig. 159 a vertical downward load W_1 is applied at the sleeve. This load is equivalent to a vertical component $W_1/2$ and a horizontal component $(W_1\tan\theta)/2$ at the centre of each ball. Consider the forces acting on the link OA and take moments about O,

$$(W_1/2+W)r+(W_1h\tan\theta)/2=W\omega^2rh/g. \quad\ldots\ldots\ldots\ldots(17)$$

This type of governor is called the **Porter governor.**

Example. *For the governor of Fig.* 159 *the dead load* W_1 *is* 60 *lb. and weight of each ball is* 10 *lb.*, $h=6$ *in.*, $r=4$ *in. and the arms are all equal in length. Find the speed of the governor if friction is neglected.*

Using the expression (17) on p. 265 and noticing that $\tan\theta = 2/3$,

$$(60/2 + 10)\tfrac{1}{3} + 60\,.\,\tfrac{1}{2}\,.\,\tfrac{2}{3}/2 = 10\,.\,\omega^2\,.\,\tfrac{1}{3}\,.\,\tfrac{1}{2}/g,$$

$$\omega = 21{\cdot}23 \text{ radians/sec.} \quad \text{or} \quad 202\ 75 \text{ r.p.m.}$$

N.B. The speed for an unloaded governor for the given configuration would have been 76·7 r.p.m.

Spring-loaded governors. Various types of spring-loaded governors are in common use, and the general principle is shown in the **Hartnell type** (Fig. 160). Two balls of equal weight are held by bell-crank levers pivoted at O and P, and the roller ends of these levers exert an upward pressure on the sleeve with increase of speed against a spring-loading on the sleeve. The spring thus replaces the central deadweight load in the Porter governor, and this spring is given an initial compression sufficient to maintain the balls at a radius r when the engine is running at normal speed.

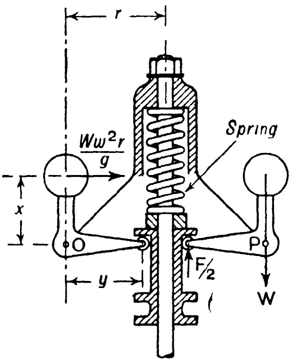


FIG. 160.

The usual problem in this type of governor is to find the necessary spring stiffness for a given speed fluctuation and sleeve movement; or alternatively, the speed fluctuation for a given spring stiffness and sleeve movement.

Let W lb. be the weight of each ball, and ω_1 and ω_2 be the engine speeds when the radii of the balls are r_1 and r_2 respectively. If F_1 is the force on the sleeve for ω_1 and r_1, and F_2 is the force on the sleeve for ω_2 and r_2, then by taking moments about O,

$$\frac{F_1}{2} \times y = \frac{W\omega_1^2 r_1}{g} \times x \quad \text{or} \quad F_1 = \frac{2W\omega_1^2 r_1 x}{gy}, \quad \text{............(16}a\text{)}$$

and

$$\frac{F_2}{2} \times y = \frac{W\omega_2^2 r_2}{g} \times x \quad \text{or} \quad F_2 = \frac{2W\omega_2^2 r_2 x}{gy}. \quad \text{............(16}b\text{)}$$

If the movement of the sleeve is h, then

$$h = (r_2 - r_1)\,y/x.$$

If s is the stiffness of the spring or the force required to compress the spring one inch, then

additional force necessary to displace sleeve by $h = hs = F_2 - F_1$

or
$$s=\frac{F_2-F_1}{h}=\frac{(F_2-F_1)x}{(r_2-r_1)y}\,; \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(16c)$$

or, substituting from (16*a*) and (16*b*),
$$s=\frac{2Wx^2(\omega_2{}^2r_2-\omega_1{}^2r_1)}{gy^2(r_2-r_1)}\,. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(16d)$$

Example. *A governor of the Hartnell type has equal balls of weight* 6 *lb. each, set initially at a radius of* 8 *in. The arms of the bell-crank lever are* $4\frac{1}{2}$ *in. vertically and* 6 *in. horizontally. Find* (*a*) *the initial compressive force on the spring if the speed for an initial ball radius of* 8 *in. is* 240 *r.p.m.* ; (*b*) *the stiffness of the spring required to permit a sleeve movement of* 0·15 *in. on a fluctuation of* $7\frac{1}{2}$ *per cent in the engine speed.*

(*a*) $\omega_1=8\pi$ rad./sec.
$$F_1=\frac{2W\omega_1{}^2r_1x}{gy}=\frac{2\times 6\times(8\pi)^2\times 8\times 4\frac{1}{2}}{32{\cdot}2\times 6\times 12}=117{\cdot}7\text{ lb.}$$

(*b*) $\omega_2=1{\cdot}075\omega_1=8{\cdot}6\pi$ rad./sec. $h=0{\cdot}15$
$$h=(r_2-r_1)y/x=(r_2-8)\,6/4\tfrac{1}{2},$$
or $r_2=8{\cdot}113$ in.
$$s=\frac{2Wx}{gyh}(\omega_2{}^2r_2-\omega_1{}^2r_1)$$
$$=\frac{2\,.\,6\,.\,4\frac{1}{2}}{32{\cdot}2\,.\,6\,.\,0{\cdot}15}\left\{\frac{(8{\cdot}6\pi)^2\,.\,8{\cdot}113-(8\pi)^2\,.\,8}{12}\right\}$$
$= 134{\cdot}9$ **lb. per inch.**

Frictional effect in governors. The frictional resistances of the various joints of the governor and the gear the governor has to operate can be conveniently reduced to a single force F acting at the sleeve in a direction opposite to that in which motion is about to take place. When the sleeve is rising, F will act downwards; when the sleeve is falling, it will act upwards. If the speed of the governor is increasing and h decreasing, the effect will be to alter the central load W_1 to W_1+F; also, when the sleeve is descending, the central load will change to W_1-F.

Sensitiveness of governors. The sensitiveness of a governor is measured by the change in the value of h due to a change of speed of 1 per cent.

Effort of a governor. The effort of a governor is the force it is capable of exerting at the sleeve for a given percentage change of speed.

Balancing. When a mass A of weight W (Fig. 161) is attached to a straight arm revolving with angular velocity ω at radius r about a centre O, the arm must exert an inward pull on the mass equal to $W\omega^2r/g$. The reaction will constitute a pull on the axis of rotation at O equal and opposite to the pull on the mass, and the *direction* of the pull on the axis will vary continuously as the mass rotates. This force on the axis can only be balanced by another mass in the same plane of mass W_1 placed diametrically opposite to W and revolving at radius r_1 with the same angular velocity ω.

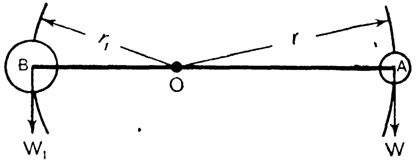
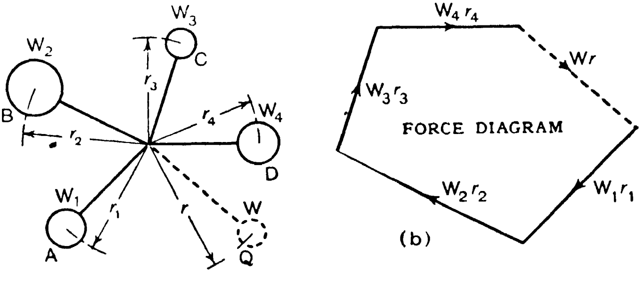


FIG. 161,

Thus, $W_1r_1\omega^2/g$ must equal $Wr\omega^2/g$, or $W_1r_1 = Wr$.

This is the condition that there shall be no unbalanced force acting at the axis O.

A large number of masses A, B, C, etc. (Fig. 162), of weights W_1, W_2, W_3, etc., respectively at distances r_1, r_2, r_3, etc., respectively from an axis O and revolving in one plane about that axis with angular velocity ω, can be balanced so that there is no force on the axis.



SPACE DIAGRAM
(a)

(b)

FIG. 162.

Suppose W is the weight of the mass at Q at radius r which will balance the masses A, B, C, etc. Since ω^2/g is common to all the forces acting on the axis, these forces due to A, B, C, etc., and Q will be proportional to W_1r_1, W_2r_2, W_3r_3, etc., and to Wr respectively. All the forces act in the same plane and at the same point O, and the problem reduces to finding the relative position, magnitude and direction of a force Wr which will constitute the equilibrant of forces W_1r_1, W_2r_2, W_3r_3, etc. To find Wr completely, draw the polygon of forces as shown in Fig. 162, and the closing line gives the equilibrant Wr. If the radius r is chosen, W can be calculated, since the product Wr is now known.

Balancing any number of revolving masses in different planes. As in the case of one plane, one condition for the perfect balance of revolving masses is

(1) the algebraic vector sum of the forces represented by W_1r_1, W_2r_2, etc., must be zero. This may be expressed briefly as

$$\Sigma(Wr)=0.$$

When the masses are not all in the same plane, a further condition must be satisfied, namely,

(2) the resultant of all the couples acting must be zero. Expressed otherwise :

$$\Sigma(Wrl)=0.$$

One rotating mass W_1 cannot be balanced by another mass W_2 revolving in another parallel plane distant l from the first, even though these masses are diametrically opposite and $W_1r_1=W_2r_2$. The forces are in equilibrium, but there is an unbalanced couple of magnitude W_1r_1l or W_2r_2l. This couple has variable direction as the masses rotate, but is constant in magnitude. The couple acts in the plane containing the two masses and their axis.

An unbalanced couple will cause vibratory effects, and is as undesirable as an unbalanced force. The magnitude of a couple can be represented by the products such as Wrl ; and since couples are vector quantities, they can be represented by straight lines, and the direction should be along the axis of the couple. It should also be noted that because the moment of a couple is the same about any point in its plane, the couple can be transferred to any other new position in its plane without affecting the result.

Example. *Four masses* A, B, C *and* D *weighing* 100, 120, 160 and *W lb. respectively are rigidly connected to a shaft at radius* 18, 15, 24 *and* 10 *in. respectively from the axis. The shaft revolves about its axis and the planes of revolution are spaced at equal intervals. Find the magnitude of W and the relative angular positions of* B, C *and* D *to* A *so that the masses may completely balance one another.*

Since the unknown mass is in the plane of D, let this plane be chosen as a plane of reference.

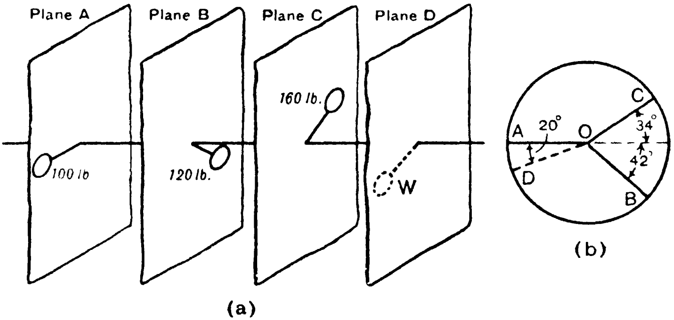


Fig. 163.

	Loads (lb.)	Radii (in.)	Forces Products	Distance from D(l)	Couples
A	100	18	1800	$3l$	$5400l$
B	120	15	1800	$2l$	$3200l$
C	160	24	3840	l	$3840l$
D	W	10	$10W$	0	0

The couples must form a closed triangle or polygon as the case may be. Therefore draw a triangle with sides to represent the couples $5400l$, $3200l$ and $3840l$ as in Fig. 164, *a*. Here *ab* represents $5400l$ and arcs are drawn with centres *a* and *b* in turn and radii $3200l$ and $3840l$ respectively to determine c. In this way the angles between the masses are found.

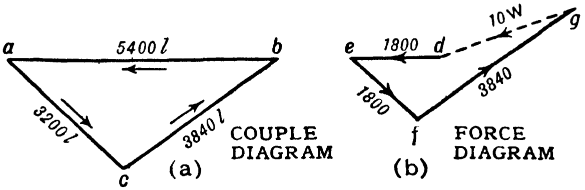


Fig. 164.

In Fig. 164, (*a*), *ab*, *ac*, *bc* represent to scale the couples due to the masses in the planes A, B and C respectively, and these masses are shown in the correct relative angular positions in Fig. 163, *b*.

The force diagram can now be drawn, Fig. 164, *b*, in which $de = 1800$, $ef = 1800$, $fg = 3840$, representing the masses in A, B and C respectively; *de*, *ef*, *fg* must be drawn respectively parallel to *ba*, *ac* and *cb*; *gd* represents the necessary balancing force in magnitude and direction. By measuring *gd* to scale and dividing by the radius 10, W can be calculated. $W = 285$ lb., and A, B, C, D must be arranged in their respective planes as shown in Fig. 163, *b*.

Simple harmonic motion (S.H.M.). When a particle moves in a straight line and its acceleration is always directed towards, and varies as the distance from, a fixed point in that line, the particle is said to move with **simple harmonic motion.** This type of motion should be thoroughly understood; it is of fundamental importance, because it is the basis of all periodic and vibratory motions.

Simple harmonic motion may be regarded geometrically as the projection of uniform circular motion on the diameter of the circle of motion. If the ball of a conical pendulum moving in a circle be viewed from a distance in the plane of the ball, the latter appears to have an oscillatory motion which to the viewer is really simple harmonic motion. Suppose the pin P to move with uniform acceleration $\omega^2 r$ in a circle of radius OP, Fig. 165, and the diameter upon which the motion is projected as AB. Let Q be the projected position of P at any given instant. Then the acceleration of Q will be the component acceleration $\omega^2 r \cos\theta$; that is, the acceleration varies directly as the distance $r\cos\theta$ from O. Therefore the projected point Q has simple harmonic motion along AB. Also the slotted bar PS will move with simple harmonic motion.

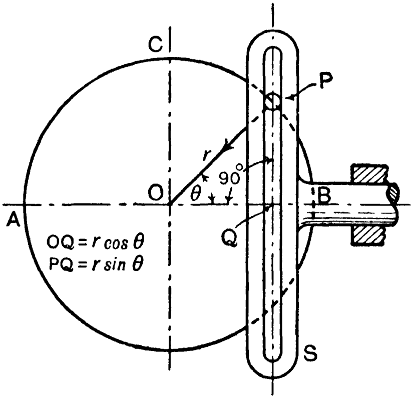


FIG. 165.

The distance the particle moves, each side of the mean position O, is called the **amplitude** of the motion and is equal to r. The time taken for one *complete* oscillation from A to B *and back to* A is called the **period** t of the oscillation, and is equivalent to one complete revolution of P, or $t = 2\pi/\omega$. The time taken for Q to travel from B to Q, that is, $\theta \times t/2\pi$, is called the **phase.**

Very often, when rapid vibrations are concerned in engineering work, the **frequency** $\omega/2\pi$ is specified instead of the period of oscillation.

At any instant the linear velocity of P is ωr, and hence the velocity of Q is the component velocity of P in the direction QO and is equal to $\omega r \sin\theta$. Therefore the velocity of Q is proportional to the distance PQ, so that at A and B the velocities of Q are zero and at O the velocity is a maximum and equal to ωr.

Analytical interpretation of S.H.M. The condition for simple harmonic motion expressed analytically is

$$\frac{d^2x}{dt^2} = -\omega^2 x = \text{acceleration.} \quad \text{.....................(18)}$$

Integrating, $$\frac{dx}{dt} = -\omega r \sin(\omega t + b) = \text{velocity.} \quad \text{...............(19)}$$

Integrating, $$x = r\cos(\omega t + b) = \text{displacement.} \quad \text{...........(20)}$$

In the above expressions r, ω and b are constants.

The *amplitude* is given when x is a maximum, that is, when $\cos(\omega t + b)$ is a maximum or unity. Therefore, the amplitude of the motion is r.

The *periodic time* of the oscillation is given when $(\omega t_2 + b)$ and $(\omega t_1 + b)$ differ by 2π. Hence, the periodic time, $t_2 - t_1$, can be obtained from

$$(\omega t_2 + b) - (\omega t_1 + b) = 2\pi, \text{ or } t_2 - t_1 = 2\pi/\omega. \quad \text{..........(21)}$$

It should be noticed that the periodic time is independent of the amplitude r. The **epoch** is denoted by the angle b, because the epoch is defined as the initial angle from which the motion of the particle is measured. In their simplest forms, the expressions for displacement, velocity and acceleration occur when $b = 0$, that is,

$$x = r\cos\omega t, \quad \frac{dx}{dt} = -\omega r \sin\omega t \quad \text{and} \quad \frac{d^2x}{dt^2} = -\omega^2 r \cos\omega t.$$

From (18), $$\frac{1}{\omega^2} = \frac{\text{displacement}}{\text{acceleration}}.$$

Therefore from (21) the period of oscillation

$$= \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{\text{displacement}}{\text{acceleration}}}.$$

Example. *The maximum velocity of a simple linear harmonic motion is* 10 *ft./sec. and the velocity at a point* 2 *ft. from the mid-position is* 6 *ft./sec. Find the amplitude of the motion, the period of oscillation and the frequency.*

Maximum velocity $= \omega r = 10$ f./s.

Velocity 2 ft. from mid-position $= \omega r \sin\theta = 10 \sin\theta = 6$.

$\therefore\ \theta = \sin^{-1} 0{\cdot}6 = 36^\circ\ 52'$.

Displacement from mid-position $= r\cos\theta = 2$ ft.

$\therefore$ Amplitude $= r = 2/\cos\theta$ or $2{\cdot}5$ ft.

$\omega = 10/r = 4$ rad./sec.

Period of oscillation $= 2\pi/\omega$

$= 2\pi/4 = 1{\cdot}57$ sec.

Frequency $= \omega/2\pi = 0{\cdot}687$.

Variable forces producing S.H.M. If W is the weight of the body moving with S.H.M., then, since at any given displacement x the acceleration is given by $-\omega^2 x$, the force acting at that displacement will be given by

$$P = Wa/g = -W\omega^2 x/g.$$

This expression can be written $-ex$, if e represents the force acting per unit displacement, namely, $W\omega^2/g$.

Springs. Since the straining force acting on a spring is proportional to the extension produced, a load W hanging on the end of a spring and vibrating vertically will move with S.H.M., because the restoring force or accelerating force causing the load W to return to its mean or mid-position is proportional to the extension or displacement and to the acceleration produced.

Let e be the force acting per unit displacement in this case; then for a displacement x the force acting is ex. The acceleration of W if the mass of the spring is neglected is therefore given by $ex = Wa/g$ or $x/a = W/eg$.

Hence, period of vibration, t,

$$= 2\pi\sqrt{\frac{\text{displacement}}{\text{acceleration}}}$$

$$= 2\pi\sqrt{\frac{W}{eg}}\,. \quad \dots\dots\dots\dots\dots\dots\dots\dots(22)$$

Example 1. *A helical spring is loaded with a* 10 *lb. weight and it is noticed when the weight is set to vibrate vertically that it makes* 50 *complete oscillations in* 20 *sec. Calculate the force required to stretch the spring* 1 *inch. Ignore the weight of the spring.*

From $t = 2\pi\sqrt{W/eg}$, $\quad e = 4\pi^2 W/gt^2$

$= 4\pi^2 \,.\, 10/g(0{\cdot}4)^2 = 76{\cdot}6$ lb.

= force to extend spring 1 ft.

$\therefore$ required result = **6·38 lb. per inch.**

Example 2. *When a load of* 1000 *lb. is placed at the centre of a light beam which is supported at its ends in a horizontal position, the central deflection is found to be* 1·6 *inch. Calculate the period of vibration if the load is set vibrating vertically.*

For small deflections, the deflection is proportional to the load producing it; so the beam can be treated as a spring.

$$e = 1000 \times 12/1{\cdot}6 \text{ lb.} = 7500 \text{ lb. per ft.}$$

$$\therefore\ t = 2\pi\sqrt{W/eg} = 2\pi\sqrt{1000/7500g} = \mathbf{0{\cdot}404\ sec.}$$

Reciprocating motion of the slider-crank mechanism. If the ratio of the length of the connecting rod l to that of the crank r is large, the motion of the piston of a reciprocating engine is approximately S.H.M.

Consider the mechanism shown in Fig. 166, in which OP is the crank of length r, PF the connecting rod of length l, F the crosshead.

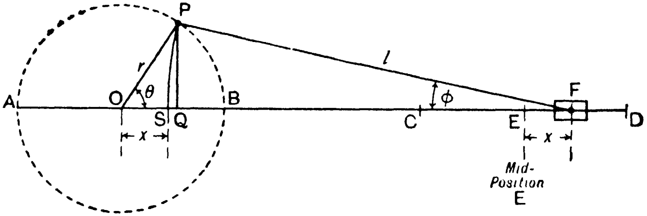


FIG. 166.

As the crank rotates about the crankshaft at O, the crosshead will move from D to C, where DC $=2r$, and back again. When the crank has turned through an angle θ from the position OB, the crosshead will have moved to a position distant x from the mean position E. If the arc PS be drawn with F as centre, then OS will also equal x.

$$\begin{aligned} x &= \mathrm{ED} - \mathrm{FD} = \mathrm{ED} - (\mathrm{OD} - \mathrm{OQ} - \mathrm{QF}) \\ &= r - (r + l - r\cos\theta - l\cos\phi) \\ &= r\cos\theta - l(1 - \cos\phi). \end{aligned}$$

Since PQ $= r\sin\theta = l\sin\phi$, then $\sin\phi = \dfrac{r\sin\theta}{l}$ and

$$\cos\phi = \sqrt{1 - \frac{r^2\sin^2\theta}{l^2}}\,.$$

Expanding the latter expression by means of the Binomial Theorem and ignoring all terms after the second,

$$\cos\phi = 1 - \frac{r^2\sin^2\theta}{2l^2} \text{ nearly.}$$

Thus,

$$x = r\cos\theta - \frac{r^2\sin^2\theta}{2l}\,. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(23)$$

Differentiating,

$$\frac{dx}{dt} = \frac{dx}{d\theta}\cdot\frac{d\theta}{dt} = -r\sin\theta\frac{d\theta}{dt} - \frac{r^2}{2l}\sin 2\theta\frac{d\theta}{dt}\,. \quad \ldots\ldots\ldots(24)$$

$$\therefore\ \frac{dx}{dt} = -\omega r\left(\sin\theta + \frac{r}{2l}\sin 2\theta\right), \text{ if } \omega = \frac{d\theta}{dt}.$$

Differentiating again,

$$\text{Acceleration, } d^2x/dt^2 = -\omega^2 r\left(\cos\theta + \frac{r\cos 2\theta}{l}\right). \quad \ldots\ldots\ldots(25)$$

It should be noticed that if the piston and crosshead have simple harmonic motion, then the acceleration at a distance x from the mean position is $-\omega^2 r\cos\theta$; so that the effect of the obliquity of the connecting rod is confined to, and mainly allowed for, in the second term of (25). If l is increased to a very large size, then obviously the second term ceases to be of any consequence.

From the expressions (23), (24) and (25) the velocity and acceleration of the piston of a reciprocating engine can be calculated fairly accurately for various points distant x from the mean piston position. Using these calculated values, diagrams of displacement, velocity and acceleration can be drawn on a time or crank-angle base for all positions of the crank. Such diagrams are very useful to the engine designer.

These analytical methods should be contrasted with the graphical methods of p. 226.

Example. *The stroke of an engine is* 18 *in. and its crank shaft is revolving at* 125 *r.p.m. Calculate the acceleration of the piston when it has moved* 4 *in. from the end of its stroke if the piston is moving,* (*a*) *with S.H.M.,* (*b*) *with reciprocating motion with the connecting rod four times as long as the crank.*

$$\omega = 250\pi/60, \quad \text{or} \quad 4\cdot167\pi \text{ radians per sec.}$$

(*a*) For S.H.M., acceleration $= -\omega^2 r\cos\theta$ and $\cos\theta = (9-4)/9$

$$= -(4\cdot167\pi)^2 \cdot \tfrac{3}{4} \cdot \tfrac{5}{9}$$

$$= -71\cdot4 \textbf{ ft. per sec. per sec.}$$

(*b*) Reciprocating motion ; where $l = 4r$.

The value of θ can be found by drawing a scale diagram for the position when $x = 9 - 4 = 5$ in. It can also be found approximately by substituting for x, l and r in (23). Employing the latter method,

$$x = r\cos\theta - \frac{r^2\sin^2\theta}{2l},$$

$$5 = 9\cos\theta - \frac{81\sin^2\theta}{72}.$$

Solving this trigonometrical equation, $\theta = 50^\circ\ 55'$.

Acceleration, $d^2x/dt^2 = -\omega^2 r\left(\cos\theta + \dfrac{r\cos 2\theta}{l}\right)$

$$-(4\cdot167\pi)^2 \cdot \tfrac{3}{4}(\cos 50^\circ\ 55' + \tfrac{1}{4}\cos 101^\circ\ 50')$$

$$= -(4\cdot167\pi)^2 \cdot \tfrac{3}{4}(0\cdot6305 - 0\cdot2051/4)$$

$$= -74\cdot42 \textbf{ ft. per sec. per sec.}$$

Simple angular harmonic oscillations. When the angular displacement of a body is proportional to the torque acting and tending to restore the body to its mid-position of oscillation, the motion is said to be **Simple Angular Harmonic Motion.** Suppose α is the angular acceleration of a body displaced through an angle ϕ radians from its vertical position of equilibrium, then if I is the moment of inertia of the body about its axis of suspension, the necessary torque

$T = I\alpha/g$, where T is the torque necessary for a displacement of 1 radian.

$$\therefore\ \phi/\alpha = I/gT.$$

$$\text{Period of vibration} = t = 2\pi\sqrt{\frac{\text{angular displacement} = \phi}{\text{angular acceleration} = \alpha}}$$

$$= 2\pi\sqrt{I/gT} = 2\pi\sqrt{Wk^2/gT}. \quad \ldots\ldots\ldots\ldots(26)$$

Here k is the radius of gyration of the body about the axis of oscillation.

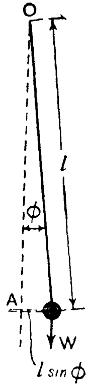


FIG. 167.

Simple pendulum. Consider a concentrated mass of weight W suspended from a fixed point O (Fig. 167) by a thread of negligible weight and length l, and slightly displaced from its mid-position in the vertical line OA. If the angular displacement of the thread is a small angle ϕ, the restoring torque is $T = Wl \sin \phi$, and the angular acceleration α is given by

$$T = Wk^2\alpha/g.$$

Thus, $$Wl \sin \phi = Wk^2\alpha/g.$$

But $k = l$, very nearly, and $\sin \phi = \phi$ nearly if ϕ is very small.

Hence, $$\phi/\alpha = l/g.$$

Therefore, period of vibration,

$$t = 2\pi\sqrt{\frac{\text{angular displacement}}{\text{angular acceleration}}}$$

$$= 2\pi\sqrt{l/g}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(27)$$

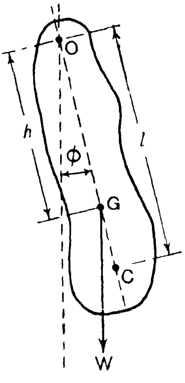


FIG. 168.

Compound pendulum. In the case of the simple pendulum, the mass is assumed to be very dense and concentrated in a small space and the weight of the cord is neglected. Many cases occur in practical work where neither condition holds and the oscillating body is called a **compound pendulum.**

Suppose an irregular mass of weight W, Fig. 168, oscillates in a vertical plane about an axis at O. Let k_g and k_0 be the radii of gyration of the mass about the centre of gravity and the axis of oscillation respectively.

Then, $$k_0^2 = h^2 + k_g^2.$$

$\therefore$ the period of complete oscillation, from (26),

$$= 2\pi\sqrt{Wk_0^2/gT}$$
$$= 2\pi\sqrt{W(h^2 + k_g^2)/gT}.$$

But, $T\phi = Wh \sin \phi$, or $T = Wh$ if ϕ is small.

$$\therefore\ t = 2\pi\sqrt{W(h^2 + k_g^2)/gWh}$$
$$= 2\pi\sqrt{(h^2 + k_g^2)/gh}. \quad \text{.........................(28)}$$

Comparing the result with the period $t = 2\pi\sqrt{l/g}$ for a simple pendulum, the length l of a simple pendulum to give the same period as the compound pendulum is given by

$$l = k_0^2/h \quad \text{or} \quad (h^2 + k_g^2)/h. \quad \text{.......................(29)}$$

In Fig. 168, $l = \mathrm{OC} = k_0^2/h$, in which OC is on the line OG and C is called the **centre of oscillation**; and a small particle placed at C with arm of negligible weight and length l will have the same period of oscillation as the compound pendulum.

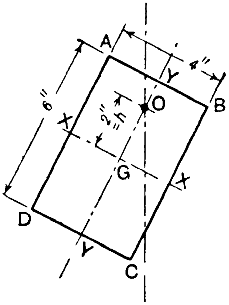


FIG. 169.

N.B.—$\mathrm{GC} = k_g^2/l$.

Example 1. *A cardboard rectangle oscillates with small amplitude about a point* O, *Fig.* 169, *in a vertical plane. Calculate the period of oscillation and check the result by experiment.*

Assuming the cardboard is uniform in mass and thickness and therefore the weight proportional to the area, the centre of gravity G will lie at the geometrical centre of the cardboard. Treating the cardboard as an area and ignoring the thickness, the polar moment of inertia,

$$I_G = I_X + I_Y = (4 \times 6^3/12) + (6 \times 4^3/12) = 104 \text{ in.}^4 \text{ units (p. 212).}$$
$$I_0 = I_G + \text{Area} \times (\mathrm{OG})^2 = 104 + (24 \times 2^2) = 200 \text{ in.}^4 \text{ units (p. 211).}$$
$$\therefore\ k_0^2 = 200/\text{Area} \qquad = 200/24 \qquad = 8\tfrac{1}{3} \text{ in.}^2 \text{ units.}$$
$$t = 2\pi\sqrt{k_0^2/gh} \qquad = 2\pi\sqrt{8\tfrac{1}{3}/32{\cdot}2 \times 12 \times 2}$$
$$= 0{\cdot}65 \text{ sec.}$$

N.B. $g = 32{\cdot}2 \times 12$ in. per sec. per sec.

When checked by experiment, the time for 15 complete oscillations was found to be 9·8 sec. Therefore, the period by experiment $= 9{\cdot}8/15$ $= 0{\cdot}65$ sec.

Example 2. *A uniform circular hoop weighing* 1·8 *lb. rests with a point on its inner circumference on a knife edge. The hoop is allowed to oscillate in a vertical plane about its point of support and the period of oscillation is* 1·38 *sec. Find the polar moment of inertia of the hoop about its centre of gravity.* $h = 10$ *in.* $= 0{\cdot}83$ *ft.*

$$t = 2\pi\sqrt{k_0{}^2/gh}, \text{ or } k_0{}^2 = ght^2/4\pi^2 = 32{\cdot}2 \times 0{\cdot}83 \times 1{\cdot}38^2/4\pi^2 = 1{\cdot}295.$$

$$k_g{}^2 = k_0{}^2 - h^2 = 1{\cdot}295 - 0{\cdot}688 = 0{\cdot}61 \text{ ft.}^2$$

$I_G = Wk_g{}^2/g = 1{\cdot}8 \times 0{\cdot}61/32{\cdot}2$ or **0·0339 in engineer's units.**

Torsional oscillations. Suppose a cylinder (Fig. 170) or other solid be firmly attached to the free end of a wire fastened at O, and the cylinder makes oscillations of small amplitude in a horizontal plane with the wire twisting and untwisting alternately, then the oscillations are called torsional because the controlling force is the resistance the wire sets up to torsion.

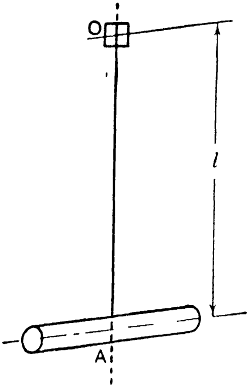


FIG. 170.

For an angular displacement ϕ of the solid about the axis OA, the angle of twist in the wire will be ϕ, and referring to p. 247,

$$\phi = Tl/CI,$$

where T = torque in the wire of length l,

J = polar moment of inertia of the wire section,

C = modulus of rigidity of the material of the wire.

This same torque T will create an angular acceleration α in the movement of the cylinder such that $\alpha = gT/I$, where I is the moment of inertia of the cylinder about the axis OA.

$$\therefore \frac{\text{displacement}}{\text{acceleration}} = \frac{\phi}{\alpha} = \frac{TlI}{gTCJ} = \frac{Il}{CJg}.$$

$$\therefore \text{ period of oscillation} = t = 2\pi\sqrt{Il/CJg}. \quad \text{............(30)}$$

It is just as important to know the natural period of oscillation for shafts as it is for springs and beams. If a revolving propellor shaft is subjected to the action of periodic impulsive twisting moments coinciding with the natural period for the shaft, then the amplitude of any oscillations set up will tend to increase and produce undue strain and fracture.

Example. *In an experiment, a loaded steel wire* 24·8 *in. long and* 0·022 *in. in diameter is loaded with a solid weighing* 3·3 *lb., the latter having a radius of gyration of* 1·878 *in. about the axis of the wire, and the period of torsional oscillation about the axis of the wire is* 10·4 *sec. Find the modulus of rigidity C of the wire.*

$$t = 2\pi\sqrt{Il/CJg} \quad \text{or} \quad C = 4\pi^2 Il/Jt^2 g.$$

Also $$I = Wk^2 = 3{\cdot}3 \times (1{\cdot}878)^2, \quad J = \pi d^4/32 = \pi \times (0{\cdot}022)^4/32.$$

$$l = 24{\cdot}8, \quad t = 10{\cdot}4, \quad g = 32{\cdot}2 \times 12 \text{ in./sec.}^2$$

This gives **C = 12,000,000 lb./in.²**

EXERCISES ON CHAPTER XI

1. Show that the acceleration of a point moving with uniform speed in a circle of radius r is v^2/r.

At what speed can a train run round a curve of 2000 ft. radius without having any thrust on the wheel flanges, when the outer rail is laid 1½ in. above the inner one, and the gauge is 4 ft. 8½ in.?

2. A motor-car is rounding a curve on a level uncambered road. Show clearly, in a diagram, the forces acting on the car. Denote carefully the direction of the forces and describe the forces, for example, the weight, etc. You may ignore the propelling and resisting forces in the direction of the car's forward motion. (U.E.I.)

3. The centre of gravity of the shaft of a rotor of a turbine is found to be 0·025 in. eccentric to the longitudinal axis of the shaft. If the rotor and shaft weigh 2576 lb. and the rotor is symmetrically placed between two bearings, find, for a speed of 3000 r.p.m., the magnitude of the maximum vertical force tending to lift the bearing caps.

Also estimate the bending moment at the central cross-section of each cap due to these vertical forces, if the holding-down bolts for the caps are at 4½ in. centres.

4. A vehicle describes a horizontal circle of 600 ft. radius with a velocity of 40 m.p.h.; find the inward or centripetal acceleration necessary to maintain the motion. Also, if the vehicle runs on rails of gauge 1 metre, how much higher ought the outer rail to be above the inner one?

5. If the radius of a level railway line is 1000 ft., find the speed which causes a locomotive to exert a side thrust on the outer rail equal to 1/100 of its own weight. For a speed of 30 m.p.h., and with rails which will withstand the same side thrust, what is the least radius which can be safely employed?

6. Find the necessary elevation for the outer rail if the resultant thrust of a train when rounding a curve of 900 ft. radius at 45 m.p.h. should be normal to the track. Gauge 4 ft. 8½ in.

7. In exercise 6 find the height of the centre of gravity if the locomotive of the train is not to overturn :

(*a*) with the elevation as in this question ;
(*b*) if there is no elevation of the outer rail.

8. Taking μ as 0·3 and the radius of the track as 200 ft., find the greatest speed a car can round the curve without sideslip if

(*a*) the track is level ;
(*b*) if the track is banked at an angle of $\tan^{-1} 0{\cdot}2$;
(*c*) if the condition of overturning is reached and the track is banked at an angle of $\tan^{-1} 0{\cdot}2$. Take $h=3$ ft., $d=4$ ft. 6 in.

9. A lorry with its load, both symmetrical about a longitudinal central plane, weighs altogether 20 tons and is travelling round a curved track of 500 ft. radius at 30 m.p.h. Given that the centre of gravity of the combined lorry and load is 4 ft. 6 in. above the track and the wheels of the lorry on the same axle are 4 ft. apart, find :

(*a*) the pressure on each wheel normal to the track when the lorry is on the level, and also when the track is banked at an angle of 10° if the speed is 30 m.p.h. ;
(*b*) the angle of banking to equalise the pressure on the wheels normal to the track at the same speed ;
(*c*) the speed at which the lorry will overturn at the angle of banking calculated in (*b*).

10. Find the greatest speed at which a car can turn a corner of 60 ft. radius when the coefficient of friction between the tyres and the road is 0·35 if there is to be no skidding :

(*a*) on the level road ;
(*b*) if the angle of banking is $\tan^{-1} 0{\cdot}25$.

11. Obtain the formula giving the height of a conical pendulum. If the revolving pendulum weighs 10 lb. and the height is 8 in., what is its speed in radians per second and r.p.m? If the ball is acted on by a vertical downward force of 10 lb. weight, what is its angular velocity for the same height?

12. What is the height of a simple governor, if friction is neglected, when the speed is 60 r.p.m.?

13. If in question 12 the weight of the balls is 2 lb. each and the sleeve carries a load of 12 lb., find the speed of running for the same height.

14. The balls and central load of a loaded governor are respectively 5 lb. each and 30 lb. At the maximum speed the radius of the ball path is 8 in., and at the minimum speed 6 in. Neglecting friction, find the extreme speeds when the arms and links are each 10 in. long.

15. Find the out-of-balance force in tons if a mass of 5152 lb. rotates at 3000 r.p.m. at a radius of 0·2 in.

16. A railway carriage wheel is found to be out of centre to the extent of 3 lb. at 18 in. radius. What will be the amount of hammer blow on the rails when running at 60 m.p.h.? Diameter of wheels = 3 ft. 6 in.

17. Explain the meaning of centripetal acceleration.

A disc makes 120 r.p.m. A pin weighing 5 lb. is connected to the disc at a radius of 16 in., and a second pin weighing $7\frac{1}{2}$ lb. at a radius of 20 in.; the weights are on radii at right angles to each other. Determine the magnitude of the unbalanced force and also the single mass at a radius of 10 in. which will balance the disc.

18. A flywheel of mass 3600 lb. is found to have its centre of gravity 0·06 in. out of centre. Determine the dynamic force on the shaft at 180 r.p.m. and the required balance weight at a radius of 28 in.

19. Revolving masses of W, 100, 120 and 116 lb. are rigidly connected to a shaft at radii 15, 12, 14 and 12 in. respectively. The planes of revolution of the masses are at equal intervals apart. Determine W and the angular positions of the other masses with respect to W in order that the masses may completely balance one another.

20. Three masses of 10, 15 and 20 lb. are attached to a balanced disc at angular distances, measured from the 10 lb. mass, of 0°, 90° and 150° and at radii of 3, 2 and $2\frac{1}{2}$ ft. respectively. Find the resultant force on the axis of the disc when it is rotating at 120 r.p.m. Determine also the angular position and magnitude of an additional mass placed at a radius of $1\frac{1}{2}$ ft. which will give running balance. (U.E.I.)

21. Balanced masses of 20, 15, W and 20 lb. at radii of 9, 12, 16 and 18 in. respectively revolve in one plane. The angles between the 20 and 15 and the 15 and W are 90°; find W and the angular positions.

22. In question 21, if the four masses were in four different planes A, B, C and D respectively, find W and the angular position relative to the first mass for complete balance when B, C and D are respectively 16, 20 and 28 in. from A.

23. Obtain the magnitude and position of the balance weights in the driving wheels of a locomotive to balance the rotating parts from the following data. The latter may be considered as 480 lb. at the centre of each crank pin and the crank radius as $10\frac{1}{2}$ in. The cylinders are 2 ft. apart and symmetrically placed and the c.g. of the balance weights 4 ft. $10\frac{1}{2}$ in. apart, and the balance weights are to be placed at a radius of 2 ft. 9 in. *N.B.* The angle between the cranks is 90°.

24. Find the equivalent simple pendulum for a uniform rod 6 ft. long oscillating about an axis through one end.

25. If a platform has a vertical S.H.M. and a travel of 6 in., find its shortest period if a weight resting on it has to remain in contact throughout the travel.

26. Find the period of oscillation of a single pendulum 1 metre in length.

27. Explain clearly the meaning of Simple Harmonic Motion, and give a few instances where we get this motion approximately.

A motor-car weighing 2 tons compresses its springs by 2 in. Find its period for one complete vertical oscillation, and the number of vertical oscillations it would make per second.

28. A body weighing 100 lb. is oscillating in a horizontal straight line with simple harmonic motion. The amplitude of the oscillation is 4 ft., and the periodic time of a complete oscillation is 10 sec. Neglecting friction, find :

(*a*) the maximum force acting on the body ;
(*b*) the velocity of the body when it is 1 ft. from the centre of oscillation ;
(*c*) the time taken to travel 2 ft. from an extreme point of the oscillation. (U.E.I.)

29. Explain briefly what you understand by Simple Harmonic Motion.

A body weighing 20 lb. reciprocates through a distance of 2 ft. with a simple harmonic motion, the number of double strokes per minute being 120. Draw a diagram of the accelerating forces acting on the body for a complete stroke, and mark on the diagram the magnitudes and directions of the forces acting at the ends of the strokes.

30. A wagon compresses its springs by 2 in. Give a formula which enables you to determine the period of vibration of the loaded springs, and find that period. Describe the experiment and give other examples of Simple Harmonic Motion.

31. A tool head of a slotting machine, assumed to be moving with simple harmonic motion, weighs 80 lb. and has a stroke of 6 in. When the tool head is $1\frac{1}{2}$ in. from its end position, the force driving it is 800 lb. If there are 60 cutting strokes per minute, find how much of the applied force is spent :

(*a*) in producing the acceleration of the tool head in this position ;
(*b*) in cutting the material and overcoming frictional resistance. (U.E.I.)

32. If an engine makes 180 double strokes per minute and the length of the stroke is 16 in., find the acceleration of the piston when it is moving towards its mid-position and is 3 in. from that position.

33. A point moves with S.H.M. with an amplitude of 1 ft. and a period of 2 sec. Find the greatest and least values of the velocity and acceleration, and the velocity and acceleration 0·5 sec. after it has left one end of its path.

34. A closely-coiled helical spring is found to be deflected 1 in. for each 20 lb. of load. Calculate (*a*) the number of complete vertical

oscillations a load of 40 lb. would make if attached to the end of this spring; (*b*) the greatest acceleration to which the load would be subjected for an amplitude of 2 in.

35. A load weighing 10 lb. is moving with S.H.M. with an amplitude of 2 ft. and a period of 3 sec. Draw diagrams to scale to show the displacement, the velocity, the acceleration, the accelerating force and the kinetic energy of the load on a time-base.

36. Use the first diagram of question 35 to plot diagrams of velocity, acceleration, accelerating force and kinetic energy of the load on a displacement-base, that is, for all points on its path.

37. A circular disc 10 in. in diameter oscillates with small amplitude in its own plane about an axis perpendicular to its plane and 3 in. from its centre. Calculate the period of oscillation.

38. A connecting rod (Fig. 171) with longitudinal axis OCP is moved to OP through 10° from its position of equilibrium OY. The connecting rod weighs 4 lb. and its centre of gravity C is 7 in. from O. If it is set free to oscillate in the plane OPY, calculate:

(*a*) the torque tending to restore the rod to its position of equilibrium;

(*b*) the kinetic energy of rotation of the rod in the position OY;

(*c*) the period of oscillation of the rod if the radius of gyration about the axis of oscillation through O is 8·5 in.

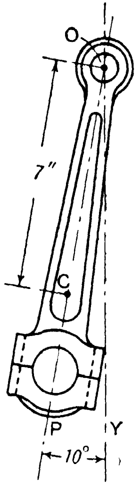


FIG. 171.

39. A uniform bar 1 ft. long weighing 12 lb. is suspended in a horizontal position by a vertical wire 30 in. long fixed at its upper end to a support and at its lower end at the c.g. of the bar. The wire is 0·1 in. in diameter. Find the modulus of rigidity of the wire, given that the square of the radius of gyration of a uniform bar about an axis normal to its length through its c.g. is $l^2/12$, where l is the length of the rod, if the period of torsional vibration about the axis of the wire is 1·94 sec.

40. Calculate the displacement, velocity and acceleration of the piston of a reciprocating engine of 20 in. stroke if the engine speed is 240 r.p.m. when the piston has moved for 0·02 sec. from its inner dead centre. The connecting rod is 40 in. long. What is the acceleration at $\frac{1}{4}$-stroke from the inner dead centre?

CHAPTER XII

HYDRAULICS

THE history of this subject covers a development over several thousands of years and there is evidence of irrigation schemes and water storage in the early life of Mesopotamia, Egypt and China. Lake Moesis, constructed more than 4000 years ago, is larger than any modern reservoir and was designed to conserve the flood waters of the Nile. The Greeks and Romans certainly knew something of aqueducts and the conveyance of water, but this knowledge appears to be based more upon experience than upon any scientific research.

Up to the fifteenth century, no record exists of any further advance in the knowledge of Hydraulics, but during this century Leonardo da Vinci, an Italian, contributed a remarkable book dealing with the motion of water and the measurement of its flow through orifices and over weirs. In the seventeenth century, Galileo developed the fundamental law for falling bodies , that is, he established that the velocity of a body falling under gravity was $V=\sqrt{2gh}$, and he and his pupil Torricelli applied this law to the flow of water through an orifice.

Storage and conveyance of water: pipes, channels, orifices and weirs. It is an established fact in the distribution of population that large communities gather near great industrial centres where the existent water supply is inadequate, and water has to be conveyed from areas where the supply is in excess of the demand to these regions of dense population. For this purpose water is collected in large catchment areas, often far removed from the locality where it is required, stored in reservoirs, and conveyed by pipe lines, or open channels, to the district where it is to be used. Similarly, hydraulic plant is served by pipe lines which convey water from a storage reservoir to the place of operation of this plant.

For these reasons it is necessary for the hydraulic engineer to study the flow of water in pipes and channels, through orifices, and over weirs, a study which is linked with the energy possessed by the fluid under varying conditions.

Energy content of a fluid. Water, or any other fluid, may possess energy, either due to its position, or to the velocity of its flow ; that is, *potential energy* due to its position, or *kinetic energy* due to its velocity of flow. A fluid may lose one form of energy when this energy is changed to the other form, or it may dissipate energy in overcoming resistance or in doing work. Such resistance may be found in the friction between pipe surfaces and the water flowing in the pipe, in bends, channels and in the surfaces over which the water has to pass. Similarly, the fluid may have to supply energy in order to raise its level against gravity, or to force its way through constrictions or other obstacles.

Pressure head, or equivalent potential energy of water. When a quantity of water is situated at a height above a certain level, it is said to possess **pressure head** and is capable of doing work in falling from the higher level to the lower, or reference level, which is known as the **datum level**, Fig. 172.

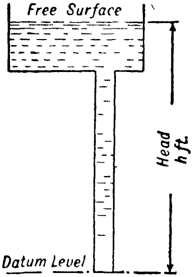


FIG. 172

Suppose a quantity of water weighing 100 lb. is stored in a tank situated 40 ft. above the ground, or datum level. This water would possess a potential energy due to its weight and height of 100 × 40, or 4000 ft. lb., and would be capable of doing this 4000 ft. lb. of work in falling from the tank to datum level.

For convenience, the pressure head of every pound of this water is expressed by the height at which it is situated, that is, 40 ft., and every pound is capable of performing 40 × 1 ft. lb. of work in falling to datum level. In other words, its potential energy is 40 ft. lb. per pound of water.

Velocity head. If water situated at a height is allowed to fall, it will give up its potential energy and acquire a certain quantity of kinetic energy due to its velocity of fall. Suppose in Fig. 173 a quantity of water weighing one pound falls from a height of h feet,

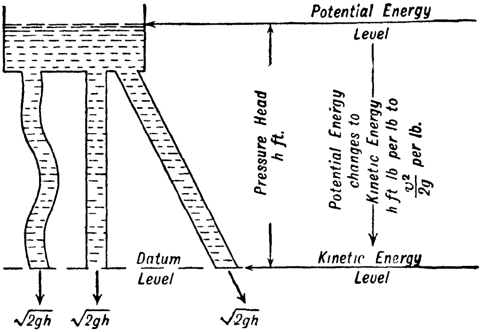


FIG. 173.

its potential energy will be converted to kinetic energy and it will acquire a velocity of $\sqrt{2gh}$ ft. per sec. at the end of the fall. It follows that a kinetic energy due to velocity may be expressed as a corresponding pressure head which would produce this velocity. Since $V = \sqrt{2gh}$, the pressure head which would produce this velocity, V may be expressed as $h = \frac{V^2}{2g}$ and is called the **velocity head.**

Example 1. *Calculate the velocity head corresponding to a velocity of* 40 *ft. per sec.*

$$V = 40 \text{ ft. per sec.}$$

$$\therefore\ h = \frac{V^2}{2g} = \frac{1600}{64\cdot4} = 24\cdot84 \text{ ft.}$$

Example 2. *Find the velocity of a stream of water which falls from a height of* 120 *ft. What is the velocity head of this water?*

$$V = \sqrt{2gh} = \sqrt{64\cdot4 \times 120} = 87\cdot91 \text{ ft. per sec.}$$

The velocity head is the head which produces this velocity, that is, 120 ft.

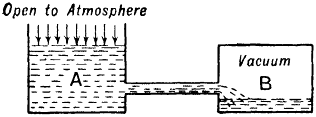


FIG. 174.

Atmospheric head. Suppose two chambers A and B are connected by a pipe (Fig. 174) and that A contains water open to the atmosphere. Then if B is exhausted of air, water will flow from A into B under the atmospheric pressure in A, if the slight difference in levels in A, and B be ignored. This atmospheric pressure may be expressed as a pressure head obtained as follows :

$$h_a = \frac{\text{atmospheric pressure in lb. per sq. ft.}}{\text{weight of 1 cubic ft. of water}} = \frac{p_a}{w},$$

and the resulting head h_a is called the **atmospheric head.**

Example. *Calculate the atmospheric head due to an atmospheric pressure of* 15 *lb. per sq. in.*

$$\text{Atmospheric pressure in lb. per sq. ft.} = 15 \times 144 \text{ lb.}$$

$$\text{Atmospheric head} = \frac{15 \times 144}{62{\cdot}3} = \mathbf{34{\cdot}67\ ft.},$$

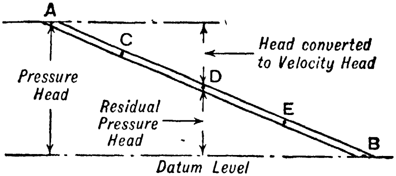
where 1 cu. ft. of water weighs 62·3 lb.

Principle of conservation of energy. It has been shown that energy cannot be destroyed, or created, but one form of energy may be converted to another form, *or* energy may be utilised in the performance of work. About the end of the eighteenth century a theorem was established by Bernoulli which gave expression to the principle of conservation of energy in its application to hydraulics.

Bernoulli's theorem. *In a steady stream of fluid in which there is no loss by friction or other causes, the sum of the atmospheric, pressure and velocity heads is constant for all sections of the stream.*

atmospheric head + pressure head + velocity head = constant.

Thus in Fig. 175, if water is flowing along the pipe from A towards B, the sum of the heads at any one section C, D or E is always the same.



Total Head at each point is constant

FIG. 175.

Example 1. *A pipe line connects two points* A *and* B *at levels differing by* 20 *ft. If the water at* A *is flowing at* 1 *ft. per sec., calculate the velocity of flow at the lower level* B, *assuming no frictional or other losses.*

At A : Total head = atmospheric + pressure + velocity heads

$$=\frac{p_a}{w}+20+\frac{V_1^2}{2g}$$

$$=\frac{p_a}{w}+20+\frac{1^2}{64\cdot 4},$$

where p_a = atmospheric pressure in lb. per sq. ft.

w = weight of 1 cu. ft. of water in lb.

At B : Total head $=\frac{p_a}{w}+0+\frac{V_2^2}{2g}$;

$$\text{then}\quad \frac{p_a}{w}+20+\frac{1^2}{64\cdot 4}=\frac{p_a}{w}+0+\frac{V_2^2}{64\cdot 4},$$

$$V_2^2-1=20\times 64\cdot 4,$$

$$\mathbf{V_2=35\cdot 9\ ft.\ per\ sec.}$$

Example 2. *The suction pipe of a pump has its open end in a sump situated* 10 *ft. below pump level. If the pressure in the pump cylinder during pumping is* 2 *lb. per sq. in., calculate the velocity at which the water enters the pump, assuming no friction or other losses. Atmospheric pressure* = 14·7 *lb. per sq. in.*

Let V be the velocity of entry in ft. per sec., and taking the pump level as datum,

$$\text{the total head at pump level}=\frac{2\times 144}{62\cdot 3}+\frac{V^2}{2g}\ \text{ft.},$$

$$\text{,, ,, ,, sump level}=\frac{14\cdot 7\times 144}{62\cdot 3}-10\ \text{ft.}$$

It follows from Bernoulli's theorem that

$$\frac{2\times 144}{62\cdot 3}+\frac{V^2}{2g}=\frac{14\cdot 7\times 144}{62\cdot 3}-10,$$

$$\frac{V^2}{2g}=\frac{12\cdot 7\times 144}{62\cdot 3}-10=19\cdot 36,$$

$$V=\sqrt{19\cdot 36\times 64\cdot 4}=\mathbf{35\cdot 31\ ft.\ per\ sec.}$$

Example 3. *A boiler having an internal gauge pressure of* 120 *lb. per sq. in. is to be supplied with feed water at an injection velocity of* 4 *ft. per sec. by a feed pump situated* 3 *ft. below the injection pipe level and having an overall efficiency of* 72%. *Calculate the pressure which will be required in the pump cylinder in order to serve this boiler.*

$$\text{Pressure head}=\frac{120\times 144}{62\cdot 3}\ \text{ft.}$$

$$\text{Velocity head} = \frac{V^2}{2g} = \frac{16}{64 \cdot 4} \text{ ft.}$$

Head due to difference in level $= 3$ ft.

Let h be the pressure head in the pump.

Then
$$h = \frac{120 \times 144}{62 \cdot 3} + \frac{16}{64 \cdot 4} + 3$$
$$= 277 \cdot 4 + 0 \cdot 248 + 3$$
$$= 280 \cdot 65 \text{ ft.}$$

$$\text{Pressure equivalent} = \frac{280 \cdot 65 \times 62 \cdot 3}{144}$$
$$= 121 \cdot 4 \text{ lb. per sq. in.}$$

Allowing for 72% overall efficiency,
$$\text{required pressure} = \frac{100}{72} \times 121 \cdot 4$$
$$= 168 \cdot 6 \text{ lb. per sq. in.}$$

Flow of water through nozzles. When a pipe is running full and the stream of water is forced through a nozzle into the atmosphere (Fig. 176), the same quantity of water must be discharged as is passing through the unrestricted pipe.

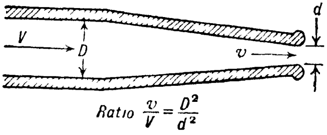


FIG 176

In order that this may occur, the velocity of discharge must be greater than that in the pipe, and since the quantities at inlet and outlet are equal, the velocity of discharge will bear a relationship to the velocity of flow in the pipe inversely proportional to the ratio of the areas.

Let D ft. be the diameter of the pipe and V ft. per sec. the velocity in the pipe.

Then quantity $Q = \frac{\pi D^2}{4} \times V$ cu. ft. per sec., a quantity which must pass the nozzle.

If d ft. is the diameter of the nozzle and v ft. per sec. the velocity of discharge,

$$v = \frac{\pi D^2 \times V \times 4}{\pi d^2 \times 4} \quad \text{or} \quad \frac{D^2}{d^2} V;$$

that is
$$\frac{\mathbf{v}}{\mathbf{V}} = \frac{\mathbf{D}^2}{\mathbf{d}^2} = \textbf{inverse ratio of areas.}$$

Example. *The supply pipe to a nozzle is 4 in. in diameter and water flows through it at 5 ft. per sec. Calculate the velocity of discharge if the diameter of the nozzle is 2 in.*

Inlet velocity to nozzle $=5$ ft. per sec. $=V$.

$$\text{Ratio of areas} = \frac{\text{area of pipe}}{\text{area of nozzle}} = \frac{\pi \times 2^2}{\pi \times 1^2} = 4.$$

Outlet velocity from nozzle $=4V=$ **20 ft. per sec.**

Effect of pressure and velocity heads in nozzle discharge. When the supply pipe to a nozzle is sloping, or there is pressure and velocity head at inlet to the nozzle, the problem becomes an application of Bernoulli's theorem.

Example 1. *A pipe 300 ft. in length and 4 in. in diameter is connected at its lower end* B *to a nozzle* $1\frac{1}{2}$ *in. in diameter which discharges into the atmosphere. If the top end of the pipe* A *is supplied from an open reservoir 35 ft. above* B, *find the velocity of discharge from the nozzle and the quantity of water discharged in gallons per minute. What is the pressure head at discharge?*

$$\text{Total head at A} = \frac{p_a}{w} + 35\,;$$

$$\text{,, \quad at B} = \frac{p_a}{w} + 0 + \frac{V^2}{2g}\,;$$

where V is the velocity at entry to the nozzle.

Then applying Bernoulli's theorem,

$$\frac{p_a}{w} + 35 = \frac{p_a}{w} + 0 + \frac{V^2}{2g},$$

$$V = \sqrt{35 \times 2g} = 47{\cdot}47 \text{ ft. per sec.}$$

The quantity of discharge $= V \times$ area

$$= 47{\cdot}47 \times \frac{\pi \times (2)^2}{144} \text{ cu. ft. per sec.}$$

$$= \frac{47{\cdot}47 \times \pi \times 4 \times 60 \times 6{\cdot}23}{144} \text{ gallons per min.}$$

Quantity = 1549 gallons per minute.

$$\text{Velocity of discharge} = V \times \frac{\text{area of pipe}}{\text{area of nozzle}}$$

$$= 47{\cdot}47 \times \frac{(4)^2}{(1{\cdot}5)^2} = 47{\cdot}47 \times \frac{64}{9}.$$

Velocity = 337·56 ft. per sec.

Since the nozzle discharges into the atmosphere, the pressure head at discharge is atmospheric head only.

Example 2. *The pressure gauge immediately before a nozzle registers 80 lb. per sq. in. gauge. The nozzle effects a reduction in pipe diameter from 3 inches to 1¼ inches and discharges into a condenser having an internal pressure of 3 lb. per sq. in. absolute. Calculate the velocity of the jet.*

Pressure head at entrance to nozzle $= \frac{80 \times 144}{62{\cdot}3}$ ft.,

then $$\frac{p_a}{w} + \frac{80 \times 144}{62{\cdot}3} = \frac{3 \times 144}{62{\cdot}3} + \frac{V^2}{2g};$$

where V would be the velocity of discharge if there were no reduction of area.

$$V^2 = 2g\left\{\frac{11{\cdot}7 \times 144}{62{\cdot}3} + \frac{80 \times 144}{62{\cdot}3}\right\},$$

$$V = \sqrt{2g\left\{\frac{91{\cdot}7 \times 144}{62{\cdot}3}\right\}} = 116{\cdot}9 \text{ ft. per sec.}$$

Next consider the effect of reduction of area.

$$\text{Velocity of discharge} = \frac{116{\cdot}9}{1} \times \frac{(3)^2}{(1\tfrac{1}{4})^2}$$

$$= \mathbf{673 \text{ ft. per sec.}}$$

Loss of head due to friction in pipes. The knowledge which at present exists on frictional losses in pipes is based upon a long period of experimental work, the results of which give certain empirical formulae and constants which vary with the condition and nature of the pipe concerned.

The general principles which these experiments have evolved can be stated as follows :

The loss of head due to friction in a pipe is *directly proportional* to

(*a*) **wetted surface of the pipe,** (*b*) **square of the velocity of flow ;** *and* **inversely proportional to**

(*c*) **area of the cross-section of the pipe at normal speeds of flow.**

It is also found that the loss of head, or energy, depends upon an empirical constant, the **coefficient of friction, or resistance,** associated with the particular material and condition of the pipe.

The first formula, which is in general use, is

$$\mathbf{h}=\frac{\mathbf{4flv^2}}{\mathbf{2gd}},$$

where h = *loss of head due to friction,*
f = *coefficient of friction,*
l = *length of pipe in ft.,*
v = *velocity of flow in ft. per sec.,*
d = *diameter of pipe in ft.*

This formula is consistent with the experimental results obtained by investigators.

The second formula was obtained very early in the researches on pipe flow and is known as the "Chezy" formula. It is

$$\mathbf{v}=\mathbf{C}\sqrt{\mathbf{mi}},$$

where C = *a constant for the pipe,*
v = *the velocity of flow in ft. per sec.,*
m = *the hydraulic mean depth,*
i = *the virtual slope.*

The **hydraulic mean depth** is the name given to the ratio

$$\frac{\textit{cross-sectional area of fluid}}{\textit{wetted perimeter of the pipe}};$$

and the **virtual slope** or **hydraulic gradient** is the ratio

$$\frac{\textit{loss of head due to friction}}{\textit{length of pipe}}.$$

An examination of the second formula in its relation to the first gives

$$v=C\sqrt{mi} \quad \text{or} \quad v^2=C^2mi.$$

It will be seen that $i=\frac{h}{l}$ and $m=\frac{\pi d^2}{4\pi d}$ for a full pipe.

Therefore $v^2=C^2\times\frac{d}{4}\times\frac{h}{l}$ and $h=\frac{4lv^2}{dC^2}$,

which when related to $h=\frac{4flv^2}{2gd}$ gives $\frac{1}{C^2}=\frac{f}{2g}$;

that is, $\mathbf{C}=\sqrt{\frac{\mathbf{2g}}{\mathbf{f}}}$.

Summarising the formulae for loss of head due to friction:

$$(a)\ h = \frac{4flv^2}{2gd}$$

$$\text{and}\quad (b)\ v = C\sqrt{mi}$$

in which, for the purposes of substitution,

$$C = \sqrt{\frac{2g}{f}} \quad \text{or} \quad f = \frac{2g}{C^2}.$$

The values of f and C depend upon pipe material and conditions, and certain representative values are tabulated as follows :

Particulars of pipe	Diameter	Velocity	C	f
Old cast iron - -	12 in.	6 ft./sec.	85	0·0089
New cast iron - -	,,	,,	112	0·0051
Steel (riveted) - -	,,	,,	104	0·0059
Smooth concrete -	,,	,,	112	0·0051

Example 1. *Calculate the hydraulic mean depth of* (*a*) *a pipe* 4 *ft. in diameter running full,* (*b*) *a rectangular channel* 8 *ft. wide,* 4 *ft. deep, running full.*

$$(a)\ m = \frac{\text{area of fluid cross-section}}{\text{wetted perimeter}} = \frac{\pi \times 2^2}{\pi \times 4} = 1\text{ ft.}$$

$$(b)\ m = \frac{8 \times 4}{2 \times 4 + 8} = \frac{32}{16} = 2\text{ ft.}$$

Example 2. *Calculate the hydraulic mean depth of a trapezoidal channel* 4 *ft. in depth, base* 6 *ft. in width, top* 9 *ft. in width, running full.*

$$\text{Area of fluid section} = 4 \times \tfrac{1}{2}\{9 + 6\}$$
$$= 30\text{ sq. ft.}$$

$$\text{Wetted perimeter} = 6 + 2\sqrt{1{\cdot}5^2 + 4^2}$$
$$= 6 + 2\sqrt{18{\cdot}25} = 14{\cdot}54\text{ ft.}$$

$$m = \frac{30}{14{\cdot}54} = 2{\cdot}06\text{ ft.}$$

Example 3. *Find the quantity of water discharged from a pipe* 5000 *ft. in length and* 16 *inches diameter when the difference in level between the ends of the pipe is* 100 *ft. and C in the Chezy formula is* 106.

$$m=\frac{\pi\times 64}{\pi\times 16}=4 \text{ in. or } \tfrac{1}{3} \text{ ft.}$$

$$i=\frac{100}{5000}=0{\cdot}02.$$

$$v=C\sqrt{mi}=106\sqrt{\tfrac{1}{3}\times 0{\cdot}02}=8{\cdot}656 \text{ ft. per sec.}$$

$$Q=8{\cdot}656\times\frac{\pi\times(\frac{2}{3})^2}{4}=3{\cdot}021 \text{ cu. ft. per sec.}$$

Virtual slope, or hydraulic gradient. When water flows from a tank down a uniform pipe (Fig. 177), the velocity of flow will be V ft. per sec., a quantity dependent upon the difference in level at A and B. If, on the other hand, the velocity of flow is controlled so that the full value of $V=\sqrt{2gh}$ is not obtained, there will be a pressure head in addition to a velocity head at every point in the pipe.

Suppose the pipe is fitted with piezometer tubes at C, D, E and F (Fig. 177), then these tubes will show a water level which is the effective pressure head at each point. If the velocity of flow is V ft. per sec., there should be a fall in level below free surface level of $V^2/2g$, the velocity head due to the velocity V. Actually, it is found that the levels of water at C, D, E and F do not follow the line S, T determined by the velocity head, but there is a fall of pressure head due to frictional losses, and the water levels give a line SR, known as the **virtual slope or hydraulic gradient.**

The head actually available as pressure head at C, D, E and F is shown by the shaded height, while the head lost due to friction is shown in black.

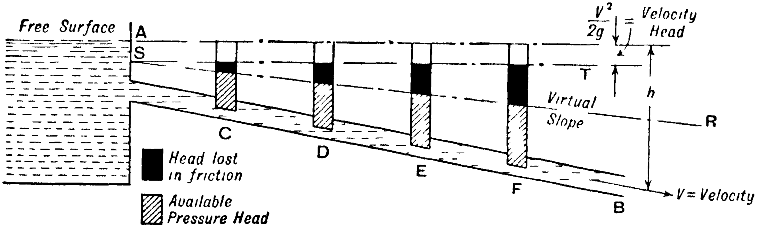


FIG. 177.

Example 1. *A pipe 4 ft. in diameter discharges 12,000 gallons per minute. If the pipe is 2000 ft. in length and the coefficient of resistance $f=0{\cdot}0075$, find the head lost in friction.*

$$Q=12{,}000 \text{ gallons per min.}=\frac{200}{6{\cdot}23} \text{ cu. ft. per sec.}$$

$$v=\frac{Q}{A}=\frac{200}{6{\cdot}23\times\pi\times2^2}=2{\cdot}555 \text{ ft. per sec.}$$

$$\text{Loss of head}=\frac{4flv^2}{2gd}$$

$$=\frac{4\times0{\cdot}0075\times2000\times(2{\cdot}555)^2}{64{\cdot}4\times4}$$

$$=\mathbf{1{\cdot}521\ ft.}$$

Example 2. *The loss of head resulting from the flow of 10,400,000 gallons of water per day of 24 hours through a pipe 1 mile in length and 2 ft. in diameter is 12 ft. Calculate the coefficient of resistance.*

$$Q=\frac{10{,}400{,}000}{24\times60\times60\times6{\cdot}23} \text{ cu. ft. per sec.}$$

$$v=\frac{Q}{A}=\frac{10{,}400{,}000}{24\times3600\times6{\cdot}23\times\pi\times1^2}=6{\cdot}15 \text{ ft. per sec.}$$

$$h=\frac{4flv^2}{2gd}, \quad \text{or} \quad f=\frac{2gdh}{4lv^2}.$$

$$f=\frac{64{\cdot}4\times2\times12}{4\times5280\times(6{\cdot}15)^2}=\mathbf{0{\cdot}00193.}$$

Example 3. *Find the velocity of flow and the maximum quantity of discharge in gallons per hour from a sewer 15 in. in diameter having a fall of one in 1200, $C=102$.*

$$v=C\sqrt{mi}=102\sqrt{\frac{1{\cdot}25}{4}\times\frac{1}{1200}}$$

$$=\mathbf{1{\cdot}646\ ft.\ per\ sec.}$$

$$Q=1{\cdot}646\times\left\{\frac{\pi\times(1{\cdot}25)^2}{4}\right\}\times60\times60\times6{\cdot}23$$

$$=\mathbf{45{,}300\ gallons\ per\ hour.}$$

Example 4. *The pressure at the pump end of an accumulator service is 500 lb. per sq. in. gauge. Calculate the pressure of delivery to the accumulator if the service pipe is 4 in. in dia. and 1000 ft. in length. $f=0{\cdot}01$, velocity of flow 3 ft. per sec.*

At the pump, pressure head $=\frac{500 \times 144}{62 \cdot 3}=1155$ ft.

$$\text{Loss of head} = \frac{4flv^2}{2gd} = \frac{4 \times 0 \cdot 01 \times 1000 \times 9 \times 12}{64 \cdot 4 \times 4}$$

$$= 16 \cdot 77 \text{ ft.}$$

$$\text{Effective head} = 1155 - 16 \cdot 77 = 1138 \cdot 2 \text{ ft.}$$

$$\text{Pressure} = \frac{1138 \cdot 2 \times 62 \cdot 3}{144}$$

$$= \mathbf{492 \cdot 3 \text{ lb. per sq. in.}}$$

Example 5. *During an experiment* 70 *gallons of water flowed through a horizontal pipe* $\frac{3}{4}$ *in. dia. in* 12·45 *minutes, and the measured difference of pressure head in a length of* 10·4 *ft. was* 1·95 *ft. Determine the coefficient of resistance to the flow.* (U.L.C.I.)

$$v = \frac{Q}{At} = \frac{70 \times 144}{6 \cdot 23 \times \pi \times (0 \cdot 375)^2 \times 747}$$

$$= 4 \cdot 905 \text{ ft./sec.}$$

$$h = \frac{4flv^2}{2gd} = 1 \cdot 95.$$

$$1 \cdot 95 = \frac{4f \times 10 \cdot 4 \times (4 \cdot 905)^2 \times 12}{64 \cdot 4 \times 0 \cdot 75}$$

$$\mathbf{f = 0 \cdot 007843.}$$

EXPT. 1. **Head lost by friction.**

OBJECTS. *To determine the values of f in the friction formula for various pipes.*

APPARATUS. The apparatus is fastened to a wall and consists of three main pipes about 1 in. to $1\frac{1}{2}$ in. dia., one of rough cast or wrought iron, another of bright solid drawn steel and the third of copper (Fig. 178).

The three pipes are each fitted with piezometer tubes at equal lengths approaching 10 feet and independent stop valves. The pipes are manifolded at the valve end to a single supply pipe, and outlet is through bends at the end of each pipe.

METHOD OF PROCEDURE. The three stop valves are regulated to give equal readings at each of the inlet piezometer tubes, that is, equal values of h_1. Then the readings of h_2 for each pipe are taken at the outlet piezometer tubes. The loss of head $h_1 - h_2$ is calculated for each pipe and represents the head lost in friction during the passage between the piezometer tubes. Since $h = \frac{4flv^2}{2gd}$ is the

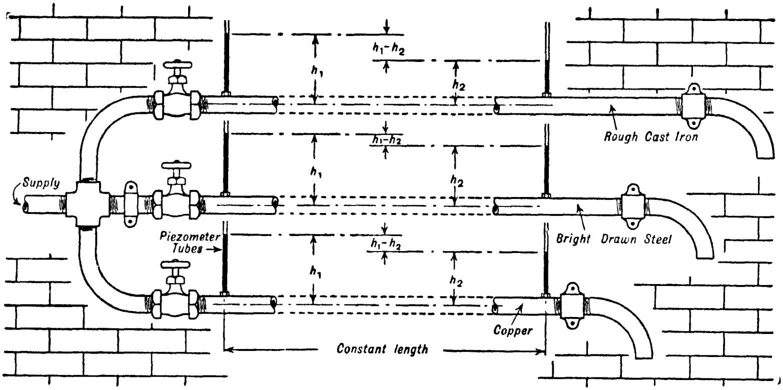


FIG. 178.

formula for loss of head due to friction and the diameter (d), length (l) and velocity (v) are constant, then the loss of head (h) is proportional to f, the friction constant for the pipe.

The absolute value of f may be calculated if the flow over a given period is measured, when the velocity of flow will be $\frac{4Q}{\pi d^2}$, where Q is the quantity in cubic feet per sec. and d is the diameter of the pipe. Using the value of v so obtained and the measured loss of head $h_1 - h_2$ in $h = \frac{4flv^2}{2gd}$, $f = \frac{2gd(h_1 - h_2)}{4lv^2}$. If the value of f is obtained for one pipe, that for the others may be calculated directly from the loss of head, to which f is proportional.

OBSERVATIONS AND DERIVED RESULTS.

l = length between piezometer tubes = ft.
d = diameter of pipe in ft. = ft.
a = area of pipe cross-section in sq. ft. = sq. ft.

Pipe	h_1	h_2	$h = h_1 - h_2$	Quan. in t sec.	t (sec.)	Q (cu. ft. per sec.)	$v = Q/a$	$f = \frac{2gd(h_1 - h_2)}{4lv^2}$
Cast iron								
Steel -								
Copper -								

Flow through open channels. The treatment of the flow of water in channels is similar to that for pipes. The Chezy formula is applicable with appropriate constants, and the quantity of flow may be deduced from the product of the velocity obtained from this formula and the cross-sectional area of the fluid stream.

Example. *Calculate the velocity and quantity of flow for an open channel, trapezoidal in shape, 8 ft. wide at the bottom with sides inclined at 45°, if the water flows 6 ft. in depth and the slope of the channel is 1 in 300. Take C as 75.*

$$m = \frac{\text{area}}{\text{wetted perimeter}} = \frac{28 \times 3}{8 + (6 \times 2) \times 1{\cdot}414} = \frac{84}{24{\cdot}97}$$

$$= 3{\cdot}364 \text{ ft.}$$

$$v = C\sqrt{mi} \qquad = 75\sqrt{3{\cdot}364 \times \tfrac{1}{300}}$$

$$= 7{\cdot}94 \text{ ft. per sec.}$$

$$Q = 7{\cdot}94 \times 84 \times 6{\cdot}23 \times 3600 \times 24$$

$$= 359{,}100{,}000 \text{ gallons per day.}$$

Effect of sudden enlargement or reduction in pipe diameter. When pipe lines are installed, it is customary to avoid, so far as is possible, any sudden alteration in diameter of pipe or any rapid change of direction of flow. The main reason for this precaution is that any such alteration produces a change of velocity with a consequent loss of energy.

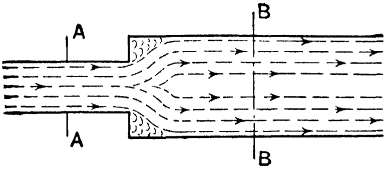


FIG. 179.

Suppose water flows as in Fig. 179 from a small to a larger section of pipe, and consider the sections AA and BB. Let the velocity at AA be V ft. per sec. and that at BB be v ft. per sec.; then if the change of velocity is instantaneous, the loss of head per lb. will be

$$\frac{(\text{change of velocity})^2}{2g}$$

or

$$h = \frac{(V - v)^2}{2g}.$$

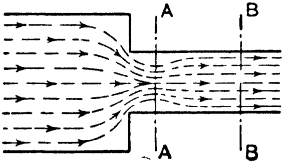


FIG. 180.

Suppose water flows as in Fig. 180 from a large to a smaller section of pipe. There will be a change of velocity and an enforced constriction due to reduction of area, which will be followed by a loss of head when the stream recovers full bore in the smaller pipe.

Let V be the velocity through the constriction AA, and v be the velocity at full bore BB. Then the loss of head

$$\mathbf{h} = \frac{(\mathbf{V} - \mathbf{v})^2}{2\mathbf{g}}.$$

The actual change of velocity can only be determined by experiment, and it is general to take the loss of head to be

$$\mathbf{h} = \frac{0{\cdot}5\mathbf{v}^2}{2\mathbf{g}},$$

where v is the velocity at full bore.

Example 1. *Calculate the loss of head due to the sudden enlargement of a pipe passing* 7000 *gallons per hour from* 4 *inches diameter to* 8 *inches diameter.*

$$V = \frac{7000 \times 144}{6{\cdot}23 \times 3600 \times \pi \times 2^2} = 3{\cdot}577 \text{ ft. per sec.}$$

$$v = V \times \frac{4^2}{8^2} = \frac{3{\cdot}577 \times 16}{64} = 0{\cdot}894 \text{ ft. per sec.}$$

$$\text{Loss of head} = \frac{(3{\cdot}577 - 0{\cdot}894)^2}{2g} = \frac{(2{\cdot}683)^2}{2g}$$

$$= \mathbf{0{\cdot}112 \text{ ft. or } 1{\cdot}34 \text{ in.}}$$

Example 2. *A pipe of* 14 *inches diameter is suddenly reduced to* 4 *inches diameter. Calculate the loss of head, assuming* $h = \frac{0{\cdot}5v^2}{2g}$, *if the pipe discharges* 2 *cu. ft. per sec.*

$$v = \frac{2 \times 144}{\pi \times 2^2} = 22{\cdot}92 \text{ ft. per sec.}$$

$$\text{Loss of head} = \frac{0{\cdot}5 \times (22{\cdot}92)^2}{2g} = \mathbf{4{\cdot}08 \text{ ft.}}$$

Measurement of flow of water. The great majority of the researches into the flow of water have taken place during the last hundred or so years. During the nineteenth century, the French engineers De Prony, Poncelet and Lesbros, with the German Eytelwein, established results which were used for half a century; they were followed by those of Bazin, Francis, Rankin, Thomson, Osborne Reynolds and Venturi, upon whose results modern practice is based. The orifice (see Fig. 184) was the oldest method for measuring water, and the sharp-edged weir was used for the flow in open channels. The Vee notch (see Figs. 181 and 186) was suggested by Thomson in 1861, and since 1905 has been standardised for commercial measurement.

Methods of measurement of fluid flow. Where moderately small quantities are involved, it is general to employ **notches** of either triangular or rectangular shape. These notches are set with the Vee or rectangular edges at right angles to the line of flow of the water, and the quantity passing through the notch can be determined. Where large quantities have to be measured, such as the flow of streams or rivers, the flow is determined by passing the water over a **calibrated weir.** The incidence of a notch in the line of flow introduces a resistance and a **coefficient of discharge,** which is the ratio between the actual flow through the notch and the normal flow of the stream to be determined. In the case of a triangular notch of 90° angle this coefficient is constant; whereas for a rectangular notch the coefficient has to be determined experimentally for the particular notch.

Rate of flow through a triangular notch. Consider an elementary strip of depth δy situated in the stream at a depth of y ft. (Fig. 181).

$$\text{Velocity of flow,} \quad v = \sqrt{2gy}.$$

$$\text{Area of strip} = \delta y \times x = A.$$

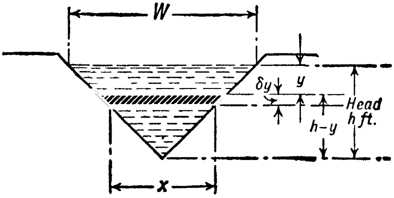


Fig. 181.

Since the notch is triangular,

$$\frac{x}{W}=\frac{h-y}{h},$$

and
$$x=\frac{(h-y)\,W}{h},$$

where h is the full head.

Quantity passing the elementary strip is Q_1.

Then
$$Q_1=vA=\sqrt{2gy}\times x\times\delta y \text{ cu. ft. per sec.}$$
$$=\sqrt{2gy}\times\frac{(h-y)\,W}{h}\,\delta y.$$

Extracting constants, $Q_1=\dfrac{W\sqrt{2g}}{h}\{h\sqrt{y}-y^{3/2}\}\,\delta y.$

Total quantity,
$$Q=\int_0^h\frac{W\sqrt{2g}}{h}\{h\sqrt{y}-y^{3/2}\}\,\delta y$$
$$=\frac{W\sqrt{2g}}{h}\left[\frac{2hy^{3/2}}{3}-\frac{2y^{5/2}}{5}\right]_0^h$$
$$=\frac{W\sqrt{2g}}{h}\left\{\frac{2h^{5/2}}{3}-\frac{2h^{5/2}}{5}\right\}=\frac{W\sqrt{2g}}{h}\left\{\frac{4h^{5/2}}{15}\right\}$$
$$=\tfrac{4}{15}\mathrm{W}\sqrt{2\mathrm{g}}\,.\,\mathrm{h}^{3/2}.$$

Special case : angle of notch is 90°.

If the angle is 90°, $W=2h,$

and $\mathrm{Q}=\tfrac{8}{15}\sqrt{2\mathrm{g}}\,.\,\mathrm{h}^{5/2}$ or $4{\cdot}281\mathrm{h}^{5/2}$.

N.B.—This is a theoretical discharge, and in actual practice a quantity less than this would flow through the notch.

The coefficient of discharge, K, is defined as the ratio

$$\frac{\text{actual discharge}}{\text{theoretical discharge}}$$

and is a quantity which must be determined by experiment.

Thus $\mathrm{Q}=\mathrm{K}\tfrac{8}{15}\sqrt{2\mathrm{g}}\,.\,\mathrm{h}^{5/2}$ for a right angular Vee notch;

or $\mathrm{Q}=\mathrm{K}\tfrac{4}{15}\mathrm{W}\sqrt{2\mathrm{g}}\,.\,\mathrm{h}^{3/2}$ for an ordinary Vee or triangular notch.

Example 1. *A right-angled V notch is used to measure the flow of water into a cooling tank. If the head is maintained at* 17 *inches and* $K=0{\cdot}7$, *calculate the quantity of cooling water in gallons per hour.*

$$Q=K\times\tfrac{8}{15}\sqrt{2g}\,.\,h^{5/2}\text{ cu. ft. per sec.}$$

$$=0{\cdot}7\times\tfrac{8}{15}\sqrt{2g}\,.\,(\tfrac{17}{12})^{5/2}=7{\cdot}155\text{ cu. ft. per sec.}$$

or $$7{\cdot}155\times6{\cdot}23\times3600\text{ gallons per hour.}$$

$$=\mathbf{160{,}400\ gallons\ per\ hour.}$$

Example 2. *Calculate the head required on a right-angled V notch to pass* 200 *gallons of water per minute if* $K=0{\cdot}7$.

$$200\text{ gallons per min.}=\frac{200}{60\times6{\cdot}23}\text{ cu. ft. per sec.}$$

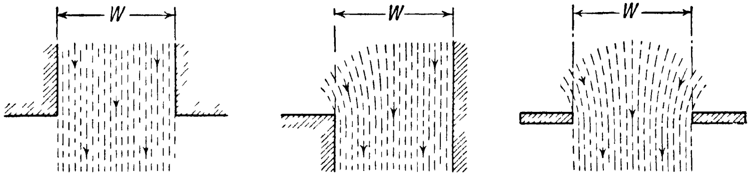
$$=0{\cdot}5351.$$

$$Q=0{\cdot}5351=0{\cdot}7\times\tfrac{8}{15}\sqrt{2g}\,.\,h^{5/2},$$

$$h=\left\{\frac{0{\cdot}5351\times15}{0{\cdot}7\times8\times\sqrt{2g}}\right\}^{2/5}=\mathbf{0{\cdot}502\ ft.\ or\ 6{\cdot}02\ in.}$$

Flow of water over a rectangular notch or weir. The quantity of water flowing over a rectangular notch is determined by a partially empirical formula known as the **Francis formula.** The application of this formula is affected by the number of end contractions determined by the number of times the stream is contracted by the walls of the notch, Fig. 182.

If water flows along a channel and enters the notch without contraction at the ends of the stream, the full width of the notch is discharged under a head *h* (Fig. 183). When the stream is contracted by the sides of the notch as is shown in Fig. 182, the effective width to discharge is reduced by the end contractions. Then if *N*



No end contractions
N=0

One end contraction
N=1

Two end contractions
N=2

FIG. 182.

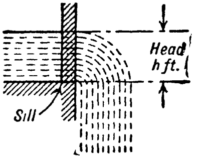


FIG. 183.

is the number of end contractions and K is an experimentally determined coefficient of discharge, the Francis formula for the quantity of flow becomes

$$Q = K . h^{3/2} \left\{ W - \frac{N}{10} h \right\}.$$

The generally accepted value of K is 3·33, and the formula with this value of K is

$$Q = 3{\cdot}33 h^{3/2} \left\{ W - \frac{N}{10} h \right\},$$

where *Q is the quantity of flow in cu. ft. per sec.*;
h is the head over the sill, or bottom, of the notch in feet;
W is the width of the notch in feet;
N is the number of end contractions.

This formula can be applied to a **weir**, which is the name given to a dam placed across a stream of water, over the sill of which the water is free to flow with, or without, end contraction of the stream.

Example 1. *Calculate the flow of water in gallons per hour over a rectangular notch having two end contractions and a width of* 30 *inches. The mean head is* 7 *inches.*

$$\begin{aligned} Q &= 3{\cdot}33 h^{3/2} \left\{ W - \frac{N}{10} h \right\} \\ &= 3{\cdot}33 \left(\frac{7}{12} \right)^{3/2} \left\{ 2{\cdot}5 - \frac{2 \times 7}{10 \times 12} \right\} \\ &= 3{\cdot}33 \times 0{\cdot}5833^{3/2} \{ 2{\cdot}5 - 0{\cdot}1167 \} \\ &= 3{\cdot}535 \text{ cu. ft. per sec.} \\ &= 3{\cdot}535 \times 3600 \times 6{\cdot}23 = \textbf{79,300 gallons per hour.} \end{aligned}$$

Example 2. *A weir under a head of* 2 *ft. discharges with two end contractions at a rate of* 100,000,000 *gallons per day. Find the length of the weir.*

$$Q = \frac{100{,}000{,}000}{24 \times 3600 \times 6{\cdot}23} = 185{\cdot}8 \text{ cu. ft. per sec.}$$

$$185{\cdot}8 = 3{\cdot}33 \times 2^{3/2} \left\{ W - \frac{2 \times 2}{10} \right\}$$

$$\left(W - \frac{4}{10} \right) = \frac{185{\cdot}8}{3{\cdot}33 \sqrt{8}} = 19{\cdot}73.$$

$$W = 19{\cdot}73 + 0{\cdot}4 = \mathbf{20{\cdot}13 \text{ ft.}}$$

Example 3. *Calculate the head over a rectangular notch sill in order that the notch may discharge* 100,000 *gallons of water per hour. The width of the notch is* 33 *inches and there are no end contractions.*

$$Q=\frac{100,000}{3600\times 6\cdot 23}=4\cdot 459 \text{ cu. ft. per sec.}$$

Since $N=0$, the formula is $Q=3\cdot 33Wh^{3/2}$.

Hence $$4\cdot 459=3\cdot 33\times \tfrac{33}{12}h^{3/2}.$$

$$h=\left(\frac{4\cdot 459}{3\cdot 33\times 2\cdot 75}\right)^{2/3}=0\cdot 62 \text{ ft. or } \mathbf{7\cdot 44\ in.}$$

Flow through orifices. A full treatment of the flow of water through orifices is given in Volume I of *Engineering Science*, pages 323–328, and this may be summarised as follows :

(1) **Theoretical velocity of flow** through an orifice under a head of h ft. $=\sqrt{2gh}$ **ft. per sec.**

(2) **Theoretical quantity** is $\mathbf{A}\times\sqrt{2gh}$ **cu. ft. per sec.**, where A is the area of the orifice.

(3) **Actual velocity of flow** is $\sqrt{2gh}\times \mathbf{C}_v$ **ft. per sec.**, where C_v *is the coefficient of velocity.*

(4) **Actual quantity** is $\mathbf{A}\times\sqrt{2gh}\times\mathbf{C}$ **cu. ft. per sec.**, where C *is the coefficient of discharge.*

(5) The stream suffers a contraction of area at the orifice and the ratio $\dfrac{\textit{area at vena contracta}}{\textit{area of orifice}}=$ **coefficient of contraction**, $\mathbf{C}_c$.

(6) **Coefficient of discharge** $\mathbf{C}=$ *coefficient of contraction* $\times$ *coefficient of velocity* ; that is,

$$\mathbf{C}=\mathbf{C}_c\times\mathbf{C}_v.$$

Revision examples.

Example 1. *Calculate the discharge from a circular orifice* $1\frac{1}{2}$ *in. in diameter under a head of* 12 *ft. if the coefficient of discharge* $C=0\cdot 62$.

$$Q=0\cdot 62\times A\times V \text{ cu. ft. per sec.}$$
$$=0\cdot 62\times\pi\times(\tfrac{1}{16})^2\times\sqrt{2g\times 12}$$
$$=\mathbf{0\cdot 2115\ cu.\ ft.\ per\ sec.}$$

Example 2. *Calculate the head required to produce a flow of* 770,000 *gallons per hour through an orifice* 2 *in. in diameter if* $C = 0{\cdot}62$.

$$Q = \frac{770{,}000}{6{\cdot}23 \times 3600} \text{ cu. ft. per sec.}$$

$$= \frac{770{,}000}{6{\cdot}23 \times 3600} = 0{\cdot}62\sqrt{2gh}.$$

$$h = \left\{\frac{770{,}000}{6{\cdot}23 \times 3600 \times 0{\cdot}62 \times \sqrt{2g}}\right\}^2$$

$$= \mathbf{47{\cdot}6\ ft.}$$

Example 3. *An orifice* 2 *in. in diameter discharges* 78·4 *gallons of water per minute under a head of* 4 *ft. Calculate the coefficient of discharge.*

$$\text{Area of orifice} = \frac{\pi}{144} \text{ sq. ft.} = A.$$

$$V = \sqrt{2gh} = 8{\cdot}025 \times 2 = 16{\cdot}05 \text{ ft. per sec.}$$

$$78{\cdot}4 = 6{\cdot}23 \times 60 \times V \times A \times C$$

$$= 373{\cdot}8 \times 16{\cdot}05 \times \frac{\pi}{144} \times C.$$

$$\mathbf{C = 0{\cdot}598.}$$

EXPT. 2. **Flow through orifices.**

OBJECTS. (*a*) *To determine the flow of water in cubic feet per second through various sharp-edged orifices under constant head.*

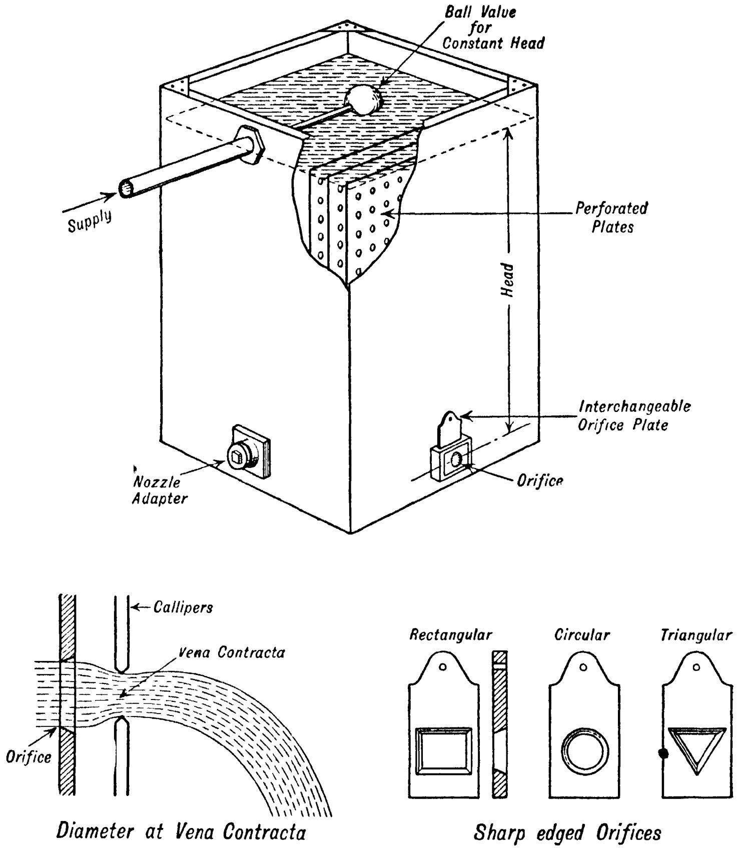
(*b*) *To determine the coefficients of discharge, contraction and velocity for each of these orifices where practicable.*

APPARATUS. A convenient apparatus for this type of experiment is shown in Fig. 184. It consists of a deep galvanised tank fitted with a supply service to the mains and a ball cock control to maintain a constant head. A series of vertical perforated baffle plates are fitted to the inside of the tank and these serve to steady the water flow to the orifice. Near the base of the tank is a fixture into which interchangeable plates can be slid, each of which presents a different shape of sharp-edged orifice. The discharge water is collected in a suitable tank calibrated in cubic feet or in gallons. A plain plate is also provided to close the orifice when required.

METHOD OF PROCEDURE. For each orifice the method of procedure is somewhat similar, except that it is only practicable to obtain the coefficient of contraction for the circular orifice. An orifice plate is fitted and water allowed to flow for a certain time, *t* sec. This water is collected in the measuring tank during discharge from the orifice and the quantity discharged measured. The quantity Q is then determined from

$$Q = \frac{\text{total quantity in cu. ft.}}{\text{time in sec.}}$$

and is in cubic ft. per second.



Diameter at Vena Contracta

Sharp edged Orifices

Fig. 184.

The **coefficient of discharge** is the ratio between the actual and theoretical discharge and this is given by

$$\mathbf{C} = \frac{\textbf{actual discharge}}{\textbf{area of orifice} \times \sqrt{2gh}},$$

since the theoretical discharge is the theoretical velocity $\sqrt{2gh} \times$ area of the orifice.

The **coefficient of contraction** for the circular orifice can be found by carefully callipering the jet at the vena contracta and calculating the area of the jet at this point.

Then $$\mathbf{C}_c = \frac{\textbf{area at vena contracta}}{\textbf{area of orifice}}.$$

Since the actual discharge depends upon the actual velocity and the area at the vena contracta, $C = C_c \times C_v$, where C_v is the **coefficient of velocity**; hence

$$\mathbf{C}_v = \frac{\mathbf{C}}{\mathbf{C}_c}.$$

OBSERVATIONS AND DERIVED RESULTS.

Shape of sharp-edged orifice	Head h ft.	Area A sq. ft.	Theor. vel. $V = \sqrt{2gh}$ ft. per sec.	Time t sec.	Act'l quan. Q_1 cu. ft.	Discharge $Q = \frac{Q_1}{t}$ cu. ft./sec.	Area at venacontracta	Coeff. of discharge $C = \frac{Q_1}{AV}$	Coeff. of contraction $C_c = \frac{AV_c}{A}$	Coeff. of vel. $\frac{C}{C_c}$
Rectangular										
Circular										
Triangular										

The values obtained for the coefficients are likely to vary with the shape and dimensions of the orifice, and each value obtained is relative to the particular conditions under which the discharge takes place.

EXPT. 3. **Flow through nozzles.**

OBJECTS. *To determine the coefficient of discharge through various types of nozzles under constant head.*

METHOD OF PROCEDURE. The tank shown in Fig. 184 is fitted with a nozzle adapter; and various shapes of nozzles (Fig. 185) can be fitted to the tank. The flow is measured into a calibrated tank for a certain period and the quantity of discharge for each nozzle determined in cubic ft. per sec. This is compared with the theoretical discharge, which is *minimum area of nozzle cross-section* $\times \sqrt{2gh}$.

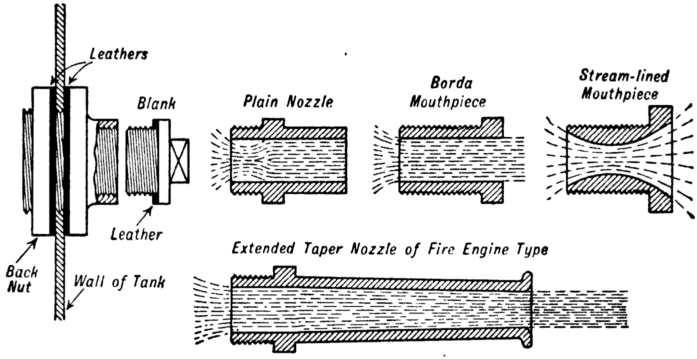


FIG. 185.

The coefficient of discharge is given by $\dfrac{\textbf{actual discharge}}{\textbf{area} \times \sqrt{\textbf{2gh}}}$ for each nozzle.

OBSERVATIONS AND DERIVED RESULTS.

Type of nozzle or mouthpiece	Head h ft.	Diam. of nozzle	Area of nozzle	Quantity cu. ft./sec.	K $\dfrac{\text{Actual dis.}}{\text{Theor. dis.}}$
Plain nozzle - -					
Borda mouthpiece -					
Stream-lined - -					
Fire engine - -					

EXPT. 4. **Flow over rectangular and Vee notches.**

OBJECT. *To determine the constants in the formulae for the flow of water over a rectangular notch and a right-angled Vee notch.*

APPARATUS. The apparatus consists of a rectangular trough fitted, at its end, with interchangeable notches, rectangular and right-angled Vee in shape. These notches are attached to the trough by countersunk screws, and if desired the rectangular notch can be made to give one or two end contractions. The trough is fitted with several perforated baffle plates to maintain a steady head and is supplied from a supply tank at such a rate that the water receives no appreciable velocity of approach. The discharge nappé is led into a calibrated tank and thus the flow in a given period of time is measured.

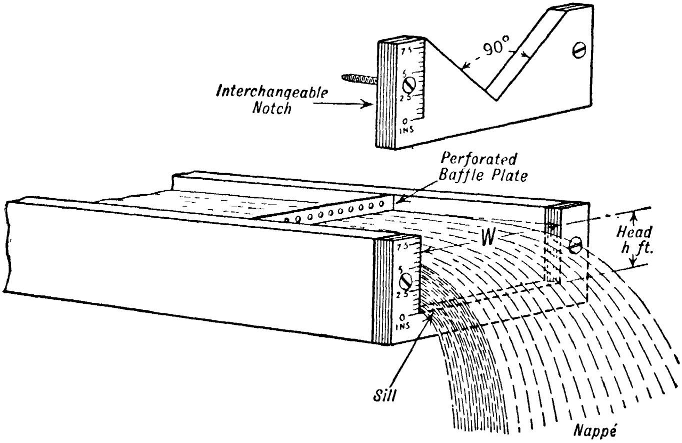


FIG. 186.

METHOD OF PROCEDURE. Adjust the supply so that water is flowing over the sill of the rectangular notch without loss of head, and note the head over the sill in inches. Measure the discharge into the calibrated tank over a given period and from this deduce the quantity Q in cu. ft. per sec. The formula for the flow over such a notch without end contractions is $Q = Kh^{3/2} \times W$, where W is the width of the notch in ft. and h is the head in ft. Equate the observed value of Q to $Kh^{3/2} \times W$, and thence determine the value of K. Repeat for other values of head. The head is usually measured by means of a hook gauge.

Replace the rectangular notch by the right-angled Vee notch and repeat the experiment. Determine the value of K in the formula $Q = K\frac{8}{15}\sqrt{2g}h^{5/2}$ for this notch.

OBSERVATIONS AND DERIVED RESULTS.

Rectangular notch : $Q = Kh^{3/2} \times W$ cu. ft. per sec.

Expt.	Head (in.)	Head (ft.)	W (ft.)	Time (sec.)	Quantity (cu. ft.)	Q (cu. ft. per sec.)	K
1	2·5	0·209					
2	5·0	0·417					
3	7·5	0·626					

Right-angled Vee notch : $Q = K\frac{8}{15}\sqrt{2g}h^{5/2}$ cu. ft. per sec.

Expt.	Head (in.)	Head (ft.)	Time (sec.)	Quantity (cu. ft.)	Q (cu. ft. per sec.)	K
1	2·5	0·209				
2	5·0	0·417				
3	7·5	0·626				

Later researches upon measurement of flow. The flow in an open channel when measured by a weir, notch, or orifice involves the piling up of the stream behind the measuring device, with a consequent loss of head. This loss cannot be avoided, as the principle involved is one in which the water is assumed to fall freely under the action of a given head. The fundamental laws, $V = \sqrt{2gh}$, and the Bernoulli theorem, have since been used in two methods of measurement known as the **Standing Wave** and **Venturi Flume** method respectively. These give very little loss of head at the device, and the piling up is negligible. In irrigation and sewage work the available head is usually small, and the development of the flume is the natural result of the difficulties experienced in the measurement of flow in these circumstances.

In essentials the standing wave and Venturi flume is little more than a stream-lined constriction in the channel so arranged that the water is gently in contact during the passage. Small constrictions are used to cause an increase of velocity in the Venturi flume, whereas larger constrictions are employed for the same purpose in the standing wave flume. In each case the constriction causes the lowering of the water surface to be followed by a gradual restoration downstream in the case of the Venturi flume, and a sudden restoration in the form of a standing wave in the case of the standing wave flume. The loss of head in the constriction is a measure of the flow of water. This flume offers no resistance to floating bodies and is thus particularly suitable for river and sewage measurement. The standing wave flume is more popular and only one depth need be measured, with one float mechanism for this purpose. The rate of flow is proportional to $H^{3/2}$, where H is the loss of head.

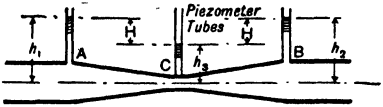


FIG. 187.

The Venturi meter. This piece of apparatus was invented by an Italian, Venturi, and is in very common use for the measurement of fluid flow. Its construction, and the principle of operation, is treated in Vol. I of *Engineering Science*, page 322, and may be revised as follows:

Let Q be the discharge in cu. ft. per sec.,
a_1 the area of cross-section in sq. ft. at A,
a_3 ,, ,, ,, ,, ,, C;

then velocity at A $=\dfrac{Q}{a_1}$ ft. per sec.,

and ,, ,, B $=\dfrac{Q}{a_3}$,, ,,

By Bernoulli's theorem, since $v=\dfrac{Q}{a}$,

$$\frac{Q^2}{2ga_1^2}+h_1=\frac{Q^2}{2ga_3^2}+h_3,$$

or

$$Q^2\left\{\frac{1}{a_3^2}-\frac{1}{a_1^2}\right\}=2gH,$$

where

$$H=h_1-h_3.$$

Therefore

$$Q=\sqrt{\frac{2gH}{\dfrac{1}{a_3^2}-\dfrac{1}{a_1^2}}}=\frac{a_1a_3}{\sqrt{a_1^2-a_3^2}}\sqrt{2gH}.$$

For a particular meter a_1, a_3 and $\sqrt{2g}$ may be regarded as constants, and the expression becomes

$$Q=\text{constant}\sqrt{H},$$

where H is the difference in heads.

Example 1. *The supply main to a hydraulic plant is* 16 *inches in diameter and is fitted with a Venturi meter having a throat diameter of*

5 inches. If the pressure head at the pipe end is 24 ft. and at the throat 18 ft., calculate the quantity of flow in gallons per hour.

$$a_1 = \frac{\pi \times 8^2}{144} = 1{\cdot}3956 \text{ sq.ft.}$$

$$a_3 = \frac{\pi \times 5^2}{4 \times 144} = 0{\cdot}1363 \quad ,,$$

Difference in heads $= H = 6$ ft.

$$Q = \frac{1{\cdot}3956 \times 0{\cdot}1363}{\sqrt{(1{\cdot}3956)^2 - (0{\cdot}1363)^2}} \times \sqrt{2g \times 6}$$

$$= \frac{0{\cdot}1903}{1{\cdot}389} \times 8{\cdot}025 \times 2{\cdot}4495$$

$$= 2{\cdot}694 \text{ cu. ft. per sec.}$$

$$= 2{\cdot}694 \times 3600 \times 6{\cdot}23 = \textbf{60,400 gallons per hour.}$$

Example 2. *A Venturi meter has a pipe diameter of 5 ft. and a throat diameter of 1·5 ft. The discharge is 700,000 gallons per hour. Calculate the difference in heads between pipe line and throat.*

$$a_1 = \frac{\pi \times 25}{4} = 19{\cdot}63 \text{ sq. ft.}$$

$$a_3 = \frac{\pi \times 2{\cdot}25}{4} = 1{\cdot}767 \text{ sq. ft.}$$

$$Q = \frac{700{,}000}{3600 \times 6{\cdot}23} = 31{\cdot}21 \text{ cu. ft. per sec.}$$

$$= \frac{a_1 a_3}{\sqrt{a_1^2 - a_3^2}} \sqrt{2gH}.$$

$$31{\cdot}21 = \frac{19{\cdot}62 \times 1{\cdot}766}{\sqrt{(19{\cdot}62)^2 - (1{\cdot}766)^2}} \times 8{\cdot}025 \sqrt{H}$$

$$= \frac{34{\cdot}64}{19{\cdot}58} \times 8{\cdot}025 \sqrt{H}.$$

$$H = \left\{\frac{31{\cdot}21 \times 19{\cdot}58}{34{\cdot}64 \times 8{\cdot}025}\right\}^2 = \mathbf{4{\cdot}84 \text{ ft.}}$$

EXPT. 5. **The Venturi meter.**

OBJECTS. *To determine the flow of water through a Venturi meter and the meter constant.*

APPARATUS. A Venturi meter, consisting of a constricted tube the major diameter of which is equal to that of the pipe line, is fitted into a supply pipe and water allowed to flow through the

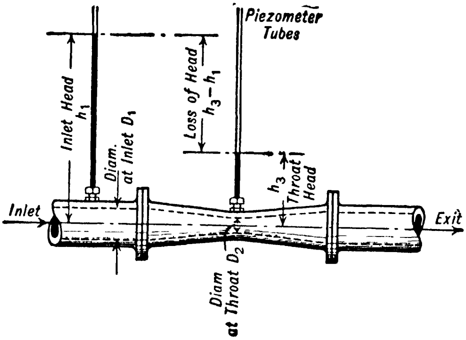


FIG. 188.

constriction or throat into a calibrated tank. Piezometer tubes are inserted at the inlet and at the throat, and these serve to measure the pressure heads at these points.

METHOD OF PROCEDURE. Allow water to flow through the Venturi meter into the calibrated tank for a given period and calculate the quantity of flow Q in cubic feet per sec. Measure the piezometer heads at inlet and throat, h_1 and h_3 respectively, and find the loss of head, h_1-h_3, at the throat. Calculate the areas of cross-section a_1 and a_3, that is, $\frac{\pi D_1^2}{4}$ and $\frac{\pi D_2^2}{4}$, at these points. Apply these observations to the formula $Q=\frac{a_1a_3}{\sqrt{a_1^2-a_3^2}}\sqrt{2gH}$, where H is the loss of head h_1-h_3, and compare this value of Q with the observed value measured in the calibrated tank.

The meter constant will be $\frac{a_1a_3}{\sqrt{a_1^2-a_3^2}}\sqrt{2g}$, and Q will be this constant multiplied by $\sqrt{H}$.

OBSERVATIONS AND DERIVED RESULTS.

Diameter at inlet $=D_1$ ft.

Area at inlet $=\frac{\pi D_1^2}{4}=$ sq. ft. $=a_1$.

Diameter at throat $=D_2$ ft.

Area at throat $=\frac{\pi D_2^2}{4}=$ sq. ft. $=a_3$.

Total quantity in t sec. = cu. ft.
Quantity in 1 sec. $= Q =$ cu. ft.
Difference in heads $= h_3 - h_1 =$ ft. $= H$.
Meter constant $= \dfrac{a_1 a_3}{\sqrt{a_1^2 - a_3^2}} \sqrt{2g} =$
Q = constant × $\sqrt{H}$ = cu. ft. per sec.

EXERCISES ON CHAPTER XII

1. Explain the terms pressure head, atmospheric head and velocity head. A storage tank is situated 320 ft. above the lowest point of a town. State the pressure head possessed by the water in this tank and find the velocity of supply to the town if 60% of the theoretical velocity is lost in pipe lines and bends.

2. Water is stored at a height at which the atmospheric pressure is 13·8 lb. per sq. in.; calculate the atmospheric head.

3. At what height must a storage tank be situated in order to supply water to a low level with a pipe velocity of 18 ft. per second, neglecting all losses due to friction and other causes.

4. State Bernoulli's theorem and give the units in which each of its terms must be expressed. A pipe line 500 ft. in length has an incline of 1 ft. in every 20 feet of length. If water enters the top end of the pipe at 1 ft. per sec., calculate the theoretical velocity (*a*) at the bottom of the pipe, (*b*) at the mid-point.

5. A pipe line 100 ft. in length with a slope of 1 ft. per 10 ft. of length discharges into a vessel at its lower end in which the pressure is maintained at 3 lb. per sq. in. If the supply to the pipe is a tank open to the atmosphere, calculate the theoretical velocity at which the water enters the low pressure vessel. Atmospheric pressure is 14·7 lb. per sq. in.

6. A pump draws water from an open sump situated 15 ft. below pump level. In the process of pumping, the pressure in the pump cylinder falls to 1·5 lb. per sq. in.; calculate the theoretical velocity at which the water enters the pump. Atmospheric pressure = 14·7 lb. per sq. in., If this pump has to deliver to a height of 120 ft. and its efficiency is 80%, calculate the pressure in lb. per sq. in. exerted by the pump plunger.

7. A boiler feed pump has to inject water at a velocity of 5 ft. per sec. against a boiler pressure of 100 lb. per sq. in. Find the pressure required on the pump plunger if the pump is situated 2 ft. below the injection pipe to the boiler and if the pump efficiency is 80%.

8. Water flows from a reservoir which is open to the air to a jet which is 15 ft. below the surface of the water in the reservoir. The jet issues into a condenser in which the pressure is 4 lb. per sq. in. absolute. Find the velocity of the water issuing from the jet. (U.E.I.)

9. At a section A in a pipe running full of water the diameter of the pipe is 2 ft. and the velocity of water is 4 ft. per sec. At another section B, which is 30 ft. below the level of A, the diameter of the pipe is 1 ft. 6 in. Determine the difference of water pressures at A and B. (U.L.C.I.)

10. A nozzle at the end of a 3-inch internal diameter water pipe has an internal diameter at its open end of $1\frac{1}{8}$ inches. When water flowed through the pipe a pressure gauge at the entrance to the nozzle registered a pressure of 75 lb. per sq. in. gauge. Calculate the velocity of the water leaving the nozzle and the number of gallons of water issuing from the nozzle per minute. (U.L.C.I.)

11. What do you understand by Bernoulli's theorem? A pipe AB is 250 ft. long and has an inclination with the horizontal of $\sin^{-1}\frac{1}{5}$. When the pipe is running full the pressure head is 35 ft. at the upper end A, where the pipe is 2 ft. in diameter, and the velocity of flow 3·5 ft. per sec. Determine the pressure head at the end B, where the pipe has been reduced to 1·5 ft. diameter. (U.L.C.I.)

12. Calculate the discharge in gallons per day from a pipe 10,000 ft. in length, 4 ft. in diameter, running full on a slope of 1 in 1000. $C = 100$.

13. Find the loss of head due to friction in a pipe 2000 ft. in length, 1 ft. in diameter, if the velocity of flow is 4·4 ft. per sec. $f = 0{\cdot}0071$.

14. A pipe one mile long, 6 inches in diameter, conveys water flowing at 3 ft. per sec. to a reservoir. Assuming $f = 0{\cdot}01$, find the frictional loss in head along the pipe. What H.P. is lost in forcing the water through the pipe. (U.E.I.)

15. If the value of f in the formula for loss of head due to friction in pipes is 0·01, find the head required to deliver 1000 gallons of water per minute through a horizontal pipe 2 miles long and 1 ft. in diameter. (U.E.I.)

16. State Bernoulli's theorem, explaining each term. A pipe, 15 in. diameter, connects two reservoirs 3 miles apart. What is the velocity of delivery and the discharge in gallons per day when the difference in the levels in the reservoirs is 30 ft.? Assume the coefficient of friction to be 0·0075. (U.E.I.)

17. Find the H.P. required to pump 360 gallons of water per minute to a height of 300 ft. through a 6-inch diameter pipe one mile long. Coefficient of resistance, $f = 0{\cdot}007$. (U.E.I.)

18. When a train scoops up water from a trough the water enters a pipe 4 in. dia. and is delivered at a point 8 ft. above the entrance. If

30% of the head due to the speed of the train is lost, what is the velocity of delivery when the train is running at 30 miles per hour? What volume of water is delivered if the trough is one mile long? (U.E.I.)

19. Define the terms " wetted perimeter ", " hydraulic mean depth ", " hydraulic gradient " and " virtual slope " as applied to the flow of liquids through pipes and channels. Calculate the hydraulic mean depth and the virtual slope for a channel having a bottom width of 6 ft., side slopes of 45° and a length of 2·5 miles, with a difference of level between the ends of 88 ft. Depth of water in the channel, 5 ft. (U.E.I.)

20. Calculate the discharge in gallons per day through a semi-circular channel 4 ft. in diameter, with a slope of 1 in 100, running full. Take a value $C=74$.

21. Calculate the quantity of water discharged in gallons per hour over a right angular V notch under a head of 10 inches. Take K as 0·7.

22. Find the head required for a flow of 25,000 gallons per day over a right angle V notch if K is 0·7.

23. A right angle V notch is used to measure the discharge of water from a machine. If the mean head of the water flowing over the notch during a period be 11·4 in. and the coefficient of discharge be 0·622, find the number of gallons of water flowing per hour. (U.L.C.I.)

24. A right angle V notch discharges 1876 gallons per hour with a mean head of 3 inches. Calculate the coefficient of discharge.

25. Find the quantity of water discharged through a rectangular notch 22 in. in width, with two end contractions, if the head is maintained at 8 in.

26. A small rectangular notch is to be installed to measure the cooling water to a condenser. If the flow is expected to be 20 gallons per minute and a head of $1\frac{1}{2}$ in. can be used, find the width of the notch, assuming no end contractions.

27. A weir with two end contractions is to discharge 1,000,000 gallons of water per day under a head of 6 in. Find the length of the weir.

28. Calculate the discharge in gallons per hour from a sharp-edged circular orifice $1\frac{1}{4}$ in. in diameter under a head of 14 ft. $C=0{\cdot}62$.

29. Find the head required to produce a discharge of 12,000 gallons per hour through an orifice $1\frac{1}{2}$ in. diameter if $C=0{\cdot}62$.

30. Find the coefficient of discharge for a sharp-edged circular orifice 1·7 in. in diameter which discharges 6790 gallons per hour under a head of 15 ft. If the coefficient of contraction is 0·87, find the coefficient of velocity.

31. Calculate the discharge through a Venturi meter if the pipe diameter is 12 ft., throat diameter 2 ft., and difference in pressure heads at pipe and throat is 7 ft.

32. A Venturi meter having a throat diameter of 2½ in. is fitted to a horizontal water main of 6 in. diameter. Find the flow of water through the main in gallons per hour when the difference between the pressure of the water entering the meter and that at the throat is equivalent to a head of 12½ ft. (U.L.C.I.)

33. A pipe is to pass 100,000 gallons of water per hour at a velocity of 2 ft. per sec. Calculate the diameter of the pipe. If this pipe is to be fitted with a Venturi meter in which the throat diameter is ⅓ of the diameter of the main, calculate the difference in heads between main and throat, and the pressure gauge equivalent to this head in lb. per sq. in.

CHAPTER XIII

PRESSURE AT A DEPTH, CENTRE OF PRESSURE, DAMS AND RETAINING WALLS FOR WATER

THE pressure of water upon an immersed surface depends upon the depth of immersion and always acts normal to the surface. Consider the solid shown in Fig. 189; the pressure at the points A, B, C, D and E depend upon the depths h_a, h_b, h_c, h_d and h_e respectively, and in each case acts at right angles to the surface at the point considered.

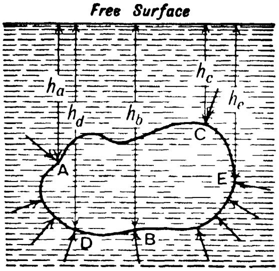


FIG. 189.

The pressure upon a surface is expressed as **pounds per sq. ft.** on the surface, and the resultant pressure upon a given area is known as the **whole pressure.**

The total force, or whole pressure, upon a horizontal surface immersed to a depth of h ft. is equal to the weight of the superimposed fluid (Vol. I, *Engineering Science*, p. 318) that is

$$\textbf{whole pressure} = \textbf{area} \times \textbf{h} \times \textbf{w lb.},$$

where w is the density of the fluid.

Inclined surfaces. The pressure upon an inclined surface is the same as that for a horizontal surface of the same area immersed to the depth of the centre of area of the inclined surface, and acts normal to the inclined surface.

Example. *Calculate the pressure upon a rectangular surface* 4 *ft.* × 2 *ft. inclined at* 45° *to the horizontal and immersed with its top* 4 *ft. edge parallel to the free surface and* 20 *ft. below it.* $w = 64$ *lb. per cu. ft.*

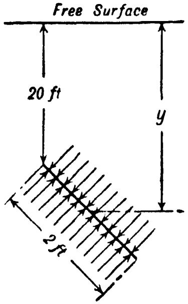


FIG. 190.

Let y (Fig. 190) be depth of centre of area.

$$y = 20 + 1 \sin 45^\circ$$
$$= 20 + 0{\cdot}7071 = 20{\cdot}7071 \text{ ft.}$$
$$\text{Pressure} = \text{area} \times y \times w$$
$$= 2 \times 4 \times 20{\cdot}7071 \times 64$$
$$= 10{,}600 \text{ lb.}$$

Centre of pressure. The point in a surface at which the resultant fluid pressure acts is known as the **Centre of Pressure.**

If an immersed surface be regarded as a surface subjected to a series or system of parallel forces, then the resultant of these forces acts at a certain point which corresponds to the centre of pressure.

Consider the bar AB shown in Fig. 191, acted upon by the forces shown, and pivoted in the free surface at A.

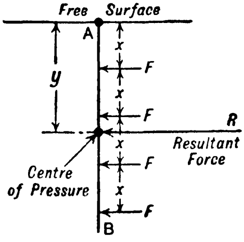


FIG. 191.

Then to find the resultant R and its position y,

$$\text{F} \times x + \text{F} \times 2x + \text{F} \times 3x + \text{F} \times 4x = \text{R} \times y\,;$$

but, $$\text{R} = \text{sum of the forces.}$$

$$\therefore\ y = \frac{\text{sum of the moments of the forces}}{\text{sum of the forces}},$$

and y is the equivalent position to that of the centre of pressure.

Centre of pressure of a rectangle immersed vertically with one edge in the free surface.

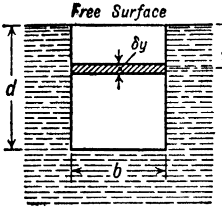


FIG. 192.

Consider an elementary strip (Fig. 192).

Area of strip $=b\times\delta y$.

Pressure on strip $=by\,\delta yw$.

$$\text{Total pressure on area}=\int_0^d by\,\delta yw$$

$$=\left[b\frac{y^2}{2}w\right]_0^d=\tfrac{1}{2}b\,d^2w.$$

Moment of pressure on strip about free surface $=by^2\,\delta yw$.

$$\text{Total moment of pressure on area}=\int_0^d by^2\,\delta yw$$

$$=\left[b\frac{y^3}{3}w\right]_0^d=\frac{bd^3}{3}w.$$

$$\text{Depth of centre of pressure}=\frac{\text{total moment}}{\text{total pressure}}$$

$$=\frac{bd^3w}{3}\times\frac{2}{bd^2w}=\frac{2}{3}\,\text{d}.$$

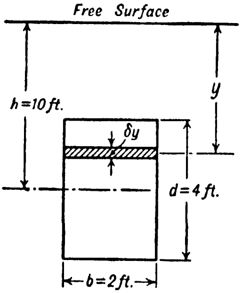


FIG. 193.

Thus the centre of pressure of a rectangle immersed vertically with one edge in the free surface is situated at $\frac{2}{3}$ of its depth.

Example 1. *Find the position of the centre of pressure of a rectangle immersed vertically in a fluid so that its centre of area is* 10 *ft. below the free surface. Dimensions of rectangle,* $b=2$ *ft.,* $d=4$ *ft.* (Fig. 193).

Consider an elementary strip.

Area of strip $=2\,\delta y$.

Pressure on strip $=2y\,\delta yw$

$$\text{Total pressure on area}=\int_8^{12} 2y\,\delta yw$$

$$=\left[y^2w\right]_8^{12}=(144-64)\,w=80w.$$

NOTE. *This is area × depth of centroid × w.*

$$\text{Moment of pressure on strip} = 2y\,\delta y w \times y.$$

$$\text{Total moment of pressure} = \int_8^{12} 2y^2\,\delta y w$$

$$= \left[\frac{2y^3}{3}w\right]_8^{12}$$

$$= \tfrac{2}{3}w\{1728 - 512\} = 810\tfrac{2}{3}w.$$

$$\text{Depth of centre of pressure} = \frac{\text{total moment}}{\text{total pressure}}$$

$$= \frac{810\tfrac{2}{3}w}{80w} = \mathbf{10{\cdot}134\ ft.}$$

Example 2. *Find the position of the centre of pressure of an equilateral triangle of side S immersed vertically in a fluid with its base in the free surface* (Fig. 194).

Consider the elementary strip (Fig. 194),

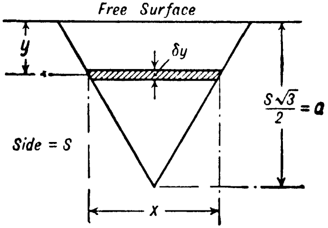


FIG. 194.

then,

$$\frac{x}{S} = \frac{\frac{S\sqrt{3}}{2} - y}{\frac{S\sqrt{3}}{2}} = 1 - \frac{2y}{S\sqrt{3}},$$

$$x = S - \frac{2y}{\sqrt{3}}$$

$$\text{Area of strip} = x\,\delta y = \left(S - \frac{2y}{\sqrt{3}}\right)\delta y.$$

$$\text{Pressure on strip} = w\left(S - \frac{2y}{\sqrt{3}}\right)y\,\delta y.$$

$$\text{Total pressure on strip} = w\int_0^a \left(Sy - \frac{2y^2}{\sqrt{3}}\right)\delta y.$$

$$P=\left[\left(\frac{Sy^2}{2}-\frac{2y^3}{3\sqrt{3}}\right)\right]_0^a \text{ and, since } a=\frac{S\sqrt{3}}{2},$$

$$=\frac{S^3\times 3}{8}-\frac{2\times S^3\times 3\sqrt{3}}{8\times 3\sqrt{3}}=\frac{S^3}{8}.$$

$$\text{Moment of pressure on strip}=w\left(S-\frac{2y}{\sqrt{3}}\right)y^2\,\delta y.$$

$$\text{Total moment of pressure}\quad =w\int_0^a\left(Sy^2-\frac{2y^3}{\sqrt{3}}\right)\delta y.$$

$$M=\left[\frac{Sy^3}{3}-\frac{2y^4}{4\sqrt{3}}\right]_0^a \text{ and, since } a=\frac{S\sqrt{3}}{2},$$

$$=\frac{S^4\times 3\sqrt{3}}{24}-\frac{2S^4\times 9}{16\times 4\sqrt{3}}=\frac{S^4\sqrt{3}}{32}.$$

$$\text{Depth of centre of pressure}=\frac{\text{moment of pressure}}{\text{pressure}}=\frac{M}{P}$$

$$=\frac{S^4\sqrt{3}}{32}\times\frac{8}{S^3}=\frac{S\sqrt{3}}{4}.$$

These examples serve to illustrate the method of obtaining the centre of pressure of an immersed surface and may now be related to the moment of inertia of the area.

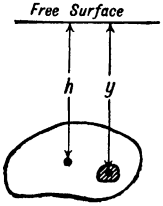


FIG. 195.

Consider any area (Fig. 195) immersed in a fluid of density w lb. per cu. ft. to a depth such that its centre of area is h ft. below the free surface. The pressure on the small shaded area is equal to *area* $\times y \times w$ *lb.*, and the moment of this pressure is *area* $\times y^2 \times w$ about the free surface, a quantity which can be identified with the **moment of inertia** or second moment of the area.

Consequently, Total pressure = sum of pressures on each small area,

and the Total moment = sum of moments of pressures on each small area.

$$\therefore \text{ Depth of centre of pressure}=\frac{\text{Total moment}}{\text{Total pressure}}=\frac{\Sigma ay^2w}{\Sigma ayw}$$

$$=\frac{\textbf{moment of inertia about free surface}}{\textbf{area}\times\textbf{depth of centre of area}}$$

Determination of moment of inertia about a parallel axis. For the purposes of the study of hydraulics, it is necessary for the student to memorise the value of the moment of inertia of certain well-known figures taken about an axis through the centre of area. These figures and the value of the moment of inertia I are shown on pages 211, 212.

When one of these areas is immersed below the free surface, the moment of inertia has to be taken about the free surface, which is generally an axis parallel to AA, through the centre of area of the figure (Fig. 196). The moment of inertia of an area immersed with its centre of area y below the free surface is given by the following expression (refer to p. 212):

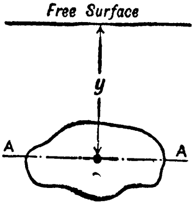


FIG. 196.

$$\mathbf{I_{free\ surface} = I_{axis\ through\ centre\ of\ area} + Area \times y^2.}$$

Example 1. *Find the moment of inertia of a circular sluice gate* 4 *ft. in diameter immersed vertically with its horizontal diameter* 10 *ft. below the free surface.*

$$\text{Moment of inertia } = \frac{\pi d^4}{64} + \text{area} \times 10^2$$

$$= \frac{\pi \times 256}{64} + \frac{\pi \times 4^2}{4} \times 100 = 404\pi$$

$$= \mathbf{1268{\cdot}56\ ft.^4\ units.}$$

Example 2. *Find the depth of the centre of pressure of the sluice gate in Example* 1.

$$\text{Depth of centre of pressure} = \frac{\text{moment of inertia}}{\text{area} \times \text{depth of centre of area}}$$

$$= \frac{404\pi}{\pi \times 2^2 \times 10} = \mathbf{10{\cdot}1\ ft.}$$

Example 3. *A rectangular gate* 10 *ft. in depth and* 4 *ft. in width covers a vertical sluice situated with its centre of area* 22 *ft. below the free surface of sea water. Find the position of the centre of pressure and the whole, or resultant pressure on the gate.*

$$\text{Depth of centre of pressure} = \frac{\text{moment of inertia}}{\text{area} \times \text{depth of centre of area}}.$$

$$I = \frac{bd^3}{12} + bd \times 22^2 = 333{\cdot}33 + 19360 = 19693{\cdot}3 \text{ ft.}^4 \text{ units.}$$

Area × depth of centre of area $=40\times22=880$ ft.^{3} units.

$$\therefore\ \bar{y}=\frac{19693{\cdot}3}{880}=22{\cdot}38\text{ ft.}$$

$$\text{Pressure}=10\times4\times22\times64\text{ lb.}$$
$$=56{,}320\text{ lb.}$$

Example 4. *A gate 4 ft. in diameter is hinged at* A *and closes the outlet of an experimental sea-water tank* (Fig. 197). *The gate is raised and lowered by a chain attached to the eye bolt* B ; *calculate the force required to raise the gate.*

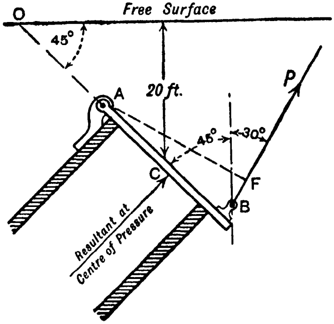


FIG. 197.

Total load on gate acting at the centre of pressure $=W$.

$$W=\pi\times2^2\times20\times64\text{ lb.}$$
$$=16{,}085\text{ lb.}$$

To find the position of centre of pressure, consider the plane containing the gate, that is, the plane OAB.

Then from O the position of the centre of pressure is

$$\frac{\text{moment of inertia}}{\text{area}\times\text{depth of centre of area}};$$

that is,

$$\frac{\dfrac{\pi\times256}{64}+\dfrac{\pi\times16}{4}\times\left(\dfrac{20}{\sin45^\circ}\right)^2}{\dfrac{\pi\times16}{4}\times\dfrac{20}{\sin45^\circ}}$$

$$=\frac{4\pi+4\pi\times\left(\dfrac{20}{\sin45^\circ}\right)^2}{4\pi\times\dfrac{20}{\sin45^\circ}}=\frac{\sin45^\circ}{20}+\frac{20}{\sin45^\circ}$$

$$=28{\cdot}31\text{ ft. from O,}$$

which is $28{\cdot}31 - \dfrac{20}{\sin 45^\circ}$ below C on OB $=0{\cdot}035$ ft.

Moments about A, $16085 \times 2{\cdot}035 = P \times AF$,

and since $AF = 4 \sin 75^\circ = 3{\cdot}8636$,

$$P = \frac{16085 \times 2{\cdot}035}{3{\cdot}8636}$$

$$= \mathbf{8468 \text{ lb.}}$$

Example 5. *A pair of lock gates, each* 30 *ft. in length, are used to close the basin of a dock. Find the total pressure on each gate when the water on the flood side is* 30 *ft. in depth and on the slack side* 10 *ft. in depth* (Fig. 198). *If each of these gates can be opened by rotating about a pivot post at the basin wall, find the turning moment necessary to open each gate against the flood water side.*

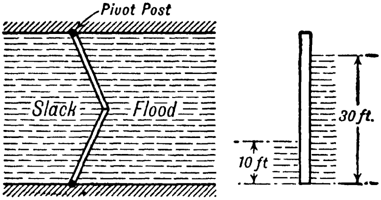


FIG. 198.

$$\text{Pressure on flood side} = 30 \times 30 \times 15w.$$

$$\text{,, \quad ,, \quad slack \quad ,,} = 10 \times 30 \times 5w.$$

The **pressure** on the slack side is in opposition to that on the flood side.

$$\text{Net pressure} = \text{pressure on flood side} - \text{pressure on slack side}$$

$$= w\{13{,}500 - 1{,}500\} = 12{,}000w$$

$$= 12{,}000 \times 62{\cdot}3 = \mathbf{747{,}600 \text{ lb.}}$$

This pressure acts on the centre of length of the gate ; therefore the turning moment to open the gate

$$= \text{net pressure} \times \tfrac{1}{2} \text{ length of gate}$$

$$= 747{,}600 \times 15 = \mathbf{11{,}214{,}000 \text{ lb. ft.}} = \mathbf{5006{\cdot}2 \text{ tons ft.}}$$

Masonry dams, sea walls and retaining walls for water. The storage of water in reservoirs, the restriction of its flow in rivers and the prevention of sea encroachment require the installation of retaining walls, dams or sea walls. These walls are scientifically designed to resist the water pressure to which they may be subjected and to offer stability to resist the overturning moment due to this water pressure.

The trapezoidal retaining wall. The commonest section which is employed for any form of retaining wall, or dam, is a trapezoid, a section which supplies an increase of dimensions at the base to meet the bending moment due to the water pressure against the face.

Consider a trapezoidal masonry dam as shown in Fig. 199, which has a depth of masonry d, maximum and minimum widths w and b respectively, and supports water to a depth h on its vertical face. The centroid of the section of the dam is at G (see construction, Vol. I, *Engineering Science*, page 190, Fig. 233).

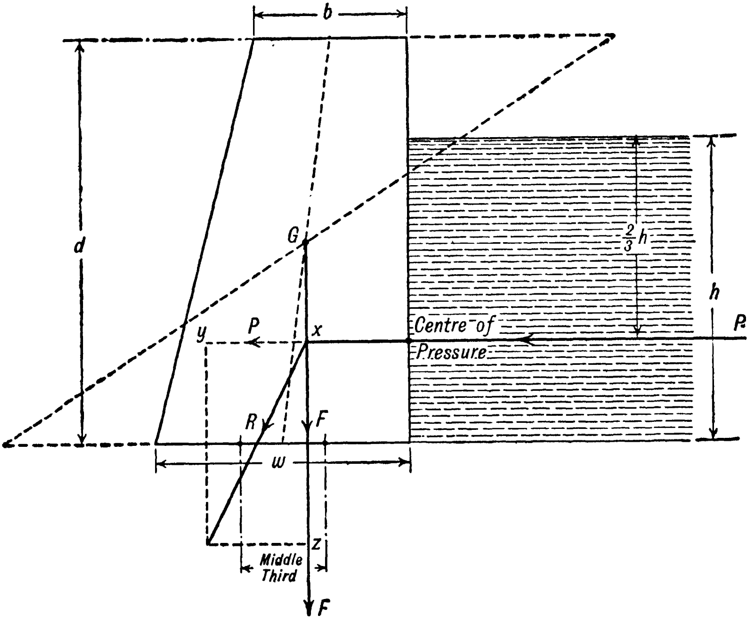


FIG. 199.

Then, for a 1 ft. length of the wall, the force due to the weight of masonry is equal to

$$\mathbf{F} = \mathbf{d}\{\mathbf{b} + \mathbf{w}\}\tfrac{1}{2} \times \mathbf{S}\ \textbf{lb.}$$

where S is the density of the masonry in lb. per cu. ft.

The pressure due to the water acts at the centre of pressure, that is, at a depth $\frac{2}{3}h$ (p. 321), and is equal to

$$\mathbf{P} = \mathbf{h} \times 1 \times \frac{\mathbf{h}}{2} \times 62{\cdot}3\ \textbf{lb.}$$

At the point x, the intersection of the lines of action of P and F, the forces due to the weight of the masonry and the water pressure, act. Draw the parallelogram of forces for P and F; that is, let xy represent P and xz represent F, then the diagonal R represents the resultant of P and F, the forces acting upon the wall.

Stability of the wall. The forces acting upon the wall may be considered as

(*a*) the pressure due to the fluid which is tending to overturn the dam, that is, an **overturning moment**;

(*b*) the weight of the dam, which exerts a **righting moment.**

If the line of action of the resultant falls outside the middle third of the width of the base, the overturning moment may be in excess of the righting moment and the dam will tend to overturn.

The **middle third** is the name given to that portion of the base between points situated at $\frac{1}{3}$ and $\frac{2}{3}$ of its width measured from one edge (see Fig. 199).

Example 1. *Find the resultant thrust on the base of the masonry dam shown in Fig. 200. Is the dam stable? Density of material composing the dam* = 155 *lb. per cu. ft.*

Consider 1 ft. length of the dam.

Weight per ft. length

$$= \tfrac{1}{2}(5 + 3) \times 15 \times 155 = 9300\ \text{lb.}$$

Water pressure per ft. length

$$= 12 \times 1 \times 6 \times 62{\cdot}3 = 4485{\cdot}6\ \text{lb.}$$

Draw to scale the cross-section of the dam and determine the position of the centroid G and the centre of pressure of the fluid pressure (Fig. 201).

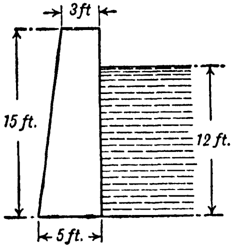


FIG. 200.

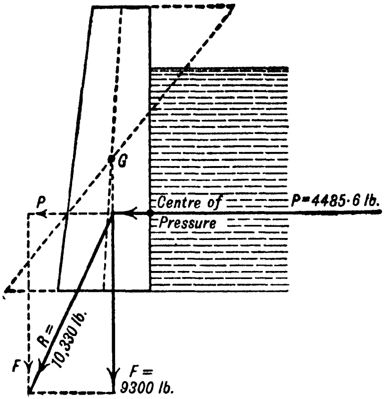


FIG. 201.

Represent P and F to scale and draw the parallelogram of forces to determine the resultant R. Then R is the resultant thrust on the base of the dam. To determine the stability, measure the distance of the point where the resultant R cuts the base from the edge of the base, and if R falls in the middle third, the dam is stable; if it is outside the middle third, it is unstable for the given conditions.

Answer. Resultant = 10,330 lb.

Unstable.

Example 2. *Find the resultant load on the foundation of the masonry dam shown in Fig.* 202, *if the masonry weighs* 170 *lb. per cu. ft. Is the dam stable?*

Area of base $= 6{\cdot}9 \times 3 = 20{\cdot}7$, area of middle $= 4{\cdot}8 \times 6 = 28{\cdot}8$,

area of top $= 3 \times 3 = 9$ sq. ft. Total area $= 58{\cdot}5$ sq. ft.

Weight per ft. of dam $= \{6{\cdot}9 \times 3 + 4{\cdot}8 \times 6 + 3 \times 3\}170$

$= \mathbf{9945}$ **lb.**

Water pressure per ft. $= 10{\cdot}8 \times 1 \times 5{\cdot}4 \times 62{\cdot}3$

$= \mathbf{3634}$ **lb.**

For position of centroid (a) take moments about base:

$$20{\cdot}7 \times 1{\cdot}5 + 28{\cdot}8 \times 6 + 9 \times 10{\cdot}5 = 58{\cdot}5\bar{y}.$$

$$\bar{\mathbf{y}} = \mathbf{5{\cdot}1\ ft.}$$

(b) take moments about water face:

$$20{\cdot}7 \times 3{\cdot}45 + 28{\cdot}8 \times 2{\cdot}4 + 9 \times 1{\cdot}5 = 58{\cdot}5\bar{x}.$$

$$71{\cdot}42 + 69{\cdot}12 + 13{\cdot}5 = 58{\cdot}5\bar{x}.$$

$$\bar{x} = 2{\cdot}632 \text{ ft.}$$

Answer. Resultant = 10,590 lb.

Stable.

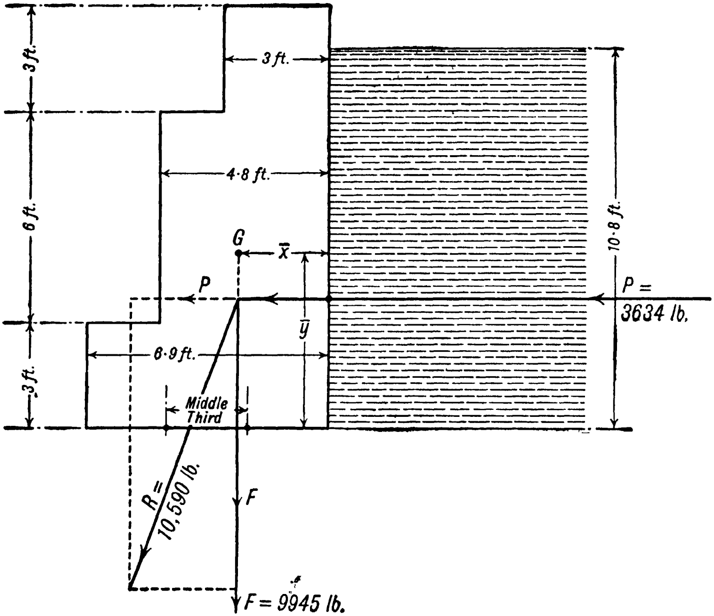


FIG. 202.

Example 3. *Find the value of the resultant thrust on the base of a reservoir wall* (Fig. 203), *supporting* 27 *ft. of water. Determine whether this wall is stable and the position where the line of action of the resultant thrust cuts the base. Density of masonry,* 170 *lb. per cu. ft.*

In this example the wall presents two sloping batters and the water surface receives a total pressure of P passing through its centre of pressure, which is at $\frac{2}{3}$ of its slanting immersed depth measured from the free surface. The water pressure is normal to the surface of the wall.

Consider a 1 ft. length of the wall.

Pressure due to water $= P = cb \times 1 \times \frac{cb}{2} \times 62{\cdot}3.$

$$cb = \tfrac{27}{30}\sqrt{10^2 + 30^2} = 0{\cdot}9 \times \sqrt{1000} = 28{\cdot}45 \text{ ft.}$$

$$P = 28{\cdot}45 \times 13{\cdot}5 \times 62{\cdot}3 = 3{,}928 \text{ lb.}$$

Weight of wall per foot $= F = \tfrac{1}{2}(24 + 10) \times 30 \times 170.$

$$\mathbf{F = 86{,}700 \text{ lb.}}$$

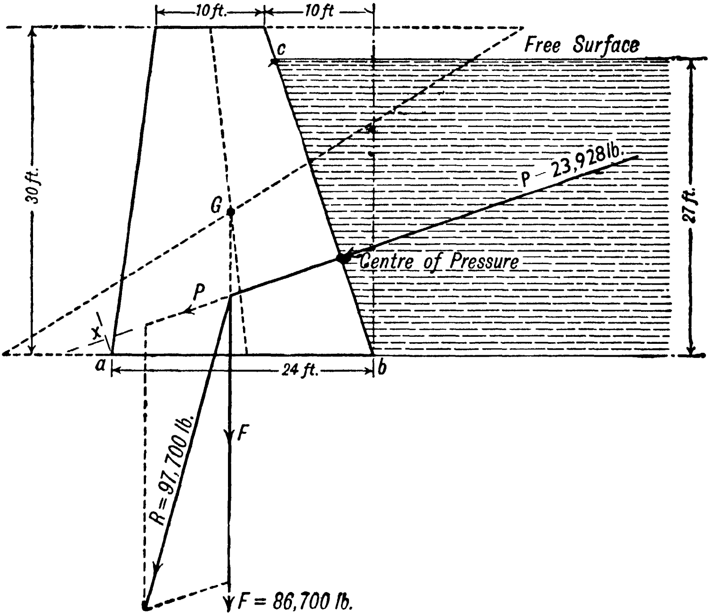


FIG. 203.

Continue the line of action of P to meet that of F and construct the parallelogram of forces for P and F.

Answer : R = 97,700 lb. and cuts the base at 9·5 ft. from a ;

the dam is therefore stable.

Example 4. *Calculate the overturning and righting moment for each of the masonry dams in Examples* 1, 2, *and* 3.

No. 1. Righting moment is the moment of the weight of the masonry taken about the extreme edge of the base = 9300 × 2·958.

Righting moment = 27,512 lb. ft.

Overturning moment is the moment of the water pressure taken about the extreme edge of the base = 4485·6 × 4.

Overturning moment = 17,942 lb. ft.

No. 2. Righting moment = 9945 × 4·268
= **42,445 lb. ft.**

Overturning moment = 3634 × 3·6
= **13,082 lb. ft.**

No. 3. Righting moment = 86,700 × 11
= **953,700 lb. ft.**

Overturning moment = $P \times ax$
= 3,928 × 1·7
= **40,678 lb. ft.**

It may be noted that in cases 2 and 3 the overturning moment is considerably less than the righting moment, whereas in case 1 the overturning moment is much nearer to the righting moment. The wall in case 1 is regarded as unstable because the resultant thrust falls outside the empirical middle third rule: actually, the dam would not overturn because the overturning moment is less than the righting moment. The student is advised to test for stability on the middle third rule, and thus ensure a margin of safety in design.

EXERCISES ON CHAPTER XIII

1. Calculate the total pressure on a steel inspection plate 1 ft. 9 in. in width and 10 in. in depth fitted to the vertical side of a storage tank so that its top edge is 20 ft. below the level of the water in the tank.

2. A circular sluice gate is 2 ft. 4 in. in diameter and is situated in a sea wall with its centre 14 ft. below sea-level. Calculate the whole pressure on the gate.

3. Find the total pressure on a triangular plate, equilateral in shape, of 2 ft. side immersed with its centre of area 20 ft. below the free surface of sea water.

4. Calculate the total pressure on the base of an oil tank 5 ft. in diameter if the oil is 18 ft. in depth and its specific gravity is 0·74.

5. Find the moment of inertia of a circle 10 in. diam. about an axis parallel to its diameter and 30 inches from it.

6. Calculate the moment of inertia of a triangle 6 in. base, altitude 8 in., about an axis parallel to the base and 20 in. from it on the opposite side to the apex.

7. Find the position of the centre of pressure of a vertical retaining wall 8 ft. in length and supporting water to a depth of 20 ft.

8. A circular sluice gate is 4 ft. in diameter and closes a sluice in a vertical wall the centre of which is 10 ft. below the free surface. Find the position of the centre of pressure.

9. Find the total load upon, and the position of the centre of pressure of, a rectangular slab of concrete forming the side of a pier, if the slab is 4 ft. in width and 8 ft. in depth, set with its top edge 25 ft. below the free surface of a river.

10. A vertical rectangular sluice gate is 4 ft. by 2 ft. and is submerged with the longer sides horizontal; the top side being 4 ft. below the surface of the water. Find the position of the centre of pressure. (U.E.I.)

11. A lock gate is 20 ft. in length and supports water to a depth of 10 ft. on the flood side and 5 ft. on the slack side. Find the total load on the gate and the position of the point of application of this load.

12. A pipe 4 ft. diameter is inclined at 45° to the horizontal and its highest point is in the free surface of a reservoir. If the pipe is closed by a gate hinged at the highest point and blanking the pipe at right angles to its axis, calculate the force required to open the gate by means of a wire rope attached to the lowest point of the gate and pulling at right angles to it.

13. A circular hatch in the vertical wall of a reservoir is 3 ft. 6 in. diameter and its centre is 7 ft. below free surface. This gate is hinged at its highest point and can be opened by a chain attached to the lowest point to pull at 60° to the vertical. Calculate the pull in the chain in order to open the gate.

14. An oil bath is 8 ft. in length, 4 ft. in width and 6 ft. in depth. It is emptied by means of opening a sluice drain in the base which is closed by a gate 1 ft. 6 in. in length and 1 ft. in width. If this gate is hinged on one of its shorter dimensions and raised by a chain attached to the other short edge, calculate the force necessary to raise the gate when the chain is vertical if the specific gravity of the oil is 0·78.

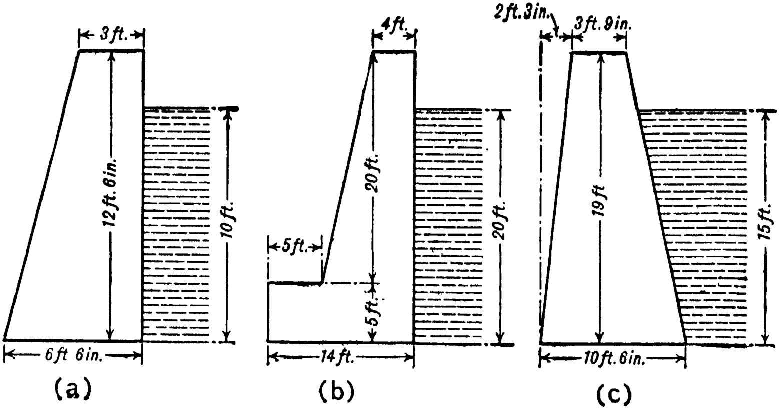


FIG. 204.

15. Find the resultant pressure on the base of each of the masonry dams shown in Fig. 204. Calculate, in each case, the righting and overturning moments and determine the stability on the middle third rule. Density of masonry = 170 lb. per cu. ft.

CHAPTER XIV

IMPACT OF WATER ON PLATES AND VANES, ROTATING MACHINES, WATER WHEELS AND WATER TURBINES

When water is delivered from a nozzle in the form of a jet, or in a directed stream, the water possesses kinetic energy, which may be utilised in the performance of work. This is effected in hydraulic prime movers, such as water wheels or turbines, by allowing the jet to strike a series of vanes and give up some of its kinetic energy to perform work on the rotating vanes. In some examples of water wheels the rotating force is supplied by the weight of falling water, and the student will be able to distinguish in the later descriptions of such machines between those operated by the gravitational effect of falling water and those worked by the partial destruction of the momentum of a jet or stream.

Impact of water on a stationary flat plate. Consider a jet of water delivering W lb. of water per sec. at a velocity of V ft. per sec. (Fig. 205). If this jet impinges on a stationary flat plate of area a sq. ft., no movement of the plate will result and there will be no work done, but the jet will lose its momentum and exert a force on the plate.

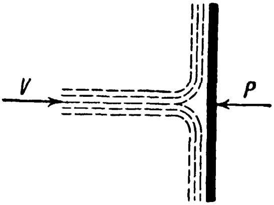


Fig. 205.

Loss of momentum = impulsive force $= \dfrac{WV}{g}$ ft. lb. sec. units, and the force P exerted upon the plate in pounds is

$$\mathbf{P} = \frac{\mathbf{WV}}{\mathbf{g}} \textbf{ lb.} = \frac{\mathbf{waV^2}}{\mathbf{g}} \textbf{ lb.}$$

where w is the weight of 1 cu. ft. of water.

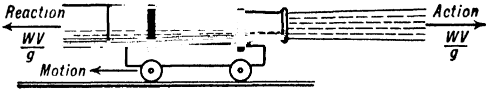


FIG. 206.

Reaction of a jet. In order that a jet of water of W lb. per sec. may be projected with a velocity V ft. per second, a force of WV/g lb. must be applied to the water. It follows, according to the third law of motion, that a reaction to this jet must be experienced equal to this force and in the opposite direction.

This reaction is well illustrated by the movement of a gun backwards when the projectile is fired, or the "kick" of a rifle against the shoulder of a rifleman when the rifle is fired. If a nozzle (Fig. 206) be securely attached to a carriage, free to move, and a jet of water is ejected from the nozzle, the carriage moves backwards under a force equal to the projecting force on the jet, and the momentum of the carriage and that of the water are equal. A simple example is the rotating spraying device used for watering lawns. The jet propulsion unit of an aeroplane is another example, but with hot gases as the working fluid.

Effect of allowing the plate to move in the direction of the velocity of the jet. Suppose the plate (Fig. 205) has a velocity of V_1 ft. per sec. in the direction of the velocity of the jet, then the whole of the momentum of the jet is not destroyed but there is a residual momentum of WV_1/g in the direction of motion.

Thus force $P =$ loss of momentum per second

$$= \frac{W(V - V_1)}{g} \text{ lb.}$$

If a is the area of cross-section of jet in sq. ft. and w is the density of water in lb. per cu. ft., the weight of water delivered per sec.

$$= W \text{ lb.} = aw(V - V_1),$$

and

$$P = \frac{aw(V - V_1)^2}{g} \text{ lb.}$$

Now since the plate is moving at V_1 ft. per sec., the work done per second

$$= PV_1 \text{ ft. lb.}$$

and the $$\text{H.P.} = \frac{PV_1}{550} = \frac{aw(V - V_1)^2 V_1}{550g}$$

Efficiency for the impact of a jet on a moving vane. The efficiency computed for any performance is the ratio of

$$\frac{\textit{work output}}{\textit{work input}} \quad \text{or} \quad \frac{\textit{useful energy}}{\textit{available energy}}.$$

In this case, the available energy is the kinetic energy of the supply jet $WV^2/2g$; and since $W = awV$,

$$\text{available energy} = \frac{awV^3}{2g} \text{ ft. lb.}$$

The work output or useful energy is the work done by the jet

$$PV_1 \quad \text{or} \quad \frac{aw(V - V_1)^2 V_1}{g}.$$

$$\text{Efficiency} = \frac{\text{useful energy}}{\text{available energy}} = \frac{PV_1 \times 2g}{awV^3}$$

$$\frac{2gaw(V - V_1)^2 V_1}{g \times awV^3} = \frac{2(V - V_1)^2 V_1}{V^3}.$$

Relationship between velocities of jet and vane for maximum efficiency.

Since $$\text{efficiency} = \frac{2(V - V_1)^2 V_1}{V^3},$$

the efficiency is a maximum when $V_1(V - V_1)^2$ is a maximum.

Let $$y = V_1(V - V_1)^2 = V_1V^2 - 2VV_1^2 + V_1^3,$$

$$\frac{dy}{dV_1} = V^2 - 4VV_1 + 3V_1^2,$$

which is equal to zero for a maximum value of y; thus

$$V^2 - 4VV_1 + 3V_1^2 = 0,$$

or $$(V - 3V_1)(V - V_1) = 0,$$

and therefore $$V = 3V_1 \text{ or } V_1.$$

Thus the maximum efficiency is obtainable when the **velocity of the jet is three times that of the plate or vane.**

Maximum efficiency when $V=3V_1$ is

$$\frac{2(V-V_1)^2V_1}{V^3}=\frac{2\times 4V_1^3}{27V_1^3}=\frac{8}{27}.$$

Example 1. *A jet of 12 sq. in. in cross-section delivers 4 cu. ft. of water per sec. and impinges on a flat plate moving in the direction of the jet at 15 ft. per sec. Calculate (a) the force exerted on the plate, (b) the maximum H.P. available, (c) the efficiency.*

$$\text{Velocity of jet}=\frac{4\times 144}{12}=48 \text{ ft. per sec.}$$

$$\text{(a) Force} \quad =\frac{aw(V-V_1)^2}{g}=\frac{12\times 62{\cdot}3(48-15)^2}{144g}$$

$$=175{\cdot}6 \text{ lb.}$$

$$\text{(b) H.P.} \quad =\frac{PV_1}{550}=\frac{175{\cdot}6\times 15}{550}=4{\cdot}79.$$

$$\text{(c) Efficiency}=\frac{2(V-V_1)^2V_1}{V^3}=\frac{2\times 33^2\times 15}{48^3}=29{\cdot}54\%.$$

Example 2. *Calculate the quantity of water to be delivered per sec. by a jet which impinges with a velocity of 60 ft. per sec. on a moving flat plate in order that a H.P. of 7·5 may be developed with maximum efficiency.*

Velocity of jet, $V=60$ ft. per sec.

For maximum efficiency, $V=3V_1$, $V_1=20$ ft. per sec.

$$\text{H.P.}=\frac{aw(V-V_1)^2V_1}{550g}.$$

$$7{\cdot}5=\frac{a\times 62{\cdot}3(60-20)^2\times 20}{550g}=\frac{a\times 62{\cdot}3\times 1600\times 20}{550g}.$$

$$a=\frac{7{\cdot}5\times 550\times g}{62{\cdot}3\times 1600\times 20}=0{\cdot}06665 \text{ sq. ft.}$$

$$W=awV=0{\cdot}06665\times 62{\cdot}3\times 60$$

$$=249{\cdot}2 \text{ lb. per sec.}$$

Effect of using a succession of vanes. If, instead of using a single flat plate moving in the direction of the jet, a series of vanes is employed, moving in succession, the whole of the water supplied by the jet will be thrown upon the vanes, and W, the quantity delivered usefully, will be awV instead of $aw(V-V_1)$, as is the case for a single vane.

Thus the work done per sec. will be Pv_1 ft. lb.; but P will be

$$\frac{awV(V-V_1)}{g}\text{ lb.}$$

$$\therefore\ \text{H.P.}=\frac{awV(V-V_1)V_1}{550g}=\frac{awVV_1(V-V_1)}{550g}.$$

$$\text{Efficiency}=\frac{2awVV_1(V-V_1)}{awV^3}=\frac{2V_1(V-V_1)}{V^2}.$$

Maximum efficiency is obtained when $V_1(V-V_1)$ is a maximum.

$$\therefore\ \text{if}\ \ y=V_1(V-V_1)=V_1V-V_1^2,$$

$$\frac{dy}{dV_1}=V-2V_1=0\ \text{for a maximum};$$

i.e. $V=2V_1$, **or, the velocity of the jet is double that of the vane.**

$$\text{Value of maximum efficiency}=\frac{2V_1(V-V_1)}{V^2}=\frac{2V_1^2}{4V_1^2}=50\%.$$

SUMMARY OF RESULTS.

Arrangement of impact	Force P on vane (lb.)	H.P.	Efficiency	Maximum efficiency	Ratio V/V_1
Stationary flat plate	$\frac{waV^2}{g}$	—	—	—	—
Moving flat plate	$\frac{wa(V-V_1)^2}{g}$	$\frac{wa(V-V_1)^2V_1}{550g}$	$\frac{2(V-V_1)^2V_1}{V^3}$	$\frac{8}{27}$	3
Succession of flat plates	$\frac{waV(V-V_1)}{g}$	$\frac{waVV_1(V-V_1)}{550g}$	$\frac{2V_1(V-V_1)}{V^2}$	$\frac{1}{2}$	2

Example. *A jet of water* 2 *inches in diameter delivers at a velocity of* 30 *ft. per sec. on to a succession of flat vanes moving in the direction of the jet at* 14 *ft. per sec. Calculate* (*a*) *the total pressure on the vanes,* (*b*) *the H.P.,* (*c*) *the efficiency.*

$$(a)\ \text{Pressure}=\frac{awV(V-V_1)}{g}=\frac{\pi\times1^2\times62{\cdot}3\times30(30-14)}{144g}$$

$$=20{\cdot}3\text{ lb.}$$

(*b*) H.P. $= \dfrac{awVV_1(V-V_1)}{550g} = \dfrac{\pi \times 1^2 \times 62\cdot3 \times 30 \times 14 \times 16}{144 \times 550g}$

$= \mathbf{0\cdot52\ H.P.}$

(*c*) Efficiency $= \dfrac{2V_1(V-V_1)}{V^2} = \dfrac{2 \times 14 \times 16}{30^2}$

$= \mathbf{49\cdot8\%}$.

Impact of a jet upon an inclined plate. Suppose a jet of water moving with a velocity of V ft. per sec. impinges upon a flat plate inclined at $\theta°$ to the direction of the jet (Fig. 207).

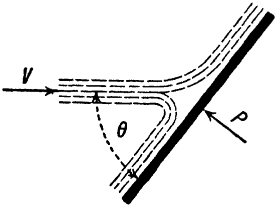


FIG. 207.

The momentum of the jet in the direction of its motion is $\dfrac{WV}{g}$ ft. lb. units.

Loss of momentum in the direction of the jet

$$= \frac{WV}{g} = \frac{waV^2}{g} = \text{force in lb.}$$

The effective thrust on the plate is P, a force which acts normal to the plate and

$$P = \frac{WV}{g} \sin\theta = \frac{waV^2}{g} \sin\theta.$$

Example 1. *Find the force exerted by a jet moving at* 60 *ft. per sec. and discharging* 7 *cu. ft. per sec. on to a stationary flat plate inclined at* 60° *to the jet.*

$$W = 7 \times 62\cdot3 = 436\cdot1 \text{ lb. per sec.}$$

$$P = \frac{WV}{g} \sin\theta = \frac{436\cdot1 \times 60}{32\cdot2} \times 0\cdot866$$

$$= \mathbf{703\cdot7\ lb.}$$

Example 2. *A plate valve retained by a spring exerting a force of* 120 *lb. at right angles to the plate is to be opened by a jet of water moving at* 30 *ft. per sec. and impinging at an angle of* 45° *to the plate. Calculate the minimum diameter of the jet.*

Let d ft. be diameter of jet.

Then weight of water per sec. $= w \times \dfrac{\pi d^2}{4} \times V = W$

$$= 62\cdot3 \times \frac{\pi d^2}{4} \times 30 \text{ lb.}$$

$$\text{Force} = \frac{WV}{g}\sin\theta = \frac{62\cdot3 \times \pi d^2 \times 30 \times 30}{4 \times 32\cdot2}\ \text{lb.}$$

$$P = 120\ \text{lb.} \quad = 1368d^2.$$

$$\therefore\ d = \sqrt{\frac{120}{1368}} \quad = 0\cdot2963\ \text{ft.} = 3\cdot56\ \text{in.}$$

Impact of water upon shaped vanes. It can be seen from the preceding work on the impact of water upon flat plates that there is a sudden deviation of the stream at impact, and the effort is applied, not as a steady force, but as an impulse or shock. Such a shock produces uneven turning moment on the rotor to which the vane is attached, and also wastes energy in the agitation of the water. It is possible to obtain the full force of the impact and the elimination of this shock by so designing the vane that the jet of water suffers a gradual deviation during its impact.

Design of vane to eliminate shock. Consider a stationary vane AB into which a jet of water is admitted tangentially at A (Fig. 208), and leaves the vane tangentially at B after being gradually deviated to avoid so far as possible shock and loss of energy. The force exerted by the water depends upon the change of momentum between A and B, and this can be determined by obtaining the change of velocity between these two points. To obtain this change of velocity, draw *op* to represent in magnitude and direction the velocity of entry *V*. Next draw *oq* to represent completely the velocity of exit *V*. Join *pq*. Then *pq* will represent the change of

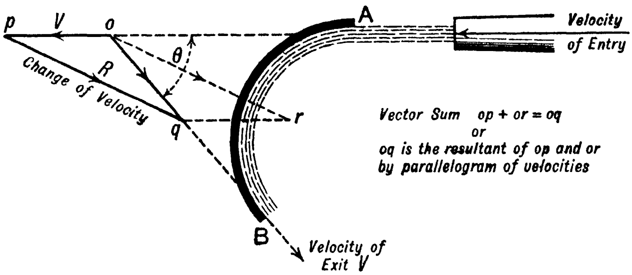


FIG. 208.

velocity R, since oq, the new velocity, is the vector sum of op and pq. pq is the change of velocity due to the effect of the vane on the jet. qp or pq reversed gives the direction of the resultant force of the water on the vane.

Alternatively, if the parallelogram of velocities be constructed to obtain the resultant and final velocity oq, it is necessary to combine R with, or to form the diagonal oq which represents, the final velocity. Hence R represents the change of velocity in magnitude and direction during the progress of the jet over the vane.

If the angle between the tangents at A and B be θ, then the angle opq will be $\frac{1}{2}\theta$. Also opq is an isosceles triangle and hence

$$R = 2V \cos \tfrac{1}{2}\theta.$$

The change of momentum is $\dfrac{2WV}{g} \cos \frac{1}{2}\theta$, where W is the weight of water supplied per second.

Thus the force on the vane will be $P = \dfrac{2awV^2}{g} \cos \dfrac{\theta}{2}$.

Example. *Calculate the force exerted upon a stationary vane which receives and discharges a jet of* 4 *cu. ft. per sec. with a velocity of* 30 *ft. per sec. if the angle between the tangents to the vane at inlet and outlet is* 72°.

$$P = \frac{2WV}{g} \cos \frac{\theta}{2}$$

$$= \frac{2 \times 4 \times 62{\cdot}3 \times 30}{32{\cdot}2} \cos 36^\circ$$

$$= 375{\cdot}6 \text{ lb}$$

Moving shaped vane. The general application of impact of water on shaped vanes is found when the vane is part of a rotating portion of a machine and has a velocity of movement independent of the jet.

Consider a vane (Fig. 209) which has a velocity of V_1 and is receiving water at a velocity V. In order that there may be no shock at entry, the vane must be so designed that the tangent at A shall be in the same direction as the relative velocity of the water and the vane.

Inlet. Draw ab to represent the velocity of entry V, and ac the velocity of the vane V_1; then draw bd parallel to ca to represent V_1 of the vane reversed. Join ad. Then ad will be the relative

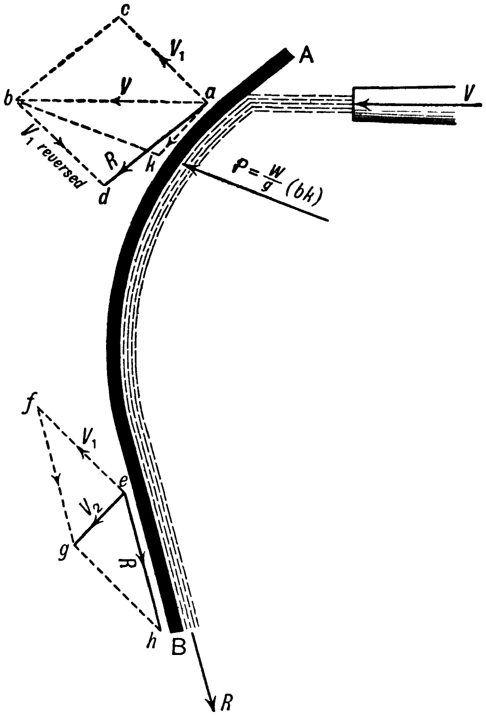


FIG. 209.

velocity R of the water to the vane, since ad is the vector sum of V and V_1 reversed, and shows the direction in which the entrant portion of the vane at A should be made.

Neglecting friction, this relative velocity is the velocity of the water at outlet when considered relative to the vane.

Outlet. Draw ef to represent the velocity of the vane V_1 and eh to represent the relative velocity R; that is, eh equal to ad and ef equal to ac. Then, on the completion of the parallelogram, the diagonal eg will represent V_2, the *absolute velocity of the water at discharge, or* by the vector diagram efg, $ef+fg=eg$, or $V_1+R=V_2$.

N.B.—V_2 *should be at right angles to* V_1 *in order that there shall be* *no component in the direction of V_1. If there is a component in the direction of V_1, this velocity is known as the* **velocity of whirl** ; *for maximum efficiency on a rotating machine the velocity of whirl must be zero.*

From *a* draw *ak* equal and parallel to *eg*, then *bk*—because it must be added vectorially to *ab* to equal *ak*—is the velocity change brought about by the passage of the water through the vane, and is the change of velocity produced by the loss of momentum of the jet.

$$\text{Force on the vane} = \mathbf{P} = \frac{\mathbf{W}}{\mathbf{g}} \times \mathbf{bk},$$

where *W* is the weight of water delivered per second.

Example. *A vane is to be designed to give no shock at entry and exit to a jet moving at 40 ft. per sec. and delivering 8 cu. ft. of water per sec. If the vane has a velocity of 20 ft. per sec. at 45° to the jet, find (a) the direction of the vane at inlet, (b) the direction of the vane at exit, to give no velocity of whirl, (c) the force exerted on the vane in pounds and the direction of this force.*

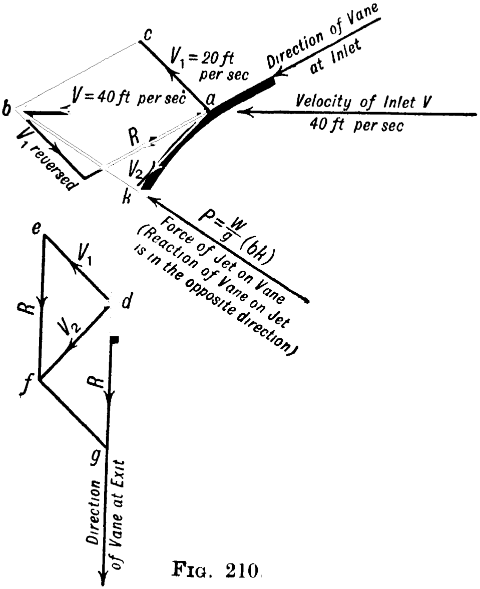


FIG. 210.

(*a*) Draw *ab* Fig. 210 to represent the velocity of inlet, 40 ft. per sec., and *ac* to represent the velocity of the vane, 20 ft. per sec.; complete the parallelogram, when R will **give the direction of the vane at inlet.**

(*b*) Since there is no velocity of whirl, set out the direction of V_2 at right angles to *cd*, which represents the velocity of the vane, 20 ft. per sec. Complete the parallelogram by making *ef* equal to R in section (*a*). Then *dg* represents the **direction of the vane at exit.**

(*c*) Set out *ak* parallel to *df* and equal to V_2; join *bk*, which will represent in magnitude and direction the change of velocity within the vane. Then

$$\text{Force } P = \frac{W}{g}(bk),$$

and since

$$W = 8 \times 62{\cdot}3 = 498{\cdot}4 \text{ lb. per sec.},$$

$$P = 498{\cdot}4\,(bk) \text{ lb.}$$

$$= 498{\cdot}4 \times 29{\cdot}4 = \mathbf{14{,}650 \text{ lb.}}$$

Hydraulic prime movers. The hydraulic prime movers may be divided into several classes.

(1) **Water wheels** in which the whole motive power is supplied by the *weight of water* fed to buckets placed on the periphery of the wheel.

(2) **Water wheels** in which the motive power is partly supplied by the *weight of water* and partly by the *impulse* of a directed stream upon vanes.

(3) **Water wheels or turbines** in which the motive power is wholly supplied by the *impulse* of a directed stream or jet upon a succession of vanes.

(4) **Reaction turbines** in which the motive power is supplied by the *reaction* of the moving water on the wheel.

The selection of suitable hydraulic prime movers for specific work is determined by the general site position at which the work has to be done, and whether or not natural resources are available to operate the machine.

Water wheels of classes (1) and (2) can often be found as established prime movers, particularly in country districts, where they have given long and valuable service in driving mills and factory equipment. Where new installations have to be considered, it is usual to instal types (3) or (4) because types (1) and (2) are now out of date, and the modern tendency is to employ machines

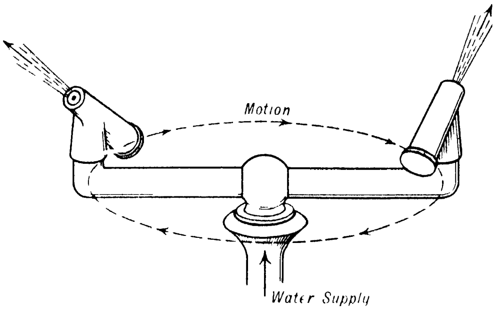


FIG. 211.

operating by impulse in preference to the cumbersome type of water wheel with gravity, or partial gravity, operation. These impulse wheels will work on almost any available head and give a high efficiency in operation.

It often becomes necessary, in order to employ these more up-to-date types, to convert the pressure head of water available to a kinetic head, in the form of a jet or stream, which may be directed on to a succession of vanes, as in the impulse or reaction type of water wheel or turbine.

Fig. 211 shows the most elementary type of reaction turbine, an adaptation of the Scotch turbine, and now used for spraying lawns and greens. It consists of a central stand-pipe the top of which is fitted with two or more offset pipes bent in an anticlockwise direction at their ends. The top is free to rotate on the stand-pipe, and when water is issuing from the jets the reaction of the jets causes the top to rotate in a clockwise direction, that is, opposite to that of the motion of the jets themselves.

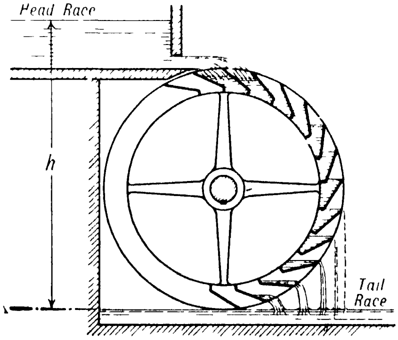


FIG. 212.

The Overshot wheel. This type of water wheel (Fig. 212) belongs to the first group of prime movers, in which practically the whole of the motive force is supplied by the weight of the falling water.

The head-race, or supply water, is led through a sluice into a series of bucket vanes on the periphery of the water wheel, and the weight of the water retained in the buckets provides the turning moment on the wheel. The effective head on such a wheel is that represented by the difference in level of the head-race and the tail-race into which the buckets discharge. In order to give a reasonable efficiency, the wheel should be nearly equal in diameter to the available head.

If W is the weight of water delivered per second, and h is the available head in feet, the horse-power $= \frac{Wh}{550}$.

It is usual to design this type of wheel for a head-race velocity of from 6 to 13 ft. per sec., and to arrange for a peripheral speed of the wheel of one half this velocity. In these circumstances the efficiency can be as high as 80%.

This type is very seldom put in as a new installation, but may frequently be found in old mill streams.

N.B.—*In this type of wheel there is a very small impulse due to the velocity of approach of the stream.*

The Breast wheel. The breast wheel (Fig. 213) is a modification of the overshot wheel, in which a close-fitting breast is placed against the buckets to retain the water in these buckets for the greatest possible time before discharge into the tail-race. The water is

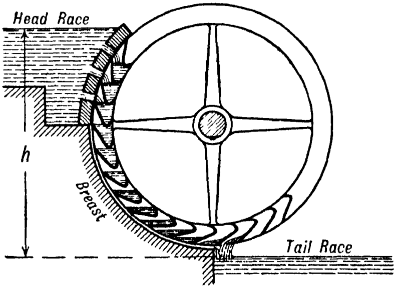


FIG. 213.

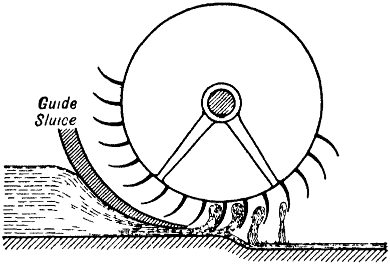


FIG. 214

brought to the vanes through a ported sluice, the ports of which are so designed that the water enters without shock. The efficiency seldom exceeds 80% and is highest in the larger diameter wheels.

As in the overshot wheels, the available head is the difference between head- and tail-race levels, and the theoretical horse-power is $\frac{Wh}{550}$.

The Undershot and Poncelet wheel. The undershot wheel (Fig. 214) works entirely by impulse provided by the stream of water passing through the guide sluice and striking the vanes. In the earlier types the vanes were radial, but in Poncelet's variation the vanes are curved and designed to eliminate shock. The effective force exerted upon the vanes depends upon the velocity of approach of the undershot stream and the loss of momentum of this stream on impact. This velocity of approach is determined by the available head of water before the guide sluice. These wheels may be used for falls up to 5 ft., and in the case of the Poncelet type to 7 ft. head. The efficiency seldom exceeds 60% and the wheel diameter should be between two and four times the available head.

As is general in wheels with a succession of vanes, the peripheral speed of the vanes should be one half of the velocity of the stream.

Force exerted on the vane of a Poncelet wheel. Consider a stream of water entering the vane of a Poncelet wheel as shown in Fig. 215. Draw *ac* to represent the velocity of the stream and *ad* the velocity of the tip of the vane in a direction parallel to the tangent to the wheel at this point. Complete the parallelogram, where *ab* or *dc*, that is, *R*, will represent the relative velocity of water and vane on entry and will show the direction in which the tangent to the vane at entry must be made to avoid shock. The water now passes up the vane and is returned towards its tip with a theoretical relative velocity of *R* ft. per sec. (actually less owing to friction) to be discharged with an absolute velocity V_2 ft. per sec. To obtain V_2, draw *ef* parallel to the tangent to the wheel at outlet and let it represent V_1, the velocity of the vane. Draw *eh* tangential to the vane at outlet and equal to *R*, that is, *dc* the relative velocity at inlet. Complete the parallelogram, when *eg*, that is, V_2, will be the absolute velocity of discharge.

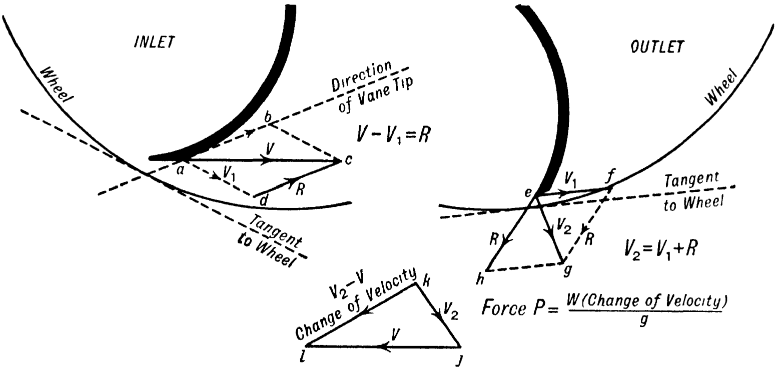


FIG. 215.

The change of momentum between inlet and outlet will depend upon the change of velocity between *V* at inlet and V_2 at discharge. Draw *jk* to represent the value of V_2 and parallel to it, that is to *eg*, and *jl* to represent *V* and parallel to it reversed, that is, *ca*. then *kl* will be the change of velocity required, that is, V_3.

$$\text{Force on the vane} = \frac{W}{g}\,(\text{change of velocity}) = \frac{WV_3}{g},$$

where *W* is the weight of water delivered per second in pounds.

The Pelton wheel. The Pelton wheel is one of the most common forms of hydraulic prime movers and is a development of a machine which was introduced into the mining districts of California about

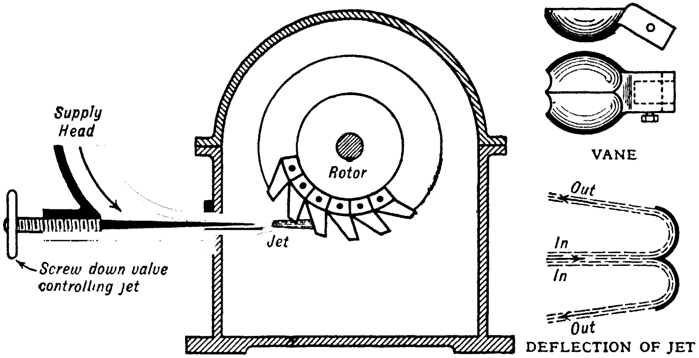


Fig. 216.

1865 and known as the **hurdy-gurdy**. This machine was a vertical wheel with flat radial vanes which received the impulse of a high velocity jet. The modern form of Pelton wheel has cup-shaped vanes (Fig. 216), which are served by a high-velocity jet operated by a screw-down valve. The vanes, *or buckets*, are usually detachable, and the jet is deflected by the bucket through an angle which approaches 180°. It is impossible, in practice, to obtain a deviation of jet equal to 180°, because by so doing the deflected jet would interfere with the projected jet.

Impact of water on a cup. If water impinges upon a fixed cup-shaped vane and is deviated through 180° (Fig. 217), its

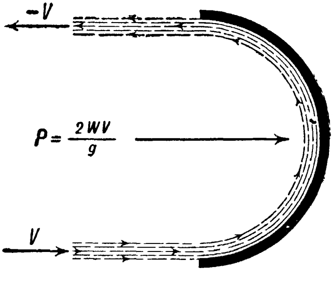


Fig. 217.

velocity is reversed and the projected stream will exert an impulse on the vane while the deviated stream exerts a force due to reaction on the same vane. These two forces will combine to provide the turning moment on the wheel to which the vane is attached.

Suppose W lb. of water to strike a cup-shaped vane at V ft. per sec. (Fig. 217), and to be deviated so that it leaves the vane parallel to projection at $-V$ ft. per sec. without loss due to shock. Then the velocity will be changed from V to $-V$ ft. per sec., and the force exerted on the vane will be proportional to the change of momentum, that is, $\frac{W}{g}(V - -V)$.

$$\text{Force } \mathrm{P} = \frac{\mathrm{W}}{\mathrm{g}}(\mathrm{V}+\mathrm{V}) = \frac{2\mathrm{WV}}{\mathrm{g}}.$$

When the velocity of the jet is not completely reversed, the active component of this velocity acts along the axis of the cup (Fig. 218)

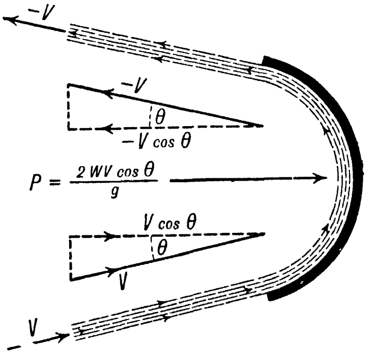


FIG. 218.

and is $V\cos\theta$ on projection and $-V\cos\theta$ after deviation. Thus

$$\text{Force } \mathrm{P} = \frac{2\mathrm{WV}\cos\theta}{\mathrm{g}}.$$

Application to the Pelton wheel. In the case of the Pelton wheel, the vane has a velocity of V_1 ft. per sec. in the direction of the jet

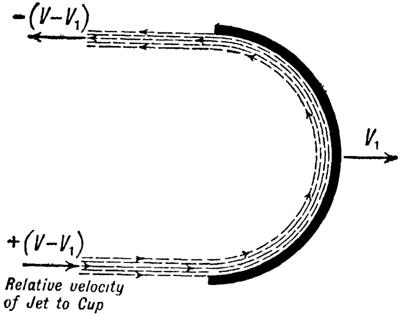


FIG. 219.

(Fig. 219). The relative velocity of impact is $V - V_1$ ft. per sec., where V is the velocity of the jet, and, neglecting friction, this is also the relative velocity with which the jet travels around the cup before leaving it. If the deviation is 180°, and taking the direction of motion of the cup as positive, the absolute velocity at discharge is $V_1 - \{V - V_1\}$ or $2V_1 - V$; because :

Velocity of cup − relative velocity of water to cup
= actual velocity of water.

The total change of velocity per lb. of water entering and leaving the cup will be : Initial velocity of water minus final velocity of water, or $V - [V_1 - \{V - V_1\}]$, that is, $2V - 2V_1$.

If, as is usually the case, the velocity of the jet is twice that of the cup, *i.e.* $V = 2V_1$, then the absolute velocity of discharge becomes $V_1 - \{2V_1 - V_1\}$ or zero.

Thus, the hydraulic efficiency of such a Pelton wheel is 100%, and the actual efficiency is between 75 and 90%.

The Pelton wheel can be used where jets of water at high velocities can be produced, and the wheel can be operated with comparatively small quantities of water.

Hydraulic efficiency and horse-power of a Pelton wheel. The efficiency of a Pelton wheel can be taken from two aspects : (*a*) the hydraulic efficiency or hydraulic performance ; (*b*) the overall efficiency, which includes those losses outside the hydraulic losses.

The hydraulic efficiency is similar to that of any other machine, that is, it is the ratio $\frac{\text{work output}}{\text{work input}}$ measured on its performance.

This can be expressed in terms of the jet energy as

$$\frac{\text{initial K.E.} - \text{final K.E.}}{\text{initial K.E.}};$$

that is, if W lb. of water be supplied per second and V is the velocity of the jet and V_1 the velocity of the buckets in feet per second,

$$e = \frac{\frac{WV^2}{2g} - \frac{W(2V_1 - V)^2}{2g}}{\frac{WV^2}{2g}}$$

$$= \frac{V^2 - V^2 + 4VV_1 - 4V_1^2}{V^2}$$

$$= \frac{4V_1(V - V_1)}{V^2}.$$

For maximum efficiency, $4V_1(V - V_1)$ must be a maximum.

Let $y = 4VV_1 - 4V_1^2$,

$$\frac{dy}{dV_1} = 4V - 8V_1;$$

which for a maximum is equal to zero;

that is, $4V - 8V_1 = 0$,

or $V = 2V_1$.

Thus, the maximum efficiency is obtained when the velocity of the jet is twice that of the vanes, and this efficiency is $\frac{4V_1(2V_1 - V_1)}{(2V_1)^2}$ $= 1$ or 100%.

Let W lb. of water strike the cups per second.

Then Horse Power $= \frac{\text{change of K.E. per sec.}}{550}$

$$\frac{\text{initial K.E.} - \text{final K.E.}}{550}$$

$$= \frac{WV^2 - W(V - 2V_1)^2}{2g \times 550}$$

$$\textbf{H.P.} = \frac{4WV_1(V - V_1)}{2g \times 550} = \frac{2WV_1(V - V_1)}{550g}.$$

Example 1. *Calculate the hydraulic efficiency of a Pelton wheel admitting water in a jet at* 130 *ft. per sec., and in which the speed of the cups is* 50 *ft. per sec.*

$$e=\frac{4V_1(V-V_1)}{V^2}=\frac{4\times50(130-50)}{130^2}$$
$$=\frac{16{,}000}{16{,}900}=94{\cdot}67\%.$$

Example 2. *The speed of the buckets of a Pelton wheel is* 100 *ft. per sec. and the jet speed is* 220 *ft. per sec. Calculate the efficiency of the wheel and the H.P. if the jet is* 2 *inches diameter.*

$$\text{Efficiency}=\frac{4V_1(V-V_1)}{V^2},$$
$$V=220,\ V_1=100 \text{ ft. per sec.}$$
$$e=\frac{4\times100(220-100)}{220^2}$$
$$=\frac{400\times120}{48{,}400}=\frac{480}{484}=99{\cdot}2\%.$$
$$\text{H.P.}=\frac{\text{change of kinetic energy in ft. lb. per sec.}}{550}$$
$$=\frac{2WV_1(V-V_1)}{g\times550},$$

in which
$$W=\frac{V\times\pi\times1^2}{144}\times62{\cdot}3 \text{ lb. per sec.}$$
$$=\frac{220\times\pi\times62{\cdot}3}{144}=298{\cdot}9 \text{ lb.}$$
$$\text{H.P.}=\frac{2\times298{\cdot}9\times100(120)}{32{\cdot}2\times550}$$
$$=405{\cdot}2.$$

Example 3. *A test of a* 14 *in. diameter Pelton wheel gave the following results: Weight of water passing through the nozzle per second,* 7·94 *lb.; velocity of issuing water,* 86·9 *ft. per sec.; revolutions of the wheel per minute,* 620; *brake horse power as measured on a rope brake,* 0·97. *Calculate (a) theoretical H.P., (b) hydraulic efficiency, (c) overall efficiency.*

Velocity of jet, V $=86{\cdot}9$ ft. per sec.

Weight of water per sec., $W=7{\cdot}94$ lb.

Velocity of cups $=\dfrac{620\times14\times\pi}{12\times60}=V_1$

$=37{\cdot}9$ ft. per sec.

(*a*) Theoretical H.P. $$=\frac{2WV_1(V-V_1)}{g\times 550}$$

$$=\frac{2\times 7\cdot 94\times 37\cdot 9\times 49}{32\cdot 2\times 550}=\mathbf{1\cdot 66}.$$

(*b*) Hydraulic efficiency $$=\frac{4V_1(V-V_1)}{V^2}$$

$$=\frac{4\times 37\cdot 9\times 49}{86\cdot 9^2}=98\cdot 3\%.$$

(*c*) Overall efficiency $$=\frac{\text{B.H.P.}}{\text{theoretical H.P.}}$$

$$=\frac{0\cdot 97}{1\cdot 66}=58\cdot 2\%.$$

Example 4. *A Pelton wheel is to develop* 450 *H.P. The velocity of the jet is* 180 *ft. per sec. and that of the buckets* 90 *ft. per sec. Calculate the number of jets each* 2 *in. in diameter if the mechanical efficiency is to be* 75%.

$$\text{Theoretical H.P.}=\frac{100}{75}\times 450-600.$$

$$600=\frac{2WV_1(V-V_1)}{g\times 550}=\frac{2W\times 90\times 90}{32\cdot 2\times 550}$$

$$W=\frac{600\times 32\cdot 2\times 550}{2\times 90\times 90}=656\text{ lb. per sec.}$$

Sectional area of one jet $=\frac{\pi\times 1^2}{144}$ sq. ft.

Weight of water per sec. per jet

$$=\frac{\pi}{144}\times\frac{180}{1}\times 62\cdot 3=244\cdot 6\text{ lb.}$$

$$\text{No. of jets}=\frac{656}{244\cdot 6}=2\cdot 68;$$

hence, **three jets** will be required.

EXPT. 6. Efficiency of a Pelton wheel.

OBJECTS. *To obtain*: (*a*) *the Brake Horse-Power of the wheel*, (*b*) *the Horse-Power given to the wheel by the water*, (*c*) *the hydraulic and overall efficiencies.*

APPARATUS. A suitable size of Pelton wheel is fitted with a brake drum on the shaft extension. This drum is then rigged as an absorption dynamometer by means of a leather, or balata, band, a

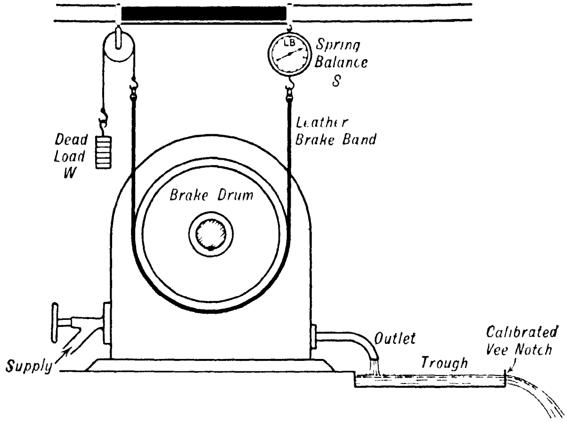


FIG. 220.

dead weight and a spring balance as shown in Fig. 220. Arrangements are then made to measure the quantity of water passing through the wheel in a given time, either by direct collection into a calibrated tank or by fitting a Vee notch, previously calibrated, to a trough on the exit side of the machine. If the latter method is used and the water in the trough is baffled to a steady head h, Q can be calculated from the formula $Q = K\frac{8}{15}\sqrt{2g}h^{5/2}$ if the notch is right-angled and K is previously determined.

METHOD OF PROCEDURE. Certain measurements will have to be taken while the wheel is opened up, or alternatively taken from the specification. These are (*a*) the diameter of the nozzle and (*b*) the mean diameter of the wheel on the bucket periphery. Calculate the area of the nozzle cross-section, and run the machine until conditions are normal. Next load the brake until it takes up the power output and produces a steady speed on the wheel. Take the following observations: (1) diameter of brake drum to centre of band; (2) dead load on brake; (3) spring balance reading; (4) speed of wheel in r.p.m.

The **Brake Horse-Power** is then obtained from the expression

$$\text{B.H.P.} = \frac{(\text{W} - \text{S})(\pi\text{D})\text{N}}{33{,}000},$$

where W and S are the dead load and spring balance readings, D is the diameter of the brake drum measured to the belt centre and N the number of revolutions per minute.

The **Horse-Power given to the wheel by the water** can be obtained by finding the weight of water delivered to the wheel per second, the velocity of the jet and the peripheral velocity of the wheel. From the Vee notch, or by direct measurement, the quantity of water Q can be obtained in cu. ft. per sec. and the velocity of the jet deduced by dividing this quantity by the area of cross-section of the jet in sq. ft. The peripheral velocity of the buckets is obtained by multiplying the circumference of the wheel by the speed in rev. per sec.

If V is the velocity of the jet, V_1 the velocity of the buckets and Q the quantity of water delivered per second, this H.P. is obtained from the expression $\frac{4 \times 62{\cdot}3QV_1(V - V_1)}{2g \times 550}$.

The **hydraulic efficiency** is given by the formula $\frac{4V_1(V - V_1)}{V^2}$.

The **overall efficiency** is the ratio between the B.H.P. and the horse-power given by the water, that is,

$$\frac{\text{B.H.P.}}{\text{Horse-power given by the water}}.$$

OBSERVATIONS.

Diameter of wheel = 18 in. = 1·5 ft.

,, ,, nozzle = $\frac{1}{2}$ in. = 0·0417 ft.

,, ,, brake drum = 12 in. = 1 ft.

N = speed of rotation = 342 r.p.m.

S = spring balance reading = 4·1 lb.

W = dead weight = 20 lb.

Quantity of water in one minute = 5·9 cu. ft.

Q = ,, ,, ,, ,, second $= \frac{5{\cdot}9}{60} = 0{\cdot}098$ cu. ft.

DERIVED RESULTS.

a = area of nozzle $= \frac{\pi}{16 \times 144} = 0{\cdot}001363$ sq. ft.

V = Velocity of jet $= \frac{Q}{a} = \frac{0{\cdot}098}{0{\cdot}001363} = 71{\cdot}9$ ft. per sec.

V_1 = ,, ,, buckets $= \frac{\pi \times 1{\cdot}5 \times 342}{60} = 26{\cdot}86$ ft. per sec.

$$\textbf{H.P. given by water} = \frac{4 \times 62{\cdot}3 \times QV_1(V - V_1)}{2g \times 550}$$

$$= \frac{4 \times 62{\cdot}3 \times 0{\cdot}098 \times 26{\cdot}86(71{\cdot}9 - 26{\cdot}86)}{2g \times 550}$$

$$= 0{\cdot}8341.$$

$$\textbf{Brake Horse-Power} = \frac{\pi \times D \times (W - S) \times N}{33{,}000}$$

$$= \frac{\pi \times 1 \times (20 - 4{\cdot}1) \times 342}{33{,}000}$$

$$= 0\ 5176.$$

$$\textbf{Hydraulic efficiency} = \frac{4V_1(V - V_1)}{V^2}$$

$$= \frac{4 \times 26{\cdot}86(71{\cdot}9 - 26{\cdot}86)}{71{\cdot}9^2}$$

$$= 93{\cdot}61\%.$$

$$\textbf{Overall efficiency} = \frac{\text{B.H.P.}}{\text{H.P. given by water}}$$

$$= \frac{0{\cdot}5176}{0{\cdot}8341} = 62{\cdot}04\%.$$

Water turbines. The full treatment of water turbines is beyond the scope of this book, but a general idea of their operation and classification may prove of interest.

A turbine consists of a **rotor** to which is attached a succession of shaped vanes, and which rotates about a central shaft bearing: and a **stator**, which surrounds the rotor and contains a series of guide passages or stationary vanes to direct the jets of water on to the rotor vanes. In almost all turbines high-velocity jets of water are used, and this water is directed on to a series of vanes, carefully designed to eliminate shock.

Broadly speaking, turbines may be classified into two types, **impulse** and **reaction** turbines. The impulse turbine is operated by the impulse due to jets of water striking the vanes and producing a turning moment on the rotor. There must be ample air space between the vanes to allow of directed movement of the jet, and the turbine must *discharge into the atmosphere.*

In the reaction turbine, the space between the vanes must be completely filled with water, in order that this water may exert a reaction on the turbine vanes and thus produce a turning moment on the rotor. Such a turbine need not discharge into atmosphere and may be immersed or *drowned.*

There is a secondary classification of turbines according to the direction of flow of the water; for example, *outward flow,* in which the flow is radial and outwards, and *inward flow,* where the flow is radial and inwards. Other types are the *axial flow,* where the jets are directed parallel to the axis of the turbine, and the *mixed flow,* where the flow is partly axial and partly radial.

EXERCISES ON CHAPTER XIV

1. Explain the energy change in a hydraulic prime mover and indicate how a jet of water may be made to impinge on vanes and provide a turning moment on the rotor to which the vanes are attached.

2. A jet of water 4 in. in diameter strikes a stationary flat plate with a velocity of 80 ft. per sec. Calculate the force exerted upon the plate.

3. If the jet in Question 2 strikes a plate which has a velocity of 20 ft. per sec. in the direction of the jet, calculate (*a*) the force exerted on the plate ; (*b*) the H.P. available ; (*c*) the efficiency.

4. Show that the maximum efficiency is obtained when the speed of a jet striking a single flat plate is three times that of the plate. What is this efficiency?

5. Calculate the quantity of water to be delivered per second by a jet which impinges with a velocity of 50 ft. per sec. on a moving flat plate in order that 10 H.P. may be developed at maximum efficiency.

6. What is the effect upon the efficiency of using a succession of flat plates moving in the direction of a jet? Show that the best efficiency is obtained in this case when the speed of the jet is double that of the vanes.

7. A jet of water 4 in. in diameter delivers on to a succession of flat vanes at 40 ft. per sec. The vanes have a velocity in the direction of the jet of 20 ft. per sec. Calculate (*a*) the pressure on the vanes ; (*b*) the H.P. available ; (*c*) the efficiency.

8. Find the quantity of water to be delivered at 20 ft. per sec. in order that a machine consisting of a succession of flat vanes may develop 4 horse-power at maximum efficiency. What will be the periphery diameter of the rotor if the speed is 60 revs. per minute?

9. A wheel fitted with a succession of flat plates rotates at 40 revs. per minute and has a periphery diameter of 4 ft. 6 in. Calculate the velocity of the jet for maximum efficiency and the horse-power of this machine if the supply of water is 1000 lb. per second.

10. Find the force exerted by a jet 4 in. diameter moving at 100 ft. per sec. and striking a stationary flat plate inclined at 50° to the axis of the jet.

11. Calculate the stiffness of the retaining spring for a plate valve which is to be opened 1 in. by a jet of water impinging upon it with a velocity of 80 ft. per sec. at 60° to the plate. Diameter of jet, 2 in.

12. Show by a diagram how the tip of a vane should be designed in order that water may enter the vane without shock when the vane is moving in the direction of the jet.

13. Determine the angle of the tip of a vane moving at 30 ft. per second in a direction inclined at 35° to the direction of a jet moving at 60 ft. per second if there is to be no shock at inlet.

14. What is meant by the absolute velocity of discharge from a shaped vane? Water for ejection to the tail-race approaches the tip of the vane of a hydraulic machine at 26 ft. per sec. If the vane is itself moving with a tip velocity of 15 ft. per sec. at 120° to the water in the vane, calculate and determine the direction of the absolute velocity of discharge.

15. Calculate the force exerted upon a vane which receives and discharges water at 40 ft. per sec. when the angle between the tangents at inlet and outlet is 244° measured clockwise. Diameter of jet is 5 in.

16. A 3-in. water pipe has a right-angled bend. Determine the vector change of velocity and momentum per second at the bend when 90 gallons of water flow through the pipe per minute. Of what value is the determination of this vector change of momentum in practice? (U.L.C.I.)

17. What is meant by velocity of whirl? Why is it necessary for the absolute velocity of discharge from a moving vane to be at right angles to the tangent to the wheel at the point of the vane?

18. A vane is to be designed to give no shock at entry and exit to a jet moving at 50 ft. per sec. and delivering 10 cu. ft. of water per sec. If the vane has a velocity of 25 ft. per sec. at 42° to the jet at inlet, find (*a*) the direction of the vane tip at inlet; (*b*) the velocity of the stream relative to that of the vane at inlet; (*c*) the direction of the vane at exit in order to give no velocity of whirl; (*d*) the absolute velocity of discharge; (*e*) the force exerted on the vane in lb.; (*f*) the direction of this force.

19. Write an account of the principal types of water prime movers and distinguish clearly between those working by (*a*) gravity ; (*b*) impulse ; (*c*) reaction.

20. Explain the term " reaction of a jet " and show how this reaction may be employed to exert a force upon a rotating vane.

21. Describe with suitable sketches the overshot wheel. An overshot wheel is supplied with 40 cu. ft. of water per second under a head of 14 ft. Calculate the theoretical horse-power of the wheel.

22. Explain the essential differences in operation between an overshot and breast wheel. What head will be required to produce a horse-power of 30 on a breast wheel to which is supplied 20 cu. ft. of water per second?

23. What is meant by an undershot wheel and in what essential respects does it differ from overshot and breast wheels? Name the variation made by Poncelet to this type of wheel, and explain the advantages which accrue from this variation.

24. The following particulars apply to a Poncelet wheel : velocity of stream, 8 ft. per sec. ; velocity of vane tip at entrance, 4 ft. per sec. at 30° to the stream ; angle between vane tip tangent at outlet and the direction of the stream at inlet, 12°. Find (*a*) the direction of the vane tip at entrance ; (*b*) the relative velocity of the water to the vane tip at entrance ; (*c*) the absolute velocity of discharge.

25. Describe briefly a Pelton wheel, and trace the path of the jet through the buckets to discharge. What velocity changes take place during the progress of the water through the buckets?

26. The speed of the buckets of a Pelton wheel is 150 ft. per sec. and the jet speed is 320 ft. per sec. Calculate the efficiency of the wheel and the horse-power if the jet diameter is $1\frac{1}{2}$ in.

27. Four jets each 2 in. in diameter operate on the buckets of a Pelton wheel under a head of 420 ft. Calculate the horse-power and efficiency of the wheel if its diameter is 5 ft. 6 in. and it has a speed of 300 r.p.m.

28. A jet of water impinges upon a series of hemispherical cups moving in the same direction and is deflected through 180°. Obtain an expression for the pressure on the cups. If the velocity of the jet is 100 ft. per sec., and that of the cups 40 ft. per sec., find the work done per lb. of water striking the cups. (U.E.I.)

29. A Pelton wheel receives 220 cu. ft. of water per minute under a head of 400 ft. and its diameter is 2 ft. 6 in. Determine its horse-power and efficiency at 520 r.p.m.

30. A test of a 12 in. Pelton wheel gave the following results : weight of water passing through the nozzle per second, 6·15 lb. ; velocity of water issuing from the nozzle, 72·5 ft. per sec. ; revolutions of wheel per minute, 310 ; brake horse power 0·46. Find the horse-power given to the wheel by the water and the efficiency of the wheel. (U.L.C.I.)

31. A Pelton wheel is to develop a brake horse-power of 920. The velocity of each jet is 230 ft. per sec. and that of the buckets 110 ft. per sec. Calculate the number of jets each $1\frac{1}{2}$ in. diameter if the mechanical efficiency is to be 70%.

32. A Pelton wheel of 3200 horse-power is driven by a jet of water $5\frac{1}{2}$ in. in diameter, which issues from a nozzle under an effective head of 850 ft. Determine the velocity of the jet, the weight of water issuing from the nozzle per minute, and the efficiency of the wheel. (U.L.C.I.)

33. One hundred cubic feet of water per minute under a head of 462 ft. are supplied to a Pelton wheel 2 ft. in diameter. Determine the horse-power of the wheel and its efficiency when running at 600 r.p.m. (U.L.C.I.)

34. A Pelton water wheel is supplied with 350 lb. of water per minute. If the diameter of the nozzle is 0·4 inch, determine (*a*) the velocity of the water issuing from the nozzle ; (*b*) the number of foot pounds of energy possessed by the water leaving the nozzle per minute. The horse-power developed is measured by a rope brake the effective diameter of which is 12 inches, and the effective load on which is 9·5 lb. at 550 r.p.m. Calculate the brake horse-power and the efficiency of the wheel under these conditions. (U.E.I.)

35. In a test on a Pelton wheel fitted with a brake band, the nozzle area and the pressure head of water were kept constant and the following data were obtained : diameter of nozzle, 0·4 in. ; diameter of brake wheel, 10·5 in. ; head of water, 95 feet ; weight of water used, 240 lb. per min. ; effective brake load, 18 lb. ; speed of wheel, 310 r.p.m. Determine (*a*) the velocity of the issuing jet ; (*b*) the horse-power supplied to the wheel ; (*c*) the efficiency of the wheel, if 1 cu. ft. of water weighs 62·3 lb. (U.E.I.)

36. A Pelton wheel 2 ft. in diameter operates under a head of 260 ft. and makes 570 r.p.m. with a supply of 100 lb. of water per minute. Calculate (*a*) the theoretical horse-power ; (*b*) the hydraulic efficiency of the wheel. If this wheel is supplied with a band brake in which the coefficient of friction is 0·5 and angle of lap 230°, find the ratio of tensions in the band and the value of these tensions if the diameter of the drum is 1 ft. 4 in. and the mechanical efficiency 65%.

HEAT ENGINES

CHAPTER XV

ENERGY ASPECT OF AN ENGINE, HEAT QUANTITIES, THE GAS LAWS AND CHARACTERISTIC EQUATIONS, WORKING CYCLES

THE term **heat engine** is generally applied to a contrivance or machine which converts heat energy into mechanical energy and makes this energy available for the performance of useful work. Broadly, heat engines may be classified into four groups :

(*a*) reciprocating steam engines,
(*b*) steam turbines,
(*c*) internal combustion engines,
(*d*) mechanical refrigeration machines.

The first three groups can be definitely associated with the general conception of a heat engine ; that is, they receive heat energy and convert it into mechanical energy ; whereas the fourth group, refrigeration machines, are supplied with mechanical energy and reverse the process by converting this mechanical energy to heat energy.

Working substances. In all forms of engines a **working substance** is employed, the nature of which is predetermined by the conditions imposed upon it in the operation of the engine. It may be accepted that the working substance is always a fluid, that is, a liquid which may be volatilised, or a gas. In the selection of a suitable substance, the following points have to be considered. The fluid chosen must be :

(*a*) Capable of receiving and rejecting heat energy in large quantities ;

(*b*) adaptable to changes of pressure and temperature within the scope of the engine;

(*c*) chemically stable, where necessary, under the temperature and pressure changes.

In the steam engine and steam turbine groups the working substance is steam, whereas in the internal combustion engines a mixture of a hydrocarbon gas and air is employed, and the gaseous products of the combustion of this mixture form the working substance. Refrigerators also require a fluid; one of the following is generally employed: ammonia, carbon dioxide or sulphur dioxide.

Cycle of operations. In all types of engines the working substance is subjected to a series of changing conditions; for example, changes in temperature and pressure, changes of state or the explosion of an explosive mixture. This series of changes is known as the **cycle of operations,** or, briefly, as the **cycle.**

In Volume II considerable attention has been given to the mechanical construction of various types of engines, the behaviour of the working substances and the energy changes involved in them. The purpose, at this stage, is to study the performance of the engines under varying conditions and to examine the theory in a more detailed manner.

RECIPROCATING STEAM ENGINES AND STEAM TURBINES

These types of engines use steam as their working substance, and the steam is prepared with a high heat content in a separate unit, known as the boiler. The function of the engine or turbine is to extract the heat energy from this steam and to convert it into mechanical energy in such a manner that the engine is available for the performance of work, that is, as a **prime mover.**

Conditions of steam. The production of steam in the boiler is affected by the requirements of the engine, and the steam, on supply, is said to possess a certain condition, which may be (*a*) *wet*, (*b*) *dry saturated*, or (*c*) *superheated* at the pressure for which the boiler is designed.

When steam is produced from water it is taken through several heating stages.

(1) The water is supplied with **sensible heat** until it reaches a temperature at which the formation of steam takes place, *a temperature which depends upon the pressure maintained in the boiler.*

(2) The water at the pressure and temperature of steam formation is provided with **latent heat**, which changes the state of the water from liquid to the vapour (steam) at the same temperature and pressure.

(3) The steam so formed may be further heated, or **superheated**, to a temperature greater than that at which steam is formed. The heat provided at this stage is known as the **heat of superheat.**

If the water is wholly converted to steam during the second stage, that is, the steam so formed is entirely free from unevaporated water in suspension, this steam is known as **dry saturated steam.** If, on the other hand, during this stage steam is produced which has unevaporated water in suspension, this steam is called **wet steam**; it has only received that portion of the latent heat required to convert a part of its content into steam. The condition of this steam is specified as having a **dryness fraction,** which is represented by the ratio

$$\frac{\textit{Weight of steam actually formed}}{\textit{Weight of wet steam}}$$

for any given quantity of wet steam. For example, if a quantity of wet steam weighing 10 lb. has a dryness fraction of 0·9, this steam will consist of $\frac{9}{10}$ of 10 lb. actual steam and $\frac{1}{10}$ of 10 lb. water in suspension. It follows that this 10 lb. of wet steam will have received only $\frac{9}{10}$ of the latent heat required to change its state from liquid to vapour.

Before superheating can be achieved, the steam must be first converted to a dry saturated state at the pressure and temperature of formation, after which it receives heat as a vapour or gas at constant pressure. The specific heat at constant pressure, c_p, for steam has a value between 0·48 and 0·66 according to pressure.

Units of heat. The measurement of heat quantities is made, either in Centigrade Heat Units or British Thermal Units.

The Centigrade Heat Unit (C.H.U.) is the mean heat required to raise the temperature of 1 lb. of water one degree Centigrade.

The British Thermal Unit (B.Th.U.) is the mean heat required to raise the temperature of 1 lb. of water one degree Fahrenheit.

The two units are of course convertible, and *one C.H.U. is equal to* $\frac{9}{5}$ *of one B.Th.U.*

Summary of results. The heat content of one pound of steam under different conditions may be summarised as follows, where :

H is the total heat per 1 lb. in C.H.U.
h is the sensible heat per lb. in C.H.U.
L is the latent heat per lb. in C.H.U.
x is the dryness fraction.
$t_{(\text{sat.})}$ is the temperature of formation in Centigrade degrees.
$t_{(\text{sup.})}$ is the temperature of superheat in Centigrade degrees.
c_p is the specific heat at constant pressure.

Condition	Total Heat	Total Heat in C.H.U. per lb.
Water at temperature $t_{(\text{sat.})}$ dry saturated steam	Sensible	$t_{(\text{sat.})}-0=\mathbf{h}$
At a temperature of $t_{(\text{sat})}$	Sensible + latent	$t_{(\text{sat.})}+L=\mathbf{h}+\mathbf{L}$
Wet steam of dryness fraction x	Sensible + x (latent)	$t_{(\text{sat.})}+xL=\mathbf{h}+\mathbf{xL}$
Superheated to $t_{(\text{sup.})}$	Sensible + latent + superheat	$t_{(\text{sat.})}+L+c_p(t_{\text{sup.}}-t_{\text{sat.}})$ $\mathbf{h}+\mathbf{L}+\mathbf{c}_p(\mathbf{t}_{\text{sup}}-\mathbf{t}_{\text{sat.}})$

If specific heat is treated as a ratio, then c_p is actually the ratio of the quantity of heat required to raise a definite mass of gas through a given temperature range at constant pressure to the quantity of heat required to raise the same mass of water through the same temperature range.

The specific heat of steam at constant pressure is the heat required to raise the temperature of 1 *lb. of dry saturated steam* 1° *Centigrade at constant pressure.*

NOTE. The value of h, as determined from steam tables, is not strictly equal to $(t_{\text{sat.}}-0)$, a variation which is due to the fluctuation of the specific heat of water during the process of heating.

Example 1. *Steam at a pressure of* 200 *lb. per square inch absolute is supplied to a turbine at a temperature of* 350° *C. and exhausted with a dryness fraction of* 0·85 *into a condenser maintained at a pressure of* 1·5 *lb. per square inch absolute. Calculate the heat drop per lb. if the mean specific heat at constant pressure is* 0·62.

From steam tables :

Temperature of formation at 200 lb. per sq. in. $= t_{\text{sat.}} = 194{\cdot}3°$ C.

Extent of superheat $= t_{\text{sup.}} - t_{\text{sat.}} = 350 - 194{\cdot}3 = 155{\cdot}7$ Cen. degrees.

$$\text{Total heat on supply} = H = h + L + 0{\cdot}62 \times 155{\cdot}7$$
$$= 197{\cdot}5 + 471{\cdot}2 + 96{\cdot}53.$$
$$H_1 = 765{\cdot}23 \text{ C.H.U. per lb.}$$

Temperature of rejection $= 46{\cdot}5°$ C.

Latent heat at 1·5 lb. per sq. in. $= 569{\cdot}65$ C.H.U. per lb.

$$\text{Total heat on rejection} = 46{\cdot}4 + 0{\cdot}85 \times 569{\cdot}65$$
$$= 46{\cdot}4 + 484{\cdot}2.$$
$$H_2 = 530{\cdot}6 \text{ C.H.U. per lb.}$$
$$\text{Heat drop} = H_1 - H_2$$
$$= 765{\cdot}23 - 530{\cdot}6$$
$$= 234{\cdot}6 \text{ C.H.U. per lb.}$$

Example 2. *Steam is produced by a boiler at* 170 *lb. per sq. in. absolute with a dryness fraction of* 0·92. *Calculate the quantity of heat which has to be supplied by the superheater per lb. of steam in order to establish a condition of* 100 *Centigrade degrees of superheat.*

Condition (*a*) : $t_{\text{sat}} = 186{\cdot}9°$ C., $L = 477{\cdot}6$.

$$H_1 = 189{\cdot}5 + 0{\cdot}92L$$
$$= 189{\cdot}5 + 439{\cdot}4 = 628{\cdot}9 \text{ C.H.U. per lb.}$$

Condition (*b*) : $t_{\text{sup.}} - t_{\text{sat.}} = 100°$ C.

From tables, $C_p = 0{\cdot}6053$,

$$H_2 = 189{\cdot}5 + 477{\cdot}6 + 0{\cdot}6053 \times 100$$
$$= 727{\cdot}6 \text{ C.H.U. per lb.}$$
$$\text{Heat to be supplied} = H_2 - H_1$$
$$= 727{\cdot}6 - 626{\cdot}3$$
$$= 101{\cdot}3 \text{ C.H.U. per lb.}$$

First Law of Thermodynamics. This law states that **heat and mechanical energy are mutually interchangeable, and that there is a rate of exchange between the two forms of energy.**

The rate of exchange between heat and mechanical energy is known as the mechanical equivalent of heat, or Joule's equivalent. In the foot pound Centigrade scale,

1 Centigrade Heat Unit is equivalent to 1400 ft. lb. of mechanical energy;

or, on the Fahrenheit scale,

1 British Thermal Unit is equivalent to 778 ft. lb. of mechanical energy.

Second Law of Thermodynamics. This law states that **if two bodies or substances are in contact, heat will flow from the body at the higher temperature to that at the lower temperature, until the temperatures are equal.**

For example, suppose steam is exhausted to a condenser which is supplied with cooling water at a lower temperature than the steam. Heat will flow from the steam, and be absorbed by the cooling water until their temperatures are equal. In practice, the rate of flow of the water is too fast for a complete equalisation of temperature, but the cooling water receives the bulk of the heat of the steam and thus produces condensation. Similarly, one of the major losses in an internal combustion engine is due to the transfer of heat from the hot gases through the cylinder walls to the cooling water, or air if it is an air-cooled engine, and although cooling is necessary for mechanical reasons it is a direct loss of energy.

Example 1. *Calculate the steam consumption in lb. per hour of a* 100 *H.P. engine receiving dry saturated steam at* 160 *lb. per sq. in. absolute and exhausting into a condenser at a pressure of* 1·25 *lb. per sq. in. absolute if the exhaust steam has a dryness fraction of* 0·9 *and* 14% *of the available heat energy is converted into mechanical energy.*

$$\text{Heat required per hour} = \frac{100 \times 33000}{1400} \times \frac{100}{14} \times \frac{60}{1} = 1{,}010{,}000 \text{ C.H.U.}$$

Condition (*a*): $H_1 = 666{\cdot}5$ C.H.U. per lb.

Condition (*b*): $H_2 = 42{\cdot}84 + 0{\cdot}9 \times 571{\cdot}56$

$= 42{\cdot}84 + 514{\cdot}4 = 557{\cdot}24$ C.H.U. per lb.

Heat drop per lb. = available heat energy $= H_1 - H_2$

$= 109{\cdot}26$ C.H.U. per lb.

Number of lb. of steam required

$$=\frac{1{,}010{,}000}{109{\cdot}26}=\mathbf{9{,}244\ lb.\ per\ hour.}$$

Example 2. *A turbine receives* 2210 *lb. of steam per hour at a pressure of* 200 *lb. per sq. inch with* 120 *Centigrade degrees of superheat. If exhaust is dry saturated at* 1·2 *lb. per sq. in. absolute, calculate* (*a*) *the heat drop per lb. of steam,* (*b*) *the theoretical velocity of the jet if the whole of this heat drop is converted to kinetic energy, and* (*c*) *the theoretical H.P. of the turbine if* 30% *of the available energy is converted into work.* (See Vol. II. p. 218.)

Condition (*a*) : $H_1=735{\cdot}5$ C.H.U. per lb. direct from steam tables for superheated steam.

Condition (*b*) : $H_2=614{\cdot}04$ C.H.U. per lb. from steam tables for saturated steam.

$$\text{Heat drop}=H_1-H_2=\mathbf{121{\cdot}46\ C.H.U.\ per\ lb.}$$

$$\text{K.E. of jet}=\frac{WV^2}{2g}=121{\cdot}46\times1400\text{ ft. lb.}$$

$$\frac{V^2}{2g}=121{\cdot}46\times1400.$$

$$\text{Velocity}=V=\sqrt{64{\cdot}4\times121{\cdot}46\times1400}$$

$$=\mathbf{3311\ ft.\ per\ sec.}$$

$$\text{Quantity of steam per minute}=\frac{2210}{60}=36{\cdot}83\text{ lb.}$$

$$\text{H.P}=\frac{36{\cdot}83\times\text{heat drop per lb.}\times1400\times30}{33000\times100}$$

$$=\frac{36{\cdot}83\times121{\cdot}46\times1400\times30}{33000\times100}$$

$$=\mathbf{56{\cdot}97\ H.P.}$$

Example 3. *The exhaust from a steam turbine is used as a make-up with live steam to supply heat for some factory processes. The live steam, which is dry and saturated and at a pressure of* 150 *lb. per sq. in. absolute, is expanded through a reducing valve to a pressure of* 60 *lb. per sq. in. absolute. The exhaust steam from the turbine is at a pressure of* 60 *lb. per sq. in. absolute and is* 2 *per cent. wet. If the proportion of live to exhaust steam is* 1·5 *to* 1, *find the final condition of the mixture. Assume the specific heat of superheated steam to be* 0·54. (U.E.I.)

Total heat per lb. of dry saturated steam at 150 lb. per sq. in. abs. $= 665{\cdot}8$ C.H.U., which is the total heat after expansion.

$$\text{Total heat per lb. of wet steam} = 145{\cdot}5 + 0{\cdot}98 \times 510{\cdot}1 \text{ C.H.U.}$$
$$= 645{\cdot}4 \text{ C.H.U.}$$
$$H \text{ per } 2{\cdot}5 \text{ lb. of mixture} = 1{\cdot}5 \times 665{\cdot}8 + 645{\cdot}4$$
$$= 1644{\cdot}1 \text{ C.H.U.}$$
$$H \text{ per 1 lb. of mixture} = 657{\cdot}6 \text{ C.H.U.}$$
$$\text{Heat of superheat} = 657{\cdot}6 - H_s \text{ at 60 lb. per sq. in.}$$
$$= 657{\cdot}6 - 655{\cdot}7 = 1{\cdot}9 \text{ C.H.U.}$$
$$\text{Degrees of superheat} = \frac{1{\cdot}9}{0{\cdot}54} = 3{\cdot}5 \text{ Centigrade degrees (approx.)}$$

Condition : Press. 60 lb. per in.^{2} abs. Temp. 148·3 Centigrade degrees with 3·5 Centigrade degrees of superheat.

Example 4. *What cooling surface would you expect in a surface condenser working under the following conditions?*

Shaft H.P. of turbine : 800.

Exhaust steam conditions in condenser : 1 *lb. per square inch absolute, dryness fraction* 0·95.

Temperature of condensate : 38° *C.*

Steam consumption : 13 *lb. per shaft horse-power per hour.*

Heat transferred per sq. ft. of cooling surface per minute under these conditions, 70 *C.H.U.* (U.E.I.)

$$\text{Heat to condenser per lb.} = 38{\cdot}62 + 0{\cdot}95(573{\cdot}84) - 38$$
$$= 545{\cdot}1 \text{ C.H.U.}$$
$$\text{Total heat to condenser per hour} = 800 \times 13 \times 545{\cdot}1 \text{ C.H.U.}$$
$$\text{Area of cooling surface} = \frac{800 \times 13 \times 545{\cdot}1}{70 \times 60} = \mathbf{1350 \text{ sq. ft.}}$$

Example 5. *What quantity of cooling water at a temperature of* 12° *C. is required per hour for a surface condenser to operate with an engine using* 30 *lb. of steam per H.P. per hour and developing* 100 *H.P., if the steam entering the condenser is* 87% *dry and the pressure in the condenser* 4 *lb. per sq. in. absolute. The temperature of the hot well discharge is* 70° *C. and of the outlet cooling water* 68° *C.*

$$\text{Quantity of steam per hour} = 30 \times 100 = 3000 \text{ lb.}$$
$$\text{Heat given up by steam} = 3000\{67{\cdot}1 + 0{\cdot}87 \times 558{\cdot}3 - 70\}$$
$$= 1{,}448{,}463 \text{ C.H.U.}$$

$$\begin{aligned}\text{Heat taken by cooling water} &= 1{,}448{,}463 \text{ C.H.U.}\\ &= w(68 - 12) \text{ C.H.U.}\\ 56w &= 1{,}448{,}463,\\ w &= 25{,}866 \text{ lb. per hour.}\end{aligned}$$

Example 6. *The exhaust steam from a* 1200 *kW. steam turbine exhausts to a condenser at a pressure of* 1·5 *lb. per sq. in. absolute and* 90% *dry.* 70,000 *gallons of cooling water are used per hour and the entry and exit temperatures are* 15° *C. and* 33° *C. Find the steam consumption of the turbine in lb. per kW. hour if the hot well temperature is* 46° *C.*

$$\begin{aligned}\text{Heat given to cooling water} &= 70{,}000 \times 10 \times (33 - 15)\\ &= 12{,}600{,}000 \text{ C.H.U. per hr.}\\ \text{Heat given up by steam} &= 1200w\{46{\cdot}37 + 0{\cdot}9 \times 569{\cdot}7 - 46\}\\ &= 615{,}720w \text{ C.H.U.}\end{aligned}$$

But $615{,}720w = 12{,}600{,}000.$

$\therefore$ **weight of steam $w = 20{\cdot}46$ lb. per kW. hour.**

Example 7. *Wet steam at* 180 *lb. per sq. in. abs. expands through a throttling calorimeter* (p. 93, Vol. II) *to atmospheric pressure and a temperature of* 114° *C. If the specific heat of superheated steam is* 0·48, *find the dryness fraction of the steam at supply.*

$$\begin{aligned}\text{Condition at supply, } H &= h + xL\\ &= 192{\cdot}3 + x \times 475{\cdot}3 \text{ C.H.U. per lb.}\\ H \text{ after throttling} &= H_{\text{sat.}} + (114 - 100)\ 0{\cdot}48\\ &= 639{\cdot}3 + 6{\cdot}7 = 646 \text{ C.H.U. per lb.}\\ 646 &= 192{\cdot}3 + 475{\cdot}3x,\\ x &= \frac{453{\cdot}7}{475{\cdot}3} = 95{\cdot}45\% \text{ dry.}\end{aligned}$$

Example 8. *The following data were obtained when a combined throttling and separating calorimeter* (p. 93, Vol. II) *was used to find the dryness fraction of a sample of steam taken from a main steam pipe. Water collected in separating calorimeter* 4·7 *lb., steam condensed after leaving throttling calorimeter* 51·4 *lb. Steam pressure* 160 *lb. per sq. in. abs., atmospheric pressure* 14·7 *lb. per sq. in. abs. Temperature of steam after throttling* 141° *C. at* 17 *lb. per sq. in. absolute. Find the dryness fraction, on supply.*

Total weight of steam = 51·4 + 4·7 = 56·1 lb., of which 51·4 lb. passes through the throttling calorimeter and 4·7 lb. of moisture is collected in the separating calorimeter.

Before throttling, $H = h + xL$.

At 160 lb. per sq. in., $H = 186{\cdot}7 + x \times 479{\cdot}8$ C.H.U. per lb.

After throttling, $H_{\text{sup}} = H_{\text{sat.}} + (141 - 104)\,0{\cdot}48$,

where 0·48 is taken as the specific heat of superheated steam.

$$H_{\text{sup.}} = 640{\cdot}94 + 17{\cdot}76 = 658{\cdot}7 \text{ C.H.U. per lb.}$$

$$658{\cdot}7 = 186{\cdot}7 + x \times 479{\cdot}8,$$

$$x = \frac{472}{479{\cdot}8} = 0{\cdot}9837.$$

Thus in each 1 lb. of steam entering the throttling calorimeter there is 1 − 0·9837 or 0·0163 lb. of water.

Hence in 51·4 lb. of steam there is 51·4 × 0·0163 or 0·8379 lb. of water.

Total water in 56·1 lb. of supply steam

$$= \text{separated water} + \text{throttled water}$$

$$= 4{\cdot}7 + 0{\cdot}8379 = 5{\cdot}5379 \text{ lb.}$$

$$\text{Dryness fraction on supply} = \frac{56{\cdot}1 - 5{\cdot}5379}{56{\cdot}1}$$

$$= 90{\cdot}1\%$$

BEHAVIOUR OF THE WORKING SUBSTANCE UNDER PRESSURE AND TEMPERATURE CHANGES

A perfect gas expanding in a cylinder *without change of temperature* would behave according to **Boyle's Law**; that is, the pressure at any stage in the expansion would be inversely proportional to the volume. This may be written algebraically as

$$\mathbf{PV = Constant},$$

where P is the pressure of the gas and V its volume. It seldom happens that a working substance obeys Boyle's Law, and a more general form of equation connecting pressure and volume is applicable. This equation is

$$\mathbf{PV^n = Constant},$$

where n is an index dependent upon the nature of the gas and the conditions under which the expansion occurs. If Boyle's Law is followed, n is equal to unity and the expression becomes

$$PV = \text{constant}.$$

A gas may take in heat without change of pressure, in which case the process is described as heating at constant pressure. If c_p is the specific heat of the gas at constant pressure, the heat is taken in at a rate equal to c_p Centigrade Heat Units or lb. calories per lb. of gas per Centigrade degree rise in temperature.

Example. *If the specific heat at constant pressure for steam at a certain pressure is* 0·6, *find the heat required to raise the temperature of this steam through* 40 *Centigrade degrees.*

$$\begin{aligned}\text{Heat required} &= 40 \times c_p \text{ per lb. of steam}\\ &= 40 \times 0{\cdot}6 = 24 \textbf{ C.H.U. per lb.}\end{aligned}$$

Similarly, heat may be absorbed at constant volume, and if c_v be the specific heat of the gas at constant volume, this heat is taken in at a rate equal to c_v lb. calories per lb. per Centigrade degree rise in temperature. The value of c_v is less than that of c_p, and the ratio $\gamma = \frac{c_p}{c_v}$ **is the ratio of the specific heats at constant pressure and constant volume and must exceed unity.** N.B. Since c_p and c_v are variable, γ is not constant.

If a perfect gas expands without taking in heat from external sources or giving out heat, that is *adiabatically*, the expression for the pressure-volume equation is $PV^n =$ Constant, where n is the ratio of the specific heats $\frac{c_p}{c_v} = \gamma$.

$$\therefore \ \mathbf{PV^{\gamma} = \text{constant}.}$$

Example. *Ten cubic feet of gas under a pressure of* 200 *lb. per sq. inch absolute expands adiabatically to a volume of* 15 *cu. ft. Calculate the final pressure if the ratio of the specific heats* $\frac{c_p}{c_v} = 1{\cdot}41$.

$$\begin{aligned}PV^{\gamma} &= \text{constant} = PV^{1\cdot41},\\ P_1V_1^{1\cdot41} &= P_2V_2^{1\cdot41},\\ 200 \times 10^{1\cdot41} &= P_2 \times 15^{1\cdot41},\\ P_2 &= \frac{200 \times 10^{1\cdot41}}{15^{1\cdot41}}\\ \log P_2 &= \log 200 + 1{\cdot}41 \log 10 - 1{\cdot}41 \log 15\\ &= 2{\cdot}3010 + 1{\cdot}41 - 1{\cdot}41 \times 1{\cdot}1761\\ &= 3{\cdot}7110 - 1{\cdot}6583 = 2{\cdot}0527.\end{aligned}$$

$$\mathbf{P_2 = 112{\cdot}9 \text{ lb. per sq. in. absolute.}}$$

In the consideration of the behaviour of steam as a working substance, the following values of n have been determined experimentally, and may be used in calculations involving the expansive working of steam.

(*a*) Dry saturated steam expanding and remaining dry saturated throughout the process : $\mathbf{n}=\frac{16}{15}$.

(*b*) Steam expanding without loss or gain in heat content, that is, *adiabatically* : $\mathbf{n}=\mathbf{1{\cdot}135}$.

(*c*) Steam expanding without change of temperature, that is, *isothermally* : $\mathbf{n}=\mathbf{1}$.

Ratio of expansion. When a working substance expands in a cylinder from a volume V_1 to a volume V_2, **the ratio of expansion is** $\frac{\mathbf{V_2}}{\mathbf{V_1}}=\mathbf{r}$, and the pressure-volume equation $PV^n=$constant for a given expansion becomes $P_1V_1^n=P_2V_2^n$ or

$$\frac{\mathbf{P_1}}{\mathbf{P_2}}=\left(\frac{\mathbf{V_2}}{\mathbf{V_1}}\right)^n=\mathbf{r}^n.$$

Example. *A gas expands from a volume of* 11 *cu. ft. to one of* 17 *cu. ft. according to the law* $PV^{1{\cdot}17}=$*constant. If the initial pressure is* 112 *lb. per sq. in. gauge, calculate the final pressure.*

$$\text{Ratio of expansion}=\tfrac{17}{11}=r=1{\cdot}546.$$

$$\frac{P_1}{P_2}=r^{1{\cdot}17}=\frac{112+14{\cdot}7}{P_2}=\frac{126{\cdot}7}{P_2}.$$

$$1{\cdot}17\log 1{\cdot}546=\log 126{\cdot}7-\log P_2.$$

$$1{\cdot}17\times 0{\cdot}1892=2{\cdot}1028-\log P_2,$$

$$\log P_2=2{\cdot}1028-0{\cdot}2214=1{\cdot}8814.$$

$$P_2=76{\cdot}1 \text{ lb. per sq. in. abs.,}$$

or **61·4 lb. per sq. in. gauge.**

Work done during expansion and compression. Consider a piston and cylinder (Fig. 221) in which a gas expands from volume V_1 cu. ft. to a volume V_2 cu. ft. under a pressure of P lb. If the pressure remains constant throughout the expansion, the work done will be $P(V_2-V_1)$ ft. lb., and the work diagram will be a rectangle the area of which will represent this work done. In an actual engine,

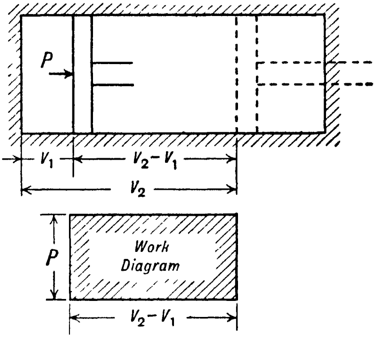


FIG. 221.

the pressure falls with the increase in volume, and the law governing the pressure and volume changes is $PV^n = \text{constant}$, where n may be (*a*) unity, for isothermal expansion, or (*b*) a value dependent upon the nature of the working substance and the conditions of working.

Case (*a*), where PV = constant or n is unity. Consider a cylinder into which a gas is admitted under an initial pressure P and with an initial volume V_1. Let the gas expand according to the law

$$PV = \text{constant}$$

until a final pressure P_2 is reached with a volume of V_2. Then the area shaded beneath the curve (Fig. 222), a rectangular hyperbola, represents the work done during expansion.

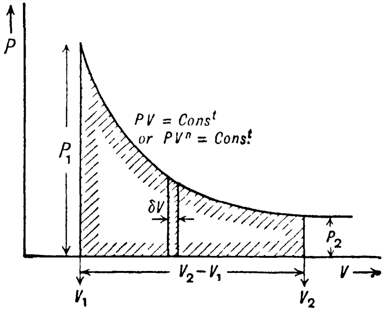


FIG. 222.

Take an elementary strip of width δv, where the pressure is p and the volume v.

$$\text{Area of this strip} = p\,\delta v.$$

$$\text{Total area} = \int_{V_1}^{V_2} p\,\delta v$$

Since $$pv = \text{constant} = K,$$

$$p = \frac{\text{constant}}{v} = \frac{K}{v}.$$

Hence $$\text{area} = K\int_{V_1}^{V_2} \frac{dv}{v} = K\Big[\log_e v\Big]_{V_1}^{V_2}.$$

Now $$K = PV,$$

$$\therefore\ \text{Area} = \text{work done} = PV\{\log_e V_2 - \log_e V_1\}$$

$$= PV\log_e\frac{V_2}{V_1} = \mathbf{PV\log_e r},$$

where r *is the ratio of expansion* $\dfrac{V_2}{V_1}$.

Case (*b*) where PV^n = constant. In this case the gas is admitted as before, and allowed to expand according to the law $PV^n = \text{constant}$.

Area of strip $= p\,\delta v$. Total area $= \int_{V_1}^{V_2} p\,\delta v$.

Since $$pv^n = \text{constant} = K,$$

$$p = \frac{K}{v^n} = Kv^{-n},$$

and $$\text{area} = K\int_{V_1}^{V_2} v^{-n}\,\delta v = K\left[\frac{v^{-n+1}}{-n+1}\right]_{V_1}^{V_2}$$

$$= K\left\{\frac{V_2^{-n+1} - V_1^{-n+1}}{-n+1}\right\}.$$

However, $K = P_1V_1^n = P_2V_2^n$.

$$\text{Area} = \frac{P_2V_2^nV_2^{-n+1} - P_1V_1^nV_1^{-n+1}}{-n+1} = \frac{P_2V_2 - P_1V_1}{1-n}.$$

$$\textbf{Work done} = \frac{\mathbf{P_1V_1 - P_2V_2}}{\mathbf{n-1}}.$$

N.B. *When the gas is compressed, work has to be provided, and the work is done in compressing the gas to the smaller volume and greater pressure.*

Units of work done, pressure and volume. In the two expressions for the work done during the expansion of a gas, the pressure is taken as absolute pressure, that is, pressure measured above vacuum pressure. If the volumes are taken in *cubic feet*, and the pressures in *lb. per sq. ft. absolute*, the resulting work done will be in *ft. lb.*

Example 1. *Calculate the work done during the expansion of a gas according to the law* $PV = $ *constant if the initial volume is* 30 *cubic feet and the ratio of expansion* 7. *Initial pressure* = 120 *lb. per sq. in. gauge.*

$$\text{Pressure} = (120 + 14\cdot7)144 \text{ lb. per sq. ft. absolute}$$

$$PV = 30 \times 144 \times 134\cdot7.$$

$$\text{Work done} = PV \log_e r = PV \log_e 7$$

$$= 30 \times 144 \times 134\cdot7 \times 1\cdot9459$$

$$= \mathbf{1{,}132{,}000 \text{ ft. lb.}}$$

Example 2. *The compression ratio in a gas engine is* 4 *and the compression law is* $PV^{1\cdot3} = $ *constant. If the final pressure is* 126 *lb. per sq. in. absolute and the final volume* 0·32 *cubic ft., find the initial pressure and the work supplied to the gas during compression.*

$$PV^{1\cdot3} = \text{constant}.$$

$$126 \times 0\cdot32^{1\cdot3} = P_1 \times 1\cdot28^{1\cdot3}.$$

$$P_1 = 126 \times \left(\frac{0\cdot32}{1\cdot28}\right)^{1\cdot3} = 126 \times \left(\tfrac{1}{4}\right)^{1\cdot3}.$$

$$\log P_1 = \log 126 + 1\cdot3 \log 0\cdot25$$

$$= 2\cdot1004 + \bar{1}\cdot2173 = 1\cdot3177.$$

$$\mathbf{P_1 = 20\cdot78 \text{ lb. per sq. in. abs.}}$$

$$\text{Work done} = \frac{P_1V_1 - P_2V.}{n - 1}$$

$$= 144 \left\{\frac{20\cdot78 \times 1\cdot28 - 126 \times 0\cdot32}{1\cdot3 - 1}\right\}$$

$$= 144 \left\{\frac{26\cdot6 - 40\cdot32}{0\cdot3}\right\}$$

$$= 144 \times -45\cdot7 = \mathbf{-6585 \text{ ft. lb.}}$$

N.B. *The negative character of this answer indicates that work has been supplied to the gas during compression. A positive value of the work done is obtained for expansion in which work is done by the gas.*

Thermal behaviour of the working substance. During the progress of the cycle for any engine, there are changes in the heat content or internal energy of the working substance, and at any given period the working substance is said to be in a certain condition. This condition refers to its pressure, temperature, volume and heat content, and must *not* be confused with the change of state which occurs when a solid is converted to a liquid, or a liquid to a gas.

It is first necessary to realise that any change of heat content in the working substance involves a loss or gain of energy, and work or heat must be either expended by the gas, or supplied to the gas, to produce this energy change. Heat, when supplied to a gas, may produce one of several results :

(*a*) It may increase the temperature of the gas with a consequent increase of pressure, but with no volume change. In this case the heat content increases, but *no external work is done* because there is no increase in volume, that is, no movement against external resistance. **Heat is then taken in at constant volume.**

(*b*) It may increase both the temperature and volume while the pressure remains unaltered. In this case *external work is done*, because there is a volume change, that is, movement against external resistance. **Heat is then taken in at constant pressure.**

The rate at which heat is absorbed in these two cases has been dealt with on pp. 366, 373, and depends upon the specific heats at constant pressure and volume.

(*c*) The third case is more complicated and more general, and is the case where temperature, pressure and volume may all, or in part, change during the supply of heat. External work may, or may not, be done, and mechanical energy may be supplied in order to produce these changes of pressure, temperature, volume and heat content.

Energy of the working substance. The stock of internal energy possessed by the working substance will be reduced if external work is done by an amount equal to the heat equivalent of this external work. In order that external work may be done, the volume of working substance must change, hence :

Heat supplied = External work done + Change of Internal Energy.

As an analogy :

Money deposited in a bank = Value of cheques drawn
+ increase or decrease of bank balance.

Joule's Law for gases states that the internal energy of a gas depends upon its temperature, and is measured as the quantity of heat possessed by the gas at a definite temperature.

Charles' Law and the corresponding Pressure Law also due to Charles.

These laws are often stated thus :

(*a*) If the volume V of a gas is kept constant, the absolute pressure P is directly proportional to its absolute temperature T, or P/T = constant when V is constant.

(*b*) If the pressure of a gas is kept constant, the volume V is directly proportional to its absolute temperature T, or V/T = constant, when P is constant.

Experiment has shown that this change of volume is $\frac{1}{273}$ of the volume at 0° C. for every degree Centigrade rise in temperature. Thus, theoretically, the volume of a gas becomes zero at a temperature of $-273°$ C., but actually the gas would liquefy before this temperature was reached. The temperature $-273°$ C. is taken as the basic temperature for heat calculations and is known as the **absolute zero of temperature.**

The algebraic statements of Charles' Law and the corresponding Pressure Law can be written thus

$$\mathbf{V} - \textbf{constant} \times \mathbf{T} \text{ and } \mathbf{P} = \textbf{constant} \times \mathbf{T} \text{ respectively,}$$

where V is the volume of the gas and T is its temperature *measured from the absolute zero* and P is the absolute pressure.

Example 1. *Heat is supplied at constant pressure to* 90 *cu. ft. of gas the temperature of which is raised from* 20° *to* 80° *C. Calculate the final volume.*

Initial volume $= 90$ cu. ft. $= V_1$.

Initial temperature $= 273 + 20 = 293 = T_1$.

Final temperature $= 273 + 80 = 353 = T_2$.

According to Charles' Law, if the pressure remains constant

$$V = \text{constant} \times T\,;$$

$$i.e. \quad \frac{V_1}{T_1} = \frac{V_2}{T_2},$$

$$\frac{90}{293} = \frac{V_2}{353}, \quad V_2 = \frac{90 \times 353}{293}$$

$$= \mathbf{108{\cdot}4\ cu.\ ft.}$$

Example 2. *In a Diesel engine heat is supplied at constant pressure and the final volume of the charge is* 1·2 *cu. ft. at a temperature of* 800° *C. Find the temperature of the charge before heat is supplied if the volume at this stage is* 0·65 *cu. ft.*

$$V_2 = 1{\cdot}2, \quad T_2 = 1073^\circ \text{ C. abs.}$$

$$V_1 = 0{\cdot}65.$$

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}.$$

$$\frac{0{\cdot}65}{T_1} = \frac{1{\cdot}2}{1073}, \quad T_1 = \frac{1073 \times 0{\cdot}65}{1{\cdot}2}$$

$$T_1 = 581{\cdot}2^\circ \text{ C. abs.}$$

$$t_1 = 581{\cdot}2 - 273 = \mathbf{308{\cdot}2^\circ\ C.}$$

Combination of Boyle's and Charles' Law. When the working substance suffers changes in temperature, pressure and volume, neither Boyle's Law, which assumes constant temperature, nor

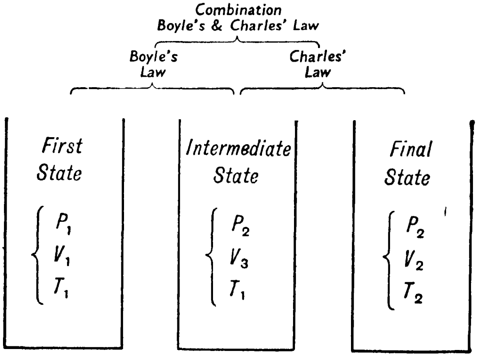


FIG. 223.

Charles' Law, which assumes alteration of temperature, with either constant pressure or constant volume, can be applied. The gas will follow a combination of the two laws, and its behaviour may well be investigated in the form of two processes, namely, a change according to Boyle's Law followed by a further change according to Charles' Law. The resultant change will then be according to a combination of the two laws (Fig. 223).

Consider a quantity of a gas at a pressure P_1, volume V_1 and absolute temperature T_1, which is allowed to expand to a final condition of pressure P_2, volume V_2 and absolute temperature T_2. This expansion may be regarded as consisting of two processes which may be regarded as taking place; (*a*) according to Boyle's Law, and (*b*) according to Charles' Law (Fig. 223).

Process (*a*) *According to Boyle' sLaw, with constant temperature* T_1.

Let the final volume at this stage be V_3; then

$$P_1V_1 = P_2V_3 \quad \text{and} \quad V_3 = \frac{P_1V_1}{P_2}.$$

Process (*b*) *According to Charles' Law, with constant pressure* P_2 *and initial volume* V_3.

In this case, $$\frac{V_3}{T_1} = \frac{V_2}{T_2},$$

but in process (*a*), $$V_3 = \frac{P_1V_1}{P_2},$$

$$\therefore \frac{P_1V_1}{T_1P_2} = \frac{V_2}{T_2} \quad \text{or} \quad \frac{\mathbf{P_1V_1}}{\mathbf{T_1}} = \frac{\mathbf{P_2V_2}}{\mathbf{T_2}}.$$

Thus the combination of Boyle's and Charles' Laws may be expressed generally as

$$\frac{\mathbf{PV}}{\mathbf{T}} = \textbf{constant.}$$

Specific volume of a gas. At a given temperature and pressure, 1 lb. of a gas will have a definite volume, which is known as the *specific volume of the gas.*

In the expression $\frac{PV}{T} = \text{constant}$, if V is the specific volume of

1 lb. of gas and P is the pressure in lb. per sq. ft., with T as the absolute temperature in ° C.,

$$\frac{PV}{T} = \text{Constant} = R,$$

or

$$\mathbf{PV = RT.}$$

The constant R has now taken a definite significance, and is known as the **gas constant** where 1lb. of gas is considered.

If w lb. of gas are employed, the equation becomes $\mathbf{PV = wRT}$, and the units of R are *ft. lb. per lb. per Centigrade degree.*

Characteristic equation of a gas. The equation

$$\mathbf{PV = wRT}$$

is known as the *characteristic equation* of a gas, and when the appropriate units of P, V and T are employed, with the weight of the gas in lb., the value of R is in ft. lb. per lb. of gas per Centigrade degree. This value of R may be converted into heat units by dividing by Joule's equivalent, that is, $\frac{R}{1400}$, which gives the value of R per lb. of gas per Centigrade degree in C.H.U.

Summary of the gas equations.

Law	Constants	Variables	Equation
Boyle's	T	P and V	$PV = \text{constant}$
Charles'	P or V	T and V or T and P	$\begin{cases} P_T = P_0 + \frac{1}{273} P_0 T \\ V_T = V_0 + \frac{1}{273} V_0 T \end{cases}$
Boyle's and Charles'	—	P, V and T	$PV = wRT$

Change in internal energy at constant pressure. When 1 lb. of gas is heated without change in pressure, the volume and temperature alter and work is done externally. This external work done is equal to the pressure multiplied by the change in volume (see p. 74, Vol. II); that is, $P(V_2 - V_1)$. The heat given to the gas at constant pressure is the specific heat at constant pressure multiplied by the change in temperature; that is, $c_p(T_2 - T_1)$. It follows from the principle of conservation of energy that

heat taken in = work done + change in internal energy, and

$$\textbf{change in internal energy} = \mathbf{c_p(T_2 - T_1) - P(V_2 - V_1)},$$

which will be in C.H.U. or ft. lb. according to the units used for P, V, and c_p.

Joule's Law (p. 379) states that the internal energy of a gas is dependent entirely upon its temperature, and is definite for a given temperature; it does not matter what happens to the volume. Hence change of internal energy can be represented by $c_v(T_2 - T_1)$ if the gas temperature changes from T_1 to T_2.

Therefore the above equality may be written

$$c_v(T_2 - T_1) = c_p(T_2 - T_1) - P(V_2 - V_1),$$

or, rearranging,

$$(c_p - c_v)(T_2 - T_1) = P(V_2 - V_1) = PV_2 - PV_1.$$

But $\quad PV_2 = RT_2 \quad \text{and} \quad PV_1 = RT_1$;

hence $\quad (c_p - c_v)(T_2 - T_1) = R(T_2 - T_1)$.

therefore $\quad \mathbf{c_p - c_v = R}$.

This is called the relationship between the gas constant and the specific heats of the gas, and has an important bearing on the work to follow.

Adiabatic expansion (to show that n = γ).

In this case the work done = loss of internal energy.

$$\text{Work done} = \frac{P_2V_2 - P_1V_1}{n - 1} \text{ per lb. (see p. 376.)}$$

$$= \frac{RT_2 - RT_1}{n - 1} = \frac{R(T_2 - T_1)}{n - 1}.$$

Loss of internal energy $= c_v(T_2 - T_1)$ per lb.

$$\therefore \; c_v(T_2 - T_1) = \frac{R(T_2 - T_1)}{n - 1},$$

and $\quad c_v = \dfrac{R}{n - 1}$.

But $\quad R = c_p - c_v$;

$$\therefore \; nc_v - c_v = c_p - c_v,$$

or $\quad \mathrm{n} = \dfrac{\mathrm{c}_p}{\mathrm{c}_v} = \gamma$.

Summary of relationship between R, c_v, c_p and γ.

In the characteristic equation of a gas.

$$(a)\ R = \frac{PV}{wT},$$

$$(b)\ R = c_p - c_v,$$

$$(c)\ \gamma = \frac{c_p}{c_v} \text{ in the adiabatic expression.}$$

Example 1. *Find the weight of gas in an ammonia cylinder of volume 3·5 cu. ft. at 0° C. and pressure 270 lb. per sq. inch gauge. R = 183·4 ft. lb. per lb. per Centigrade degree.*

$$P = (270 + 14{\cdot}7) \times 144 = 284{\cdot}7 \times 144 \text{ lb. per ft.}^2$$

$$T = 273^\circ \text{ C. abs.}$$

$$V = 3{\cdot}5 \text{ cu. ft.}$$

$$PV = wRT.$$

$$w = \frac{PV}{RT} = \frac{284{\cdot}7 \times 144 \times 3{\cdot}5}{183{\cdot}4 \times 273} = 2{\cdot}866 \text{ lb.}$$

Example 2. *An air compressor receives a charge of 6·3 cu. ft. at a temperature of 18° C. and a pressure of 20 lb. per sq. in. absolute. This charge is then compressed to a volume of 1·4 cu. ft. with a temperature rise to 210° C. Calculate the final pressure and weight. R for air is 95·8 ft. lb. per lb. per Centigrade degree.*

(a) Initial condition	(b) Final condition
$V_1 = 6{\cdot}3$ cu. ft.	$V_2 = 1{\cdot}4$ cu. ft.
$T_1 = 273 + 18 = 291^\circ$ C. abs.	$T_2 = 273 + 210 = 483^\circ$ C. abs.
$P_1 = 20 \times 144 = 2880$ lb./ft.^{2}.	$P_2 = ?$

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2},$$

$$\frac{2880 \times 6{\cdot}3}{291} = \frac{P_2 \times 1{\cdot}4}{483},$$

$$P_2 = \frac{2880 \times 483 \times 6{\cdot}3}{1{\cdot}4 \times 291} = 21{,}510 \text{ lb. per ft.}^2.$$

$$= 149{\cdot}4 \text{ lb. per in.}^2.$$

Weight of charge = w.

$$w = \frac{PV}{RT} = \frac{21{,}510 \times 1{\cdot}4}{95{\cdot}8 \times 483} = 0{\cdot}65 \text{ lb.}$$

Example 3. *A charge of methane of* 8 *cu. ft. volume has a temperature of* 30° *C. and a pressure of* 100 *lb. per sq. in. gauge. Find its volume at N.T.P. (normal temperature and pressure). Find (a) the value of R for methane and the weight of the charge if* $c_p = 0{\cdot}591$ *and* $c_v = 0{\cdot}451$.

Initial condition.	Final condition (N.T.P.).
$V_1 = 8$ cu. ft.	$V_2 = ?$
$P_1 = 114{\cdot}7 \times 144$ lb. per ft.^{2}.	$P_2 = 14{\cdot}7 \times 144$ lb. per ft.^{2}.
$T_1 = 273 + 30 = 303°$ C. abs.	$T_2 = 273°$ C. abs.

$$\frac{PV}{T} = K = \frac{114{\cdot}7 \times 144 \times 8}{303} = \frac{14{\cdot}7 \times 144 \times V_2}{273},$$

$$V_2 = \frac{114{\cdot}7 \times 8 \times 273}{303 \times 14{\cdot}7} = \mathbf{56{\cdot}26\ cu.\ ft.}$$

$R = c_p - c_v$ in C.H.U.

$= 0{\cdot}591 - 0{\cdot}451 = 0{\cdot}140$ C.H.U.

$= 0{\cdot}14 \times 1400 =$ **196 ft. lb. per lb. per Centigrade degree.**

$$w = \frac{PV}{RT} = \frac{114{\cdot}7 \times 144 \times 8}{196 \times 303} = \mathbf{2{\cdot}22\ lb.}$$

Example 4. *A boiler receives* 20,000 *lb. of air per hour at a temperature of* 18° *C. and a pressure of* 16 *lb. per sq. in. abs. Find the volume of air to be handled by the intake fan per minute if* $R = 95{\cdot}8$ *ft. lb. per Centigrade degree.*

$$PV = wRT.$$

$$w = \frac{20{,}000}{60} = 333\tfrac{1}{3}\ \text{lb. per min.}$$

$$V = \frac{wRT}{P} = \frac{333{\cdot}3 \times 95{\cdot}8 \times (273 + 18)}{16 \times 144}$$

$= \mathbf{4033\ cu.\ ft.}$

Example 5. *A gas engine consumes* 3·26 *cu. ft. of coal gas per minute at a temperature of* 20° *C. and pressure of* 15·5 *lb. per sq. in. absolute. Calculate the temperature of the gas prior to explosion if the ratio of compression is* 4·5 *and the final pressure* 160 *lb. per sq. in. absolute.*

(*a*) Primary conditions.	(*b*) Final conditions.
$V_1 = 3{\cdot}26$ cu. ft.	$V_2 = \dfrac{3{\cdot}26}{4{\cdot}5} = 0{\cdot}725$ cu. ft.
$P_1 = 15{\cdot}5 \times 144$ lb. per ft.^{2}.	$P_2 = 160 \times 144$ lb. per ft.^{2}.
$T_1 = 273 + 20 = 293°$ C. abs.	$T_2 = ?$

$$\frac{15{\cdot}5 \times 144 \times 3{\cdot}26}{293} = \frac{160 \times 144 \times 0{\cdot}725}{T_2},$$

$$T_2 = 672{\cdot}6\ °\text{C. abs.},\ \mathbf{t_2 = 400°\ C.}$$

Example 6. *A volume of 10 cubic ft. of gas at 170 lb. per sq. in. abs. and temperature 140° C. is heated at constant pressure until the volume becomes 14 cu. ft. If $c_p = 0{\cdot}251$ and $c_v = 0{\cdot}173$, find (a) the weight of the charge, (b) the external work done, (c) the change in internal energy.*

$$R = c_p - c_v = 0{\cdot}251 - 0{\cdot}173 = 0{\cdot}078 \text{ heat units.}$$

(*a*) Weight $w = \frac{PV}{RT} = \frac{170 \times 144 \times 10}{0{\cdot}078 \times 1400 \times \{273 + 140\}} = \mathbf{5{\cdot}428\ lb.}$

(*b*) External work done $= p(v_2 - v_1)$

$$= 170 \times 144(14 - 10) = \mathbf{97{,}920\ ft.\ lb.}$$

Heat taken in at constant pressure $= c_p w(T_2 - T_1)$.

$$\frac{P_1V_1}{T_1} = \frac{1700}{413} = \frac{P_1V_2}{T_2},$$

$$T_2 = \frac{170 \times 14 \times 413}{170 \times 10} = 578{\cdot}2° \text{ C. abs.}$$

$$H_{c_p} = 0{\cdot}251 \times 5{\cdot}428(578{\cdot}2 - 413)$$

$$= 0{\cdot}251 \times 5{\cdot}428 \times 165{\cdot}2 = 225{\cdot}1 \text{ C.H.U.}$$

By the principle of conservation of energy :

Heat taken in = change in internal energy + work done.

$$225{\cdot}1 = E + \frac{97920}{1400}.$$

(*c*) $E = 225{\cdot}1 - 69{\cdot}94 = \mathbf{155{\cdot}16\ C.H.U.}$

The Comparative Cycles

It has been stated that in an engine operation the working substance is submitted to a series of changing conditions known as the **working cycle.** The cycle of operations may be an **ideal cycle,** in which the working substance is conceived to undergo a series of changes which give a maximum theoretical performance, or may be the **actual cycle,** which is the cycle used in the actual engine performance.

The ideal cycles, notably the **Carnot cycle** and the **Otto cycle,** are used as standard efficiency cycles against which the performance of actual engines are compared.

It is general to represent any cycle of operations as applied to engine performance by a **pressure-volume diagram** for the working substance, and to illustrate on this diagram the changes in the state of the substance during the operations.

The Carnot cycle. This cycle is the recognised standard of comparison for all heat engines, and gives the maximum efficiency which any heat engine may attain with a perfect working substance under perfect working conditions. It is an ideal cycle which cannot be obtained in practice, but it serves as a comparative standard in all heat engine performances.

It consists of four essential operations: (1) isothermal expansion, (2) adiabatic expansion, (3) isothermal compression, (4) adiabatic compression. Thus the working substance is submitted to two stages of expansion, followed by two of compression, after which it is restored to the condition, or state, which it possessed before expansion.

Imagine one pound weight of a perfect gas to be in a cylinder made of a material which is a complete heat insulator (Fig. 224).

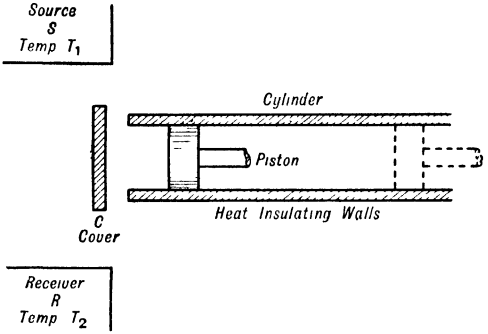


FIG 224

The end of this cylinder can be closed at will by a perfect heat insulating cover C. Heat is supplied from a heat source S, which is maintained at a constant temperature T_1, and rejected into a receiver R which is maintained at a constant temperature T_2. The working substance is now operated upon in four separate stages :

Stage 1. Heat is applied from S, the source, to the working substance in the cylinder, and the substance expands *isothermally*, that is, at constant temperature T_1.

Stage 2. The heat in the cylinder is now trapped by applying the insulating cover C, and expansion is continued without gain or loss of heat to external sources, that is, *adiabatically*, the temperature falling during the process to that of the receiver T_2.

Stage 3. The cover is removed and the substance compressed *isothermally* at a temperature T_2, rejecting heat to the receiver R.

Stage 4. Again the heat in the cylinder is trapped by applying C, and the substance is compressed *adiabatically* until it reaches the temperature of the source T_1.

The cycle is now complete, and it is necessary to investigate the energy changes during the cycle, which may be summarised as follows.

Stage	Energy Change
1	Heat supplied at constant temperature.
2	No heat change to external sources.
3	Heat rejected at constant temperature.
4	No heat change to external sources.

The Carnot cycle for a perfect gas. Fig. 225 shows the pressure-volume diagram for the Carnot cycle in which AB represents the first stage, BC the second stage, CD the third stage, and DA the final stage to complete the cycle.

At A the condition of the gas is $P_1V_1T_1$, which changes to $P_2V_2T_1$ at B under isothermal expansion at constant temperature T_1. *Heat supplied* $= PV \log_e r$ (see p. 376), which for 1 lb. of gas becomes

$$RT_1 \log_e r \quad \text{or} \quad \mathrm{RT_1} \log_e \frac{\mathrm{V_2}}{\mathrm{V_1}}.$$

At B the state of the gas is $P_2V_2T_1$, which changes to $P_3V_3T_2$ at C under adiabatic expansion *without change of heat content.*

At C the state of the gas is $P_3V_3T_2$, which changes to $P_4V_4T_2$ at D under isothermal compression at constant temperature T_2. *Heat rejected* $= PV \log_e r$, which for 1 lb. of gas becomes

$$RT_2 \log_e r \quad \text{or} \quad \mathrm{RT_2} \log_e \frac{\mathrm{V_3}}{\mathrm{V_4}}.$$

N.B. *Compression is arranged to finish at such a point D that the adiabatic compression which follows will bring the gas to its initial condition* $P_1V_1T_1$ *at A. This is achieved if the ratios of expansion and compression in the isothermal stages* 1 *and* 3 *are equal.*

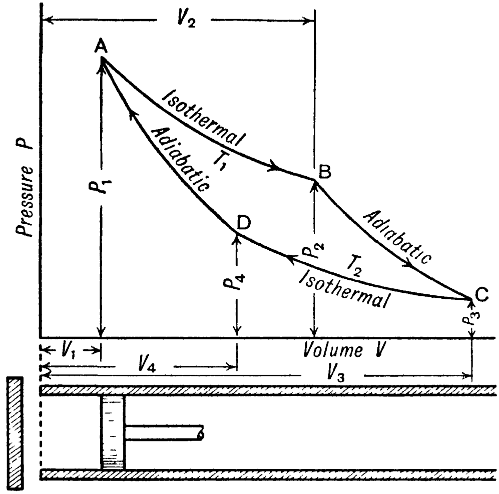


FIG. 225.

At D the state of the gas is $P_4V_4T_2$, which changes to $P_1V_1T_1$ at A under adiabatic compression *without change of heat content.*

By the principle of conservation of energy the work done must be equal to the heat energy change over the four stages ; that is,

$$\text{Work done} = \text{heat taken in} - \text{heat rejected}$$

$$= RT_1 \log_e \frac{V_2}{V_1} - RT_2 \log_e \frac{V_3}{V_4},$$

and since r, the ratio of expansion or compression in the isothermal stages $= \frac{V_2}{V_1} = \frac{V_3}{V_4}$, this work done is equal to $\mathbf{R}\,\{\mathbf{T}_1 - \mathbf{T}_2\}\,\mathbf{log}_e\,\mathbf{r}$.

$$\text{Efficiency of the cycle} = \frac{\text{work done}}{\text{heat supplied}}$$

$$= \frac{R(T_1 - T_2)\log_e r}{RT_1 \log_e r}$$

$$= \frac{\mathrm{T}_1 - \mathrm{T}_2}{\mathrm{T}_1}.$$

Thus *the efficiency of the perfect heat engine working on the Carnot cycle depends upon the absolute temperatures of supply and rejection,* *and the greater the temperature range the higher the efficiency.* 100 per cent. efficiency can only be obtained if the working substance can be expanded down to absolute zero, or $t_2 = 273°$ C., i.e. $T_2 =$ zero. This is, of course, an unworkable condition in any engine.

Carnot cycle for vapour. When a vapour is employed as a working substance, in place of a perfect gas, the vapour is expanded or compressed isothermally not only at constant temperature, but also at constant pressure. The isothermal lines in the *P v. V* diagram (Fig. 226) thus become horizontal. The stages in this form of the cycle are :

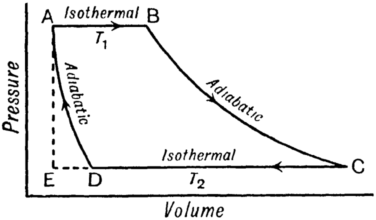


FIG. 226.

Stage 1, AB. At *A* the substance is liquid, and also during isothermal expansion to *B*, when it becomes dry vapour at the same pressure and temperature ; *latent heat* L_1 is taken in.

Stage 2, BC. Adiabatic expansion of the dry vapour follows to *C*, where the substance becomes wet vapour with the fall in temperature ; there is *no change in internal energy.*

Stage 3, CD. Isothermal compression follows in this stage, in which the condensation of the wet vapour at *C* is continued and *latent heat* L_2 is rejected.

Stage 4, DA. Adiabatic compression follows, and the mixture of vapour and liquid at *D* is completely condensed and restored to the temperature T_1 and pressure P_1 at *A*. Again there is *no change in internal energy* during compression.

In this cycle the efficiency is still $\frac{T_1 - T_2}{T_1}$, and the work done in terms of the latent heat L_1 of the substance on supply or at temperature T_1 is $L_1 \left(\frac{T_1 - T_2}{T_1}\right)$ **heat units per lb.**

The Rankine cycle. The practical operation of the steam engine is improved by the use of a separate condenser, and a standard comparative cycle for steam engine performance is Rankine's modification of the Carnot vapour cycle known as the **Rankine Cycle**, a cycle which is applicable to an engine working with a separate condenser.

The essential modifications to the Carnot cycle evident in Rankine's Cycle are :

(*a*) The continuance of the isothermal compression stage until the vapour mixture is completely condensed to a volume equal to that occupied by the substance at the commencement of the cycle ; that is, the extension of Stage 3 (Fig. 226) to *E*.

(*b*) The supply of heat in order to raise the temperature of the condensate at *E* from T_2 to T_1, the temperature at *A*.

This cycle is represented by *ABCE* in Fig. 226, and although the efficiency falls short of $\frac{T_1 - T_2}{T_1}$ to the extent of the heat supplied to the condensate, it provides a comparative cycle for steam engine and turbine performance because of its acceptance of the condition of complete condensation and reheating at the end of the cycle.

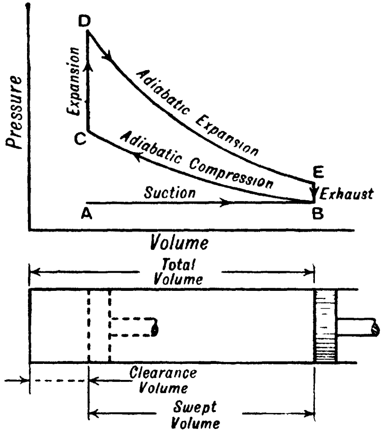


FIG. 227. OTTO CYCLE.

The Otto or constant volume cycle. This cycle is the comparative cycle for internal combustion engine performance, and it accepts the principle that the heat shall be received and rejected at constant volume. The cycle is represented in the *P v. V* diagram (Fig. 227) by *BCDE*, and *AB* is the suction line by which the drawing of the charge into the cylinder is represented.

Let the state of the charge at *B*, *C*, *D* and *E* be respectively $P_1V_1T_1$, $P_2V_2T_2$, $P_3V_3T_3$ and $P_4V_4T_4$, where $V_1 = V_4$ = total volume and $V_2 = V_3 =$ clearance volume. Then since BC and DE represent adiabatic or constant heat processes, heat is taken in during the stage CD, when combustion occurs, and rejected during the stage EB, that is, exhaust, both reception and rejection of heat being at constant volume.

Stage 1, BC. This stage is adiabatic according to the law

$$PV^{\gamma} = \text{constant},$$

where γ is the ratio of the specific heats $\frac{c_p}{c_v}$.

Then $$P_1V_1^{\gamma} = P_2V_2^{\gamma},$$

or $$(P_1V_1)V_1^{\gamma-1} = (P_2V_2)V_2^{\gamma-1},$$

that is, $$RT_1V_1^{\gamma-1} = RT_2V_2^{\gamma-1},$$

which may be written

$$\frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{\gamma-1} = \left(\frac{1}{r}\right)^{\gamma-1}$$

where r is the ratio of compression.

Stage 2, CD. Heat is taken in at constant volume on the combustion of the charge, so that if c_v is the specific heat at constant volume,

$$\textit{Heat supplied} = c_v(T_3 - T_2).$$

Stage 3, DE. This stage is adiabatic expansion, and according to the reasoning employed for Stage 1, $\frac{T_4}{T_3} = \left(\frac{1}{r}\right)^{\gamma-1}$.

Since, in Stage 1, $\frac{T_1}{T_2} = \left(\frac{1}{r}\right)^{\gamma-1}$,

$$\frac{T_4}{T_3} = \frac{T_1}{T_2} \quad \text{and} \quad T_4 = \frac{T_1T_3}{T_2}.$$

Stage 4, EB. This stage is rejection of heat at constant volume.

$$\therefore \textit{Heat rejected} = c_v(T_4 - T_1).$$

$$\eta = \text{Efficiency} = \frac{\text{work done}}{\text{heat supplied}} = \frac{\text{heat supplied} - \text{heat rejected}}{\text{heat supplied}}$$

$$= \frac{c_v(T_3 - T_2) - c_v(T_4 - T_1)}{c_v(T_3 - T_2)}$$

$$= 1 - \frac{T_4 - T_1}{T_3 - T_2}.$$

Now $$T_4 = \frac{T_1T_3}{T_2};$$

$$\therefore\ \eta = 1 - \frac{\dfrac{T_1T_3}{T_2} - T_1}{T_3 - T_2} = 1 - \frac{T_1}{T_2}\left(\frac{T_3 - T_2}{T_3 - T_2}\right)$$

$$= 1 - \frac{T_1}{T_2}.$$

But $$\frac{T_1}{T_2} = \left(\frac{1}{r}\right)^{\gamma-1};$$

$$\therefore\ \eta = \textbf{efficiency} = 1 - \left(\frac{1}{r}\right)^{\gamma-1}.$$

It is common to regard this cycle as operating with air as the working substance, in which case $\gamma = 1{\cdot}404$, and the

$$\textbf{Air standard efficiency} = 1 - \left(\frac{1}{r}\right)^{0{\cdot}404}.$$

Summary of the standard cycles

Cycle	Standard for	Remarks
Carnot - -	All heat engines	Efficiency $= \frac{T_1 - T_2}{T_1}$.
Carnot vapour -	Engines using liquid and vapour	” $= \frac{T_1 - T_2}{T_1}$
Rankine - -	Steam engines and turbines	Complete condensation and reheating of condensate (see p. 415 for further treatment)
Otto or constant volume	Internal combustion engines	Efficiency $= 1 - \left(\frac{1}{r}\right)^{\gamma-1}$ Air standard efficiency $= 1 - \left(\frac{1}{r}\right)^{0{\cdot}404}$

Example 1. *A steam engine receives dry saturated steam at 220 lb. per sq. in. absolute and exhausts into a condenser at 2 lb. per sq. in. absolute. Calculate the efficiency on the Carnot cycle.*

From steam tables :

$$T_1 \text{ at } 220 \text{ lb. per in.}^2 = 471{\cdot}9^\circ \text{ C. abs.}$$

$$T_2 \text{ at } 2 \text{ lb. per in.}^2 = 325{\cdot}4^\circ \text{ C. abs.}$$

$$\text{Efficiency, } \eta = \frac{T_1 - T_2}{T_1} = \frac{471{\cdot}9 - 325{\cdot}4}{471{\cdot}9} \quad 31{\cdot}1\%.$$

Example 2. *A gas engine develops a maximum temperature of* 1230° *C. and exhausts at* 18° *C. Calculate its efficiency on the Carnot cycle. If the ratio of expansion is* 4·6 *and* $\gamma = 1{\cdot}3$, *what is the efficiency on the Otto cycle, and how does this compare with the air standard efficiency?*

(*a*) Carnot cycle :

$$\text{Efficiency} = \frac{(1230 + 273) - (18 + 273)}{(1230 + 273)}.$$

$$= \frac{1503 - 291}{1503} = 80{\cdot}6\%.$$

(*b*) Otto cycle :

$$\text{Efficiency} = 1 - \left(\frac{1}{r}\right)^{\gamma - 1}$$

$$= 1 - (1/4{\cdot}6)^{0{\cdot}3} = 36{\cdot}7\%.$$

(*c*) Air standard cycle :

$$\text{Efficiency} = 1 - (1/4{\cdot}6)^{0{\cdot}404} = 46\%.$$

Example 3. *Calculate the air standard efficiency of a gas engine of* 16 *in. stroke,* 12 *in. diam. of cylinder and* 480 *cu. in. of clearance volume if* $\gamma = 1{\cdot}4$. *Determine the gas consumption per I.H.P. per hour, assuming the gas to have a calorific value of* 300 *C.H.U. per cu. ft. and the ratio of the thermal efficiency of the engine to its air standard efficiency to be* 55 : 100. (U.E.I.)

Swept volume of cylinder $= \pi \times 6^2 \times 16 = 1809$ cu. in.

Clearance volume $= 480$ cu. in.

Total volume $= 1809 + 480 = 2289$ cu. in.

$$r = \frac{2289}{480} = 4{\cdot}778.$$

Air standard efficiency $= 1 - (1/4{\cdot}77)^{0{\cdot}4} = 46{\cdot}47\ \%$.

Thermal efficiency $= \frac{55}{100} \times 46{\cdot}45 = 25{\cdot}55\%$.

1 H.P. hour in work units $= 33000 \times 60 = 1{,}980{,}000$ ft. lb.

$$\text{in heat units} = \frac{1{,}980{,}000}{1400} \text{ C.H.U.} = 1414{\cdot}3 \text{ C.H.U.}$$

Gas consumption $= \frac{1414 \cdot 3}{300}$ cu. ft. on 100% efficiency

$= 4 \cdot 714$ cu. ft. per I.H.P. hour.

Actual gas consumption $= 4 \cdot 714 \times \frac{100}{25 \cdot 55}$

$= 18 \cdot 45$ cu. ft. per I.H.P. hour.

N.B. The ratio between the actual thermal efficiency and the air standard cycle efficiency of the I.C. engine is often referred to as the *relative efficiency*.

EXERCISES ON CHAPTER XV

1. Explain clearly the purpose of a heat engine and the nature of the working substance. What is meant by a cycle of operations?

2. State the various conditions in which steam can be supplied to an engine, and trace the increase of heat content as superheated steam is raised from water. Write down an expression for the total heat of superheated steam which will indicate the heat content of the steam at each stage in its formation.

3. Use steam tables to find the total heat of steam under each of the following conditions :

(*a*) Dry saturated at atmospheric pressure.

(*b*) Dry saturated at 100 lb. per sq. in. abs.

(*c*) Wet steam with a dryness fraction of 0·93 at 160 lb. per sq. in. absolute.

(*d*) Superheated to 400° C. if the pressure is 200 lb. per sq. in. gauge and atmospheric pressure is 15 lb. per sq. in. absolute. Mean specific heat of steam at constant pressure = 0·6.

(*e*) Wet steam in a condenser with a vacuum of 28·57 inches when the barometer reading is 30 inches and the dryness fraction 0·85.

Explain the meaning of " dryness fraction " as used in these determinations.

4. Steam at a pressure of 250 lb. per sq. in. absolute is supplied to a turbine at a temperature of 360° C. and exhausted into a condenser, with 0·9 dryness fraction, at a pressure of 1·8 lb. per sq. in. absolute. Calculate the theoretical heat drop per lb. of steam and the velocity of the steam jet if 50% of this heat drop is converted to kinetic energy. $c_p = 0 \cdot 6$.

5. A boiler produces steam 0·96 dryness fraction at a pressure of 150 lb. per sq. in. abs. Find the heat to be supplied by the superheater per lb. of steam in order to give a temperature of 200° C., $c_p = 0 \cdot 58$.

6. Steam is received by a turbine at 250 lb. per sq. in. abs. with 100° of superheat, and exhausted 0·93 dry into a condenser pressure of 2 lb. per sq. in. abs. Calculate

(*a*) The heat drop per lb. if $c_p = 0{\cdot}6$.

(*b*) The velocity of the jet if 50% of the heat drop is converted to kinetic energy.

(*c*) The probable H.P. output if the overall efficiency is 27%, and the steam consumption is 28,000 lb. per hour.

7. Find the cooling surface for a surface condenser under the following conditions: H.P. of engine = 300, exhaust steam 0·9 dry at 1·5 lb. per sq. in. abs., temperature of condensate 47° C., steam consumption 19 lb. per H.P. hour; 64 C.H.U. are transferred per sq. ft. of the cooling surface per minute.

8. For what horse-power turbine is the following condenser suited? Area of cooling surface, 1000 sq. ft.; rate of heat transfer 72 C.H.U. per sq. ft. per min.; vacuum 28 inches with a barometer of 30 inches. Assume the engine to use 20 lb. of steam per H.P. hour and to discharge with a dryness fraction of 0·92.

9. What quantity of cooling water is required per hour for a condenser to operate with an engine using 24 lb. of steam per H.P. hour and developing 200 H.P. if the steam entering the condenser has a dryness fraction of 0·84 and the pressure is 2 lb. per sq. in. absolute. The temperature of the hot well is 52° C., and that of the inlet and outlet cooling water 16° C. and 50° C. respectively.

10. A condenser receives steam from a steam turbine at a pressure of 1·7 lb. per sq. in. abs. and dryness fraction 0·91. If 50,000 gallons of cooling water are used per hour, and the entry and exit temperatures are 16° C. and 41° C., find the probable kW. output of the turbine. The hot well temperature is 49° C. and the steam consumption is known to be 22 lb. per kW. hour.

11. Wet steam at 160 lb. per sq. in. abs. is expanded through a throttling calorimeter at an internal pressure of 18 lb. per sq. in. abs. and the temperature is found to be 115° C. Find the dryness fraction of this steam if c_p for superheated steam is 0·485.

12. Sketch and describe a throttling calorimeter and explain the principle of its operation and the limits to which it may be used.

Wet steam at 200 lb. per sq. in. abs. expands into a throttling calorimeter to atmospheric pressure and a temperature of 120° C. If the specific heat of superheated steam is 0·48, find the dryness fraction of the steam before expansion. (U.E.I.)

13. A steam turbine when supplied with 9 lb. of steam per second at 200 lb. per sq. in. abs. superheated 100 Centigrade degrees develops 3000 H.P. Find (*a*) total heat, (*b*) condition, (*c*) the specific volume of

the steam at exhaust from the turbine at 1·3 lb. per sq. in. absolute. Take the specific heat of superheated steam as 0·56. Assume that there are no losses in the turbine. (U.E.I.)

14. What advantages are gained by fitting a condenser in a steam engine plant? Substantiate your answer by reference to typical indicator diagrams.

Exhaust steam 0·9 dry enters a surface condenser at 2 lb. per sq. in. pressure. It is there condensed to water at 40° C. The steam used per hour is 1800 lb. Calculate the weight of circulating water required per minute and the number of tubes per pass if they are $\frac{1}{2}$ in. internal diameter, the rate of flow is 4 ft. per second and the temperature rise for the circulating water is 20 Centigrade degrees. (1 cu. ft. of water weighs 62·3 lb.)

15. The exhaust steam from a 2000 kW. steam turbine passes to a condenser at a pressure of 2 lb. per sq. in. abs. and 93% dry. 106,000 gallons of cooling water enter the condenser per hour at a temperature of 16° C. and leave at 32° C. If the hot well temperature is 32° C., find the steam consumption of the turbine in lb. per kWh. (U.E.I.)

16. 80 cubic ft. of gas are expanded adiabatically from a pressure of 270 lb. per sq. inch absolute to a volume of 113 cu. ft. Find the final pressure if $\frac{c_p}{c_v} = 1{\cdot}38$.

17. The ratio of expansion in a certain engine is 4·9 and the expansion law is $PV^{1\cdot14} = \text{constant}$. Calculate the pressure before expansion if the final pressure is 27 lb. per sq. in. abs.

18. An engine of cylinder volume 8·1 cu. ft. cuts off steam supply at $\frac{1}{4}$ of the stroke. Calculate the work done during expansion if the initial pressure is 80 lb. per sq. in. and the clearance volume is equal to $\frac{1}{12}$ of the stroke. Expansion (*a*) according to Boyle's Law, (*b*) according to $PV^{1\cdot17} = \text{constant}$.

19. The compression ratio in a gas engine is 4·75. Find the work supplied during compression if the final pressure is 174 lb. per sq. in. abs. with a volume of 0·41 cu. ft. Compression law $PV^{1\cdot3} = \text{constant}$.

20. Distinguish clearly between isothermal and adiabatic expansion of a gas. An internal combustion engine has a cylinder 6 in. diameter and a stroke of 8 in. The pressure and temperature of the gas at the commencement of the compression stroke are 14 lb. per sq. in. absolute and 92° C. respectively. If the clearance volume is 76 cu. in. and the law of the compression $PV^{1\cdot3} = \text{constant}$, find the pressure and temperature at the end of compression. (U.E.I.)

21. What do you understand by the internal and external energy of a gas? 6 cu. ft. of gas at 120 lb. per sq. in. absolute and 180° C. are heated at constant pressure until the volume is doubled. If $c_p = 0{\cdot}239$ and $c_v = 0{\cdot}169$, find the change in internal energy and the external work done during the supply of heat. (U.E.I.)

22. Calculate the change of internal energy and work done when 17 cubic feet of chlorine at 100 lb. per sq. in. absolute and 140° C. is heated at constant pressure to a temperature of 290° C. $c_p = 0{\cdot}116$ and $\gamma = \frac{c_p}{c_v} = 1{\cdot}34$.

23. A compressor deals with 2400 cu. ft. of air per minute which it compresses to 100 lb./sq. in. from a state where $P = 14{\cdot}7$ lb. per sq. in. abs., $t = 15°$ C., according to the law $PV^{1{\cdot}3} =$ constant. Calculate (*a*) the final volume, (*b*) the ratio of compression, (*c*) the work supplied during compression, (*d*) the final temperature.

24. 1400 cu. ft. of ammonia is compressed adiabatically from 14·7 lb. per sq. in. abs. to a pressure of 350 lb. per sq. in. abs. Calculate the final volume and the work done during compression if the compression law is $PV^{1{\cdot}336} - K$. What would have been the work done if the process was isothermal?

25. State Charles's Law. An engine charge is allowed to expand while heat is supplied at constant pressure from a volume of 1·9 cu. ft. to one of 2·7 cu. ft., the initial temperature being 300° C. Calculate the final temperature, the weight of the charge, the external work done and the change in internal energy if the following constants apply: $P = 120$ lb. per sq. in. absolute, $\gamma = 1{\cdot}3$, $c_p = 0{\cdot}24$.

26. Obtain an expression for the combination of Boyle's and Charles's Laws for a perfect gas. An air compressor receives a charge of air at a temperature of 20° C. and a pressure of 22 lb. per sq. in. absolute. Calculate the volume of the charge at admission if the final volume is 2·84 cu. ft., the final pressure 164 lb. per sq. in. abs., and the final temperature 200° C.

27. At the commencement of the compression stroke of a gas engine the pressure is 14·7 lb. per sq. in. and the temperature is 90° C. If the compression follows the law $PV^{1{\cdot}3} =$ constant, find the pressure and temperature of the gas when it is compressed to one-third of its volume. (U.L.C.I.)

28. One pound of gas at a pressure of 15 lb. per sq. in. absolute and a temperature of 15° C. is compressed adiabatically to 400 lb. per sq. in. absolute. Assuming that $PV = 96T$ and that the index for adiabatic compression is 1·408, calculate the final volume and temperature. What is the ratio of compression, and how much work has been done during compression? (U.L.C.I.)

29. What is meant by the characteristic equation of a gas? A charge of 1 lb. of gas at 80 lb. per sq. in. abs. is compressed adiabatically to a volume of 0·7 cu. ft. If the initial temperature is 120° C., find (*a*) the initial volume, (*b*) the compression ratio, (*c*) the final pressure and temperature, (*d*) the work done during compression. $R = 96$. $c_v = 0{\cdot}17$. Calculate the density of the gas after compression.

30. Seven cubic feet of a gas expand in a cylinder from a condition $P=270$ lb. per sq. in. abs., $T=513°$ C. abs., to one of 38 lb. per sq. in. absolute according to a law $PV^{\gamma}=\text{constant}$, where $c_p=0{\cdot}254$ and $c_v=0{\cdot}173$. Find the heat interchange between the charge and the cylinder walls.

31. Write down the characteristic equation of a gas, and state clearly the meaning of each symbol used.

Given 1 cu. ft. of air at 0° C. and at a pressure of 14·7 lb. per sq. in. absolute weighs 0·0807 lb., what is the constant in the equation referred to above?

A certain air reservoir contains 12 lb. of air at a pressure of 100 lb. per sq. in. absolute and a temperature of 30° C. The reservoir is connected to another by a short pipe so as to exhaust into it. When conditions are equal in both, the pressure is 40 lb. per sq. in. absolute and temperature 10° C. Find the volume of each vessel, given that at the commencement the pressure in the second reservoir was zero absolute. (U.E.I.)

32. The piston displacement in a petrol engine is 24 cu. in., and the compression space 5·6 cu. in. The pressure and temperature at the end of expansion is 44 lb. per sq. in. absolute and 1080° C. Calculate the weight of the cylinder contents if the constant in the gas equation is 100 ft. lb. per lb. per Centigrade degree.

33. An oil engine draws in air at 14 lb. per sq. in. and 32° C., and compresses it adiabatically into the clearance volume before the oil is injected. If the stroke volume is 1·182 cu. ft. and the clearance volume 0·12 cu. ft., find the pressure and temperature at the end of the compression and the mass of air compressed. $\gamma=1{\cdot}4$ and $R=96$ ft. lb. units. (U.E.I.)

34. What is meant by a "cycle." Sketch a *P v. V* diagram for the Carnot cycle, and deduce the efficiency of the cycle.

35. An engine receives steam at a pressure of 200 lb. per sq. in. with 100 Centigrade degrees of superheat, and discharges into a condenser at a pressure of 2 lb. per sq. in. Use steam tables to determine the required temperatures, and find the theoretical efficiency under the Carnot cycle.

36. Explain the modification of the Carnot cycle necessary to produce the Rankine cycle, and show that the latter cycle more nearly corresponds with steam engine and turbine performance. Sketch the *P v. V* diagram for the Rankine cycle.

37. Deduce the efficiency of an engine working on the Otto or constant volume cycle, and show that this efficiency is mainly dependent upon the ratio of expansion. What is the effect on the efficiency of increasing the ratio of expansion?

38. What is the air standard cycle, and in what way can it be identified with the constant volume cycle?

A gas engine develops a maximum temperature of 1100° C. and exhausts at 27° C. Calculate its efficiency on the Carnot cycle. If the ratio of expansion is 4·2 and $\gamma = 1{\cdot}3$, what is the efficiency on the Otto cycle? Determine the air standard efficiency for this engine.

39. What is meant by "air standard efficiency" as applied to an engine working on the Otto cycle?

Develop an expression for the thermal efficiency of this cycle. This expression should indicate that the efficiency will rise with increase of compression ratio. Why, then, are compression ratios not higher than their current values in engines working on this basic cycle? (U.E.I.)

CHAPTER XVI

ENTROPY, CHANGES OF ENTROPY AND TEMPERATURE, REFRIGERATION

Entropy and entropy-temperature diagrams. The treatment of many heat engine problems, particularly those involving heat quantity and temperature changes, can be simplified by the use of entropy-temperature charts and a knowledge of entropy. It is very difficult to define entropy, and its conception is gained largely as a result of working examples and utilising the charts to follow the various processes operable in engines and in the standard cycles.

The ability to convert heat energy into mechanical energy depends to a large extent upon the temperature at which the heat is supplied ; for example, the heat given to high-pressure steam, where the temperature is high, is much more readily converted to mechanical energy than the same heat given to steam at relatively low pressures. The efficiency of the Carnot cycle, that is $\eta = \frac{T_1 - T_2}{T_1}$, depends upon the range of temperature, and the higher the initial temperature relative to the final temperature the greater the efficiency.

Take the particular case of an engine working on the Carnot cycle between two temperatures T_1 and T_2, where $T_1 - T_2 = 1°$ C. Then the work done by the working substance is

$$\text{Heat supplied} \times \text{efficiency}$$

that is ,, ,, $\times \dfrac{T_1 - T_2}{T_1}$

or, when $T_1 - T_2 = 1$, ,, ,, $\times \dfrac{1}{T_1}$

$$= \frac{\text{Heat supplied}}{\text{abs. temp. at which heat is supplied}}$$

$= \dfrac{H}{T}$, if T is written for T_1.

But this is the work done by the working substance per degree fall of absolute temperature, and this quantity may be referred to as the **change of entropy**; this change can thus be measured by the quantity of heat supplied divided by the absolute temperature at which the heat is received, or $\frac{\mathrm{H}}{\mathrm{T}}$.

The unit of entropy is the **rank** and

$$\text{change in entropy} = \frac{\text{heat change in C.H.U.}}{\text{absolute temp. in } ^\circ\text{C.}},$$

a quantity which is measured in ranks.

Entropy may thus be regarded as the work-producing capacity of heat energy; when heat is supplied at a high temperature, entropy change is small, while heat supplied at a low temperature gives a large change of entropy.

HEAT SUPPLY → AT HIGH TEMPERATURE { ENTROPY CHANGE SMALL

HEAT SUPPLY → AT LOW TEMPERATURE { ENTROPY CHANGE LARGE

Entropy-temperature diagram. When entropy is plotted against absolute temperature (Fig. 228) during the process of heating a substance, the area of the diagram represents the **heat change**. Thus,

$$\text{entropy change} \times T = \text{heat change},$$

and

$$\textbf{entropy change} = \frac{\textbf{heat content change}}{\textbf{absolute temperature}}.$$

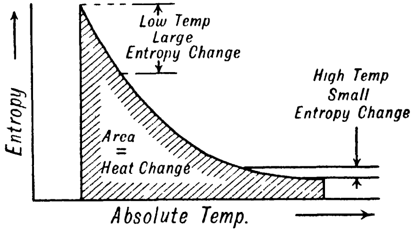


FIG. 228.

Example *Heat is given to a gas at a constant temperature of* 400 *Centigrade degrees absolute. Find the quantity of heat supplied if the change in entropy is* 0·17 *rank.*

$$0\cdot 17 = \frac{\text{change in heat content}}{\text{absolute temp.}} = \frac{H}{400},$$

$$H = 400 \times 0\cdot 17 = \mathbf{68\ C.H.U.}$$

NOTE. *The change in entropy is independent of the units chosen for heat content and absolute temperature, and is the same when B.Th.U. and Fahr. degrees are employed in the heat calculations.*

Thus, in the previous example, converting to Fahr. degrees and reworking :

$$68 \text{ C.H.U.} = \tfrac{9}{5} \times 68 = 122\cdot 4 \text{ B.Th.U.}$$

and $$400 \text{ Cen. degrees} = 492 + \tfrac{9}{5}(400 - 273) = 720\cdot 6 \text{ Fahr. degrees.}$$

$$\text{Change in entropy} = \frac{122\cdot 4}{720\cdot 6} = \mathbf{0\cdot 17 \text{ rank as before.}}$$

Units in entropy calculation. It is very infrequent that any absolute calculation of entropy is required, but the change in entropy is more often demanded. This change must be measured with definite datum levels, and these may be summarised as follows :

The change of entropy is calculated for 1 lb. mass of the working substance, the heat supplied may be in either C.H.U. or B.Th.U,. and the temperature is measured in degrees absolute, either Centigrade or Fahrenheit, *but consistent with the heat units employed*, that is, Centigrade degrees for C.H.U. and Fahrenheit degrees for B.Th.U.

Changes in entropy with heat and temperature changes. It may be accepted that any change in the heat content of a body, or substance, is accompanied by a change of entropy, although it does not necessarily follow that the change of heat content is associated with a temperature change. In adiabatic processes, since no heat change takes place, the entropy is constant, whereas in all processes involving a loss or gain of heat content there is an entropy change.

N.B. *Entropy increases when heat is supplied and decreases when heat is removed, irrespective of temperature changes.*

HEAT CONTENT CHANGES ——————→ENTROPY CHANGES

CONSTANT HEAT ——→ ADIABATIC PROCESSES ——→ ENTROPY CONSTANT

In a process such as throttling, in which no work is done but there is a temperature change, there is an increase of entropy.

Consider 1 lb. of a working substance in which the temperature change is from T_1 to T_2 and c is the specific heat of the substance in its particular condition.

Then if ϕ is the entropy and H the heat content and a small amount of heat δH is given to the substance, the increase of entropy will equal $\delta\phi$.

By definition, $\delta\phi = \frac{\delta H}{T}$, in which the change in temperature is small enough to be considered as maintaining the temperature constant.

But $\delta H = c\delta T$, where δT is the temperature change.

$$\therefore \ \delta\phi = \frac{c\,\delta T}{T}.$$

$$\text{Change of entropy} = \phi_1 - \phi_2 = c\int_{T_1}^{T_2} \frac{dT}{T}$$

$$= c\left\{\log_e T\right\}_{T_1}^{T_2}$$

$$= \mathbf{c}\ \mathbf{log}_e \frac{\mathbf{T}_2}{\mathbf{T}_1}.$$

Thus, **change in entropy** $= \mathbf{c}\ \mathbf{log}_e \frac{\mathbf{T}_2}{\mathbf{T}_1}$.

Example 1. *Calculate the change in entropy when* 1 *lb. of a substance is heated at constant pressure from* 180° *C. to* 260° *C.* $c_p = 0{\cdot}21$.

$$\phi_2 - \phi_1 = 0{\cdot}21 \log_e \left\{\frac{260+273}{180+273}\right\} = 0{\cdot}21 \log_e \frac{533}{453}$$

$$= 0{\cdot}21 \times \log_e 1{\cdot}177 = \mathbf{0{\cdot}0344\ rank.}$$

Example 2. *Heat is supplied at constant volume to* 1 *lb. of a working substance at* 100° *C. If the increase of entropy is* 0·06 *rank and* $c_v = 0{\cdot}14$, *find the final temperature.*

$$\phi_2 - \phi_1 = 0{\cdot}14 \log_e \cdot \frac{t+273}{373} = 0{\cdot}06 \ \mathbf{rank.}$$

$$\log_e \frac{t+273}{273} = 0{\cdot}4285.$$

$$t + 273 = 273 \times 1{\cdot}535 = 419{\cdot}06.$$

$$\mathbf{t = 146°\ C.}$$

Example 3. *Find the change of entropy when superheated steam at 100 lb. per sq. in. abs. is further superheated from 170° C. to 230° C.* $c_p = 0{\cdot}54$.

$$\phi_2 - \phi_1 = c_p \log_e \frac{T_2}{T_1} = 0{\cdot}54 \log_e \left\{\frac{273+230}{273+170}\right\}$$

$$= 0{\cdot}54 \log_e \frac{503}{443} = \mathbf{0{\cdot}0686\ rank.}$$

Entropy of water. The entropy of water is taken as zero when the temperature is 0° C., and entropy changes are calculated upon the absolute temperature corresponding to 0° C., that is, 273 Centigrade degrees absolute. Thus entropy of water at T Cent. degrees abs. is $c \log_e \frac{T}{273}$, and when $T = 273$ Cent. degrees abs.,

$$\log_e \frac{T}{273} = \log_e 1 = 0.$$

Entropy is given to water when the temperature of the water is raised from T_1 to T_2 according to the expression

$$\phi_2 - \phi_1 = c \log_e \frac{T_2}{273} - c \log_e \frac{T_1}{273}$$

$$= c \log_e \left(\frac{T_2}{273} \div \frac{T_1}{273}\right) = c \log_e \frac{T_2}{T_1}.$$

But c, the specific heat of water, is unity, so that

$$\text{entropy change for water} = \phi_2 - \phi_1 = \log_e \frac{\mathrm{T}_2}{\mathrm{T}_1}.$$

Entropy of dry saturated steam. When steam is formed from water, latent heat is given to the newly-formed steam without temperature change. If L is the latent heat per lb. in C.H.U., then change of entropy during evaporation is $\frac{\mathrm{L}}{\mathrm{T}}$, where T is the absolute temperature of formation of steam. The entropy of saturated steam which is raised from water at 0° C. is

$$\log_e \frac{\mathrm{T}_s}{273} + \frac{\mathrm{L}}{\mathrm{T}_s},$$

where T_s is the absolute temperature of steam formation.

Entropy of wet steam. If, during the process of steam formation only a portion of the water at the temperature of formation is converted into steam, that is, if wet steam of dryness fraction x is formed, then the second term in $\log_e \frac{T_s}{273} + \frac{L}{T_s}$ is modified according to the fraction x of the latent heat which is given to the steam, and the entropy of wet steam is thus

$$\log_e \frac{T_s}{273} + \frac{xL}{T_s}.$$

Entropy of superheated steam. When dry saturated steam at a temperature T_s absolute is superheated at constant pressure to $T_{\text{sup.}}$ absolute, the entropy increase is $c_p \log_e \frac{T_{\text{sup.}}}{T_s}$, where c_p is the specific heat at constant pressure ; thus the entropy of superheated steam is

$$\log_e \frac{T_s}{273} + \frac{L}{T_s} + c_p \log_e \frac{T_s}{T_{\text{sup.}}}.$$

Summarising these results for water and steam :

Condition	Heat Content (approximate for low pressures)	Entropy
Water at steam formation temperature	$(T_s - 273)$ C.H.U. per lb.	$\log_e \frac{T_s}{273}$
Saturated steam at formation temperature	$(T_s - 273 + L)$ C.H.U. per lb.	$\log_e \frac{T_s}{273} + \frac{L}{T_s}$
Wet steam at formation temperature	$(T_s - 273 + xL)$ C.H.U. per lb.	$\log_e \frac{T_s}{273} + \frac{xL}{T_s}$
Superheated steam at $T_{\text{sup.}}$, temperature of superheat	$T_s - 273 + L + c_p(T_{\text{sup.}} - T_s)$	$\log_e \frac{T_s}{273} + \frac{L}{T_s} + c_p \log_e \frac{T_{\text{sup.}}}{T_s}$

Example 1. *Calculate the entropy change in each of the following cases for* 1 *lb. of the fluid :*

(*a*) *Water heated from* 30° *C. to* 110° *C.*

(*b*) *Steam dry saturated at* 100 *lb. per sq. in. absolute raised from water at* 16° *C.*

(*c*) *Steam as in* (*b*), *but of dryness fraction* 0·9.

(*d*) *Steam at* 120 *lb. per sq. in. superheated* 100° *C. and raised from water at* 16° *C.*

$c_p = 0{\cdot}52.$

(*a*) $\phi_2 - \phi_1 = \log_e \left(\frac{273 + 110}{273 + 30}\right) = \log_e \frac{383}{303} = 0{\cdot}2343$ **rank.**

(*b*) $T_s = 437{\cdot}5^\circ$ C., $T_1 = 289^\circ$ C., $L = 495{\cdot}7$.

$$\phi_2 - \phi_1 = \log_e \frac{437{\cdot}5}{289} + \frac{495{\cdot}7}{437{\cdot}5}$$

$$= 0{\cdot}4147 + 1{\cdot}133 = \mathbf{1{\cdot}5477\ ranks.}$$

(*c*) $\phi_2 - \phi_1 = 0{\cdot}4147 + 0{\cdot}9 \times 1{\cdot}133 = \mathbf{1{\cdot}4344\ ranks.}$

(*d*) $T_s = 444{\cdot}9$, $T_1 = 289$, $L = 489{\cdot}9$, $c_p = 0{\cdot}52$.

$$\phi_2 - \phi_1 = \log_e \frac{444{\cdot}9}{289} + \frac{489{\cdot}9}{444{\cdot}9} + 0{\cdot}52 \log_e \frac{589{\cdot}9}{444{\cdot}9}$$

$$= 0{\cdot}4311 + 1{\cdot}101 + 0{\cdot}1467 = \mathbf{1{\cdot}6788\ ranks.}$$

Example 2. *Steam at 250 lb. per sq. in. absolute superheated to 320° C. is supplied to a turbine and exhausted at a pressure of 1·5 lb. per sq. in. abs. with a dryness fraction of 0·9. Calculate (a) the heat drop, (b) the entropy change per lb. of steam.* $c_p = 0{\cdot}64$.

Find the necessary dryness fraction of this exhaust steam in order to give no change of entropy.

(*a*) At admission $H_1 = 670{\cdot}6 + 0{\cdot}64(320 - 204{\cdot}9)$

$= 670{\cdot}6 + 73{\cdot}66 = 744{\cdot}26$ C.H.U.

At exhaust $H_2 = 46{\cdot}37 + 0{\cdot}9 \times 569{\cdot}65$

$= 46{\cdot}37 + 512{\cdot}69 = 559{\cdot}06$ C.H.U.

Heat drop $= H_1 - H_2 = \mathbf{185{\cdot}2}$ **C.H.U. per lb.**

(*b*) At admission $T_1 = 593$, $T_s = 478{\cdot}1$, $L - 461{\cdot}5$.

At exhaust $T_2 = 319{\cdot}6$, $L = 569{\cdot}7$, $x = 0{\cdot}9$.

$$\phi_1 = \log_e \frac{478{\cdot}1}{273} + \frac{461{\cdot}5}{478{\cdot}1} + 0{\cdot}64 \log_e \frac{593}{478{\cdot}1} = 1{\cdot}664.$$

$$\phi_2 = \log_e \frac{319{\cdot}6}{273} + 0{\cdot}9 \left(\frac{569{\cdot}7}{319{\cdot}6}\right) = 1{\cdot}7614.$$

Entropy change $= \phi_2 - \phi_1 = \mathbf{0{\cdot}0974\ rank.}$

(*c*)
$$\phi_3 = \log_e \frac{319{\cdot}6}{273} + x \left(\frac{569{\cdot}7}{319{\cdot}6}\right) = \phi_1 = 1{\cdot}664.$$

$$0{\cdot}1570 + x \times 1{\cdot}782 = 1{\cdot}664.$$

$$x = \mathbf{84{\cdot}6\%}.$$

The entropy-temperature chart. A chart showing, diagrammatically, the temperature and entropy of water and steam is published by H.M. Stationery Office, and is reproduced on a reduced scale by courtesy of the publishers on p. 408. This chart (Fig. 229) has an entropy base and a temperature ordinate, and the curves

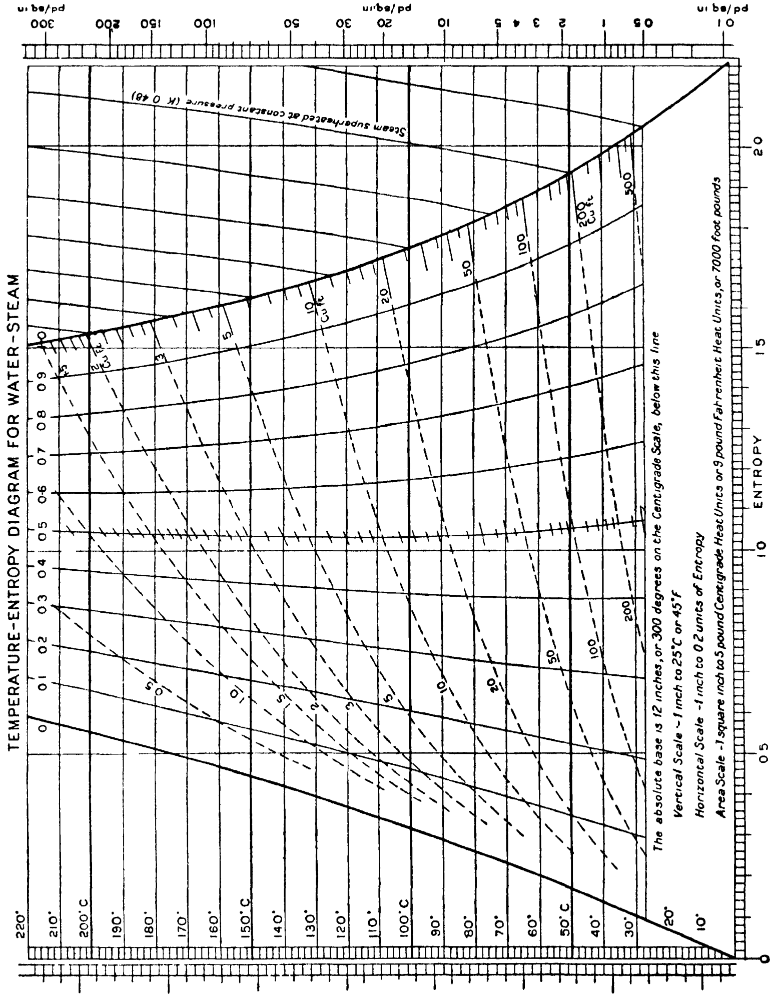

TEMPERATURE-ENTROPY DIAGRAM FOR WATER-STEAM
Steam superheated at constant pressure (K 0·48)
The absolute base is 12 inches, or 300 degrees on the Centigrade Scale, below this line
Vertical Scale - 1 inch to 25°C or 45°F
Horizontal Scale - 1 inch to 0·2 units of Entropy
Area Scale - 1 square inch to 5 pound Centigrade Heat Units or 9 pound Fahrenheit Heat Units, or 7000 foot pounds
ENTROPY
0
0·5
1·0
1·5
2·0
lbs/sq in
300
200
150
100
50
30
20
10
5
4
3
2
1
0·5
0·1
Cu ft
TEMPERATURE
F
400°
350°
300°
250°
200°
150°
100°
50°
220°
210°
200° C
190°
180°
170°
160°
150° C
140°
130°
120°
110°
100° C
90°
80°
70°
60°
50° C
40°
30°
20°
10°
0
0·1
0·2
0·3
0·4
0·5
0·6
0·7
0·8
0·9
1·0
0·5
1
2
3
5
10
20
50
100
200
500

FIG. 229.

shown indicate the temperature-entropy relations for water and steam. The following illustrations will show the methods of interpolation of this chart (Fig. 230):

(*a*) Water at 160° C. Entropy indicated at *A*.

(*b*) Water at 200° C. Entropy indicated at *B*.

(*c*) Dry saturated steam at 200° C. Entropy indicated at *C*.

(*d*) Change of entropy between water at 200° C. and dry saturated steam at 200° C. is indicated by *BC* on the entropy scale.

Entropy *v.* temperature curves are shown for each 0·1 interval of dryness fraction between the water and steam curves, and are marked accordingly 0, 0·1, 0·2, ... 0·9.

(*e*) Wet steam at 200° C., 0·7 dry. Entropy indicated at *D*.

(*f*) Change of entropy between water at 200° C. and wet steam at 200° C. and 0·7 dry is indicated by *BD* on the entropy scale.

The fields in the temperature *v.* entropy charts. The curve on the left of the temperature *v.* entropy chart is known as the **water line,** and is the graph of T *v.* ϕ for water, assuming a constant specific heat of unity, that is, the curve $\phi = \log_e \frac{T}{273}$. The curve on the right of the diagram is the **saturated steam line,** and is determined by adding the entropy due to the latent heat intake to that possessed

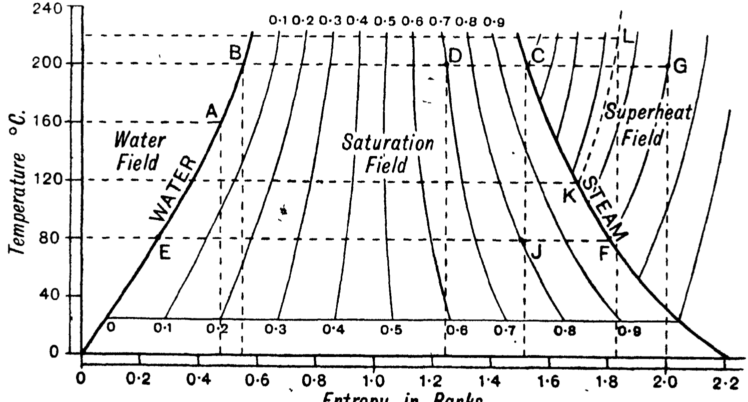


FIG. 230.

by the water at each temperature ; for example, at a temperature of 200° C. the distance *BC* is the entropy due to the latent heat intake on evaporation, that is, $\frac{L}{T}$ or $\frac{L}{(273+200)}$.

The portion to the left of the water line is known as the **water field,** that between the water and steam lines as the **saturation field,** and that to the right of the steam line as the **superheat field.** The superheat field is intersected by a series of curves which show the entropy increase with superheating from the saturation temperature. These curves are derived from the expression

$$\phi_{\text{sup.}} - \phi_{\text{sat.}} = c_p \log_e \frac{T_{\text{sup.}}}{T_{\text{sat.}}},$$

which is the entropy increase due to superheat.

Example 1. Take water at a temperature of 80° C. ; then the horizontal line through the 80° point on the temperature scale will give :

at *E* the entropy of water at 80° C.
J ,, ,, wet steam at 80° C. and 0·8 dry.
F ,, ,, dry saturated steam at 80° C.
G ,, ,, superheated steam, superheated to 200° C.

from a saturation temperature of 80° C.

The entropy of superheated steam is determined by following the superheat curve from the temperature of saturation to the level of the superheat temperature and taking the value on the entropy scale.

Example 2. Take water at a temperature of 120° C. which is converted into steam with a superheat temperature of 220° C. The entropy is obtained by drawing the horizontal through the 120° C. point on the temperature scale to meet the saturated steam line at *K*. From *K* the curve *KL* is drawn in the correct relative position to the superheat curves and concentric with them on the chart to meet the horizontal through 220° C. at *L*. The entropy is measured on the entropy scale vertically below *L*.

Use of the temperature *v.* entropy (T *v.* ϕ) chart. The following examples illustrate the use of the (T *v.* ϕ) diagram for the determination of the entropy of water steam under various conditions.

Examples. *Read off from the temperature entropy chart the entropy of water steam under the following conditions.*

(*a*) Water at 176° C. - - - -	Entropy = **0·475 rank.**
(*b*) Dry saturated steam at 176° C. -	Entropy = **1·57 ranks.**
(*c*) Steam at 150° C., 95% dry - -	Entropy = **1·56 ranks.**
(*d*) Steam at 140° C. saturation temperature superheated to 200° C.	Entropy = **1·74 ranks.**
(*e*) Steam at a pressure of 2 lb. per sq. in. abs., 0·85 dry - - -	Entropy = **1·66 ranks.**

NOTE. *This value can be determined by using the equivalent pressures scale on the extreme right of the diagram.*

The volume lines. If the published form of the T *v.* ϕ diagram is examined, it will be found that a series of chain lines (Fig. 231)

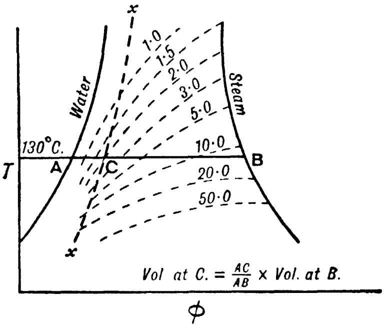


FIG. 231.

intersect the saturation field, and that these lines are marked with volumes in cubic feet. These **volume lines** may be used to determine the volume of 1 lb. of steam under any condition between water and dry saturation.

In Fig. 231 the chain lines across the saturation field indicate a constant volume condition.

Thus, if the evaporation of water from 130° C. is examined, the line AB on the T *v.* ϕ diagram will represent the process of steam formation, and the volume of 1 lb. of saturated steam can be read off at B, and is approximately 11 cu. ft. If steam tables are referred to, this volume is given as 10·72 cu. ft. Now during the formation of this steam, as latent heat is supplied and the dryness fraction approaches unity, so the volume will increase to a maximum at B. If at any given period the dryness fraction is x, then only x of every 1 lb. of water will be converted to steam and the volume will be $x \times 11$ cu. ft.

Example. *Find the volume of* 1 *lb. of steam* 0·7 *dry produced at a temperature of* 150° *C.*

Vol. of dry saturated steam at 150° C. = 6·1 cu. ft.

Vol. of wet steam = 0·7 × 6 272 = **4·39 cu. ft.**

NOTE. *In this procedure the volume of the water which is unformed steam is neglected, but this volume is relatively small and negligible.*

Summary of lines of constant condition. It will be seen from the foregoing examples that certain lines on the T v. ϕ diagram indicate a constant condition ; that is, the condition of the steam remains the same, in one aspect, during a temperature change (Fig. 232).

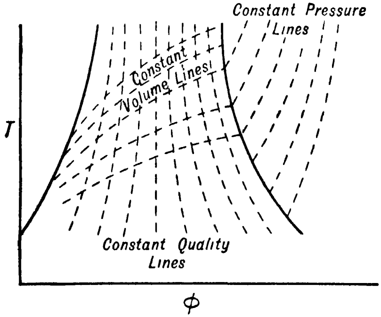


FIG. 232.

(*a*) **Constant pressure lines.** These occur in the superheat field, and indicate a change of temperature and entropy with a condition of *constant pressure.*

(*b*) **Constant dryness lines.** These lines occur in the saturation field, and indicate *constant quality* or dryness with a change of entropy and temperature.

(*c*) **Constant volume lines.** The lines occur in the saturation field, and indicate a change of entropy and temperature with *constant volume.*

Interpretation of operations on the T *v.* ϕ diagram. The ordinary processes employed in steam engine performance may be traced on the T *v.* ϕ diagram, and if the processes in a working cycle are plotted on the diagram the area of the diagram may, with suitable scales, be taken as the work done. The scales in general use on the T *v.* ϕ diagram are :

Entropy scale, 1 inch represents 0·2 rank.

Temperature scale, 1 inch represents 25 Cent. degrees.

It follows that **1 square inch** represents $0{\cdot}2 \times 25 = 5$ **C.H.U.**, since

$$\phi = \frac{H}{T} \quad \text{and} \quad H = \phi T.$$

∴ the area represented by 1 sq. in. of the T *v.* ϕ diagram can be taken as equivalent to

$$5 \text{ C.H.U.} \quad \text{or} \quad 5 \times 1400 = 7000 \text{ ft. lb.}$$

which is known as the **work scale.**

The processes. An **adiabatic process** is one in which there is no loss or gain of heat, and thus the entropy is constant ; hence the difference between the internal energies at the commencement and end of the process is equal to the work done during the process. Thus the expansion curve on a T *v.* ϕ diagram is a *vertical straight line* indicating change of temperature without change of entropy.

An **isothermal process** is at constant temperature, so that for such a process the curve on the T *v.* ϕ diagram is a horizontal straight line.

Thus the T *v.* ϕ diagram for the Carnot cycle is a rectangle $ABCD$ (Fig. 233), in which the work done is represented by the area of $ABCD$, or $(T_2 - T_1)(\phi_2 - \phi_1)$, and the heat supplied is $T_2(\phi_2 - \phi_1)$, since $\phi = \frac{H}{T}$. The efficiency is therefore

$$\frac{\text{work done}}{\text{heat supplied}}, \quad \text{and} \quad \eta = \frac{T_2 - T_1}{T_2}.$$

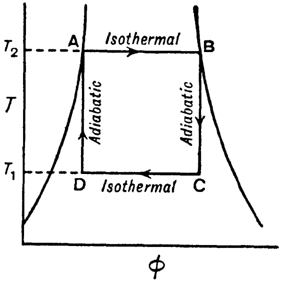


FIG. 233. CARNOT'S CYCLE.

Example 1. *Find the dryness fraction after adiabatic expansion of* 1 *lb. of steam from a temperature of* 200° *C., pressure* 80 *lb. per sq. in. abs. to a pressure of* 3 *lb. per sq. in. abs.*

Determine, on the T v. ϕ diagram, the position which represents the entropy, pressure and temperature before expansion. Draw a vertical line, for adiabatic expansion, to meet the horizontal through 3 lb. per sq. in. Read off the dryness fraction on the constant quality lines.

Answer 0·87.

Example 2. *Use the T v. ϕ diagram to find the work done by* 1 *lb. of steam operated on the Carnot cycle if the temperature range is from* 180° *C. dry saturated to* 40° *C. Find the dryness fraction of the steam after adiabatic expansion.*

In Fig. 234 the expansion commences with water at 180° C. at A and continues isothermally during steam formation to B, and adiabatic expansion of steam to C.

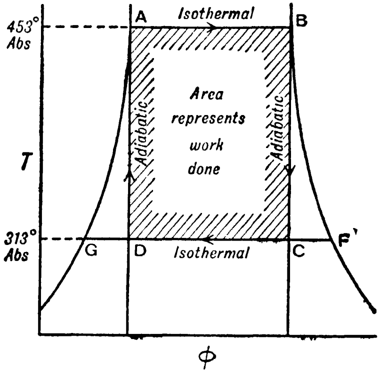


FIG. 234.

Then $ABCD$ is the work diagram for the Carnot cycle.

From the chart :

$$\text{change of temperature} = 180 - 40 = 140^\circ \text{ C.}$$

$$\text{,, ,, entropy} = 1{\cdot}565 - 0{\cdot}51$$

$$= 1{\cdot}055 \text{ ranks.}$$

$$\text{Area} = 140 \times 1{\cdot}055 = 147{\cdot}7 \text{ C.H.U.}$$

or

$$\frac{140}{25} \times \frac{1{\cdot}055}{0{\cdot}2} = 29{\cdot}54 \text{ sq. in.}$$

Since

$$1 \text{ sq. in.} = 5 \text{ C.H.U.,}$$

$$\text{work done} = 147{\cdot}7 \text{ C.H.U.} = \mathbf{206{,}780 \text{ ft. lb.}}$$

$$\text{Dryness fraction after adiabatic expansion} = \frac{GC}{GF} = \mathbf{0{\cdot}775.}$$

Example 3. *Steam at* 140 *lb. per sq. in. abs. and having* 30 *Centigrade degrees of superheat, is supplied to a turbine nozzle and there expanded adiabatically to* 2 *lb. per sq. in. abs. Determine from the entropy chart the final dryness fraction of the steam and calculate the heat drop per lb. of steam during expansion.*

90 *per cent. of this heat drop is converted into kinetic energy of the issuing jet as it leaves the nozzle. Calculate the velocity of the jet.* (U.E.I.)

The dryness fraction is obtained by determining the position on the saturated steam line corresponding to 140 lb. per sq. in. and following the constant pressure line to give 30° of superheat. Then, since the expansion is adiabatic, a vertical straight line on the diagram to meet the horizontal through 2 lb. per sq. in. will give the entropy after expansion. Read off on the constant dryness lines the final dryness fraction, which is **0·83.**

From steam tables:

$$\begin{aligned}\text{Heat drop} &= \text{Initial total heat} - \text{final total heat}\\ &= 682{\cdot}3 - \{52{\cdot}16 + (0{\cdot}83 \times 566{\cdot}51)\}\\ &= 682{\cdot}3 - 522{\cdot}4 = \mathbf{159{\cdot}9\ C.H.U.}\end{aligned}$$

Velocity of jet $= V$.

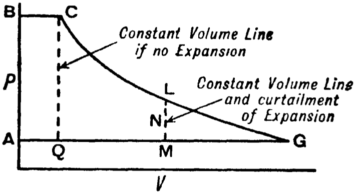
$$\frac{V^2}{2g} = \frac{9}{10} \times \frac{159{\cdot}9 \times 1400}{1},$$

$$V = \mathbf{3602\ ft.\ per\ sec.}$$

THE RANKINE CYCLE.

It has been explained that a modification of the Carnot cycle, namely the Rankine cycle, is more nearly consistent with ordinary steam-engine working and forms a valuable efficiency criterion. This cycle may be operated with saturated or superheated steam, and can be traced on the T v. ϕ diagram as shown in Fig. 235.

The cycle for superheated steam commences with the feed water at a temperature T_1 indicated at A on the diagram. This water is heated to T_2, the temperature of steam formation, B, on the diagram. Evaporation now takes place at constant temperature T_2 until saturation point C is reached. From C to D indicates the supply of superheat, and at D the adiabatic expansion commences, to be completed at the end of the vertical line DE where condensation starts. The steam at this point is generally wet, of dryness fraction $\dfrac{AE}{AF}$, and EA represents the period of isothermal expansion, or condensation, to feed-water temperature.


B
C
Constant Volume Line if no Expansion
P
L
Constant Volume Line and curtailment of Expansion
N
A
Q
M
G
V

CLG represents Expansion to Back Pressure

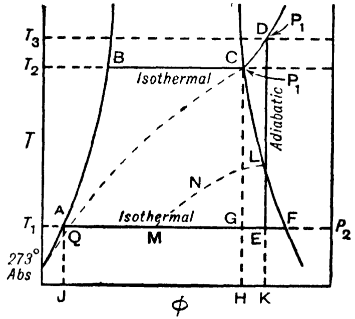


Fig. 235. Rankine Cycle. $ABCDE$ represents work done.

Summarising this :

T v. ϕ	Change in Condition	Process in Cycle
AB	Water at T_1 to water at T_2	Heat to feed water to formation temp.
BC	Water at T_2 to steam at T_2	Evaporation to saturated steam
CD	Steam at T_2 to superheat at T_3	Superheating
DE	Superheated steam to exhaust steam	Adiabatic expansion
EA	Condensation	Isothermal expansion and condensation

From A to D the pressure remains constant at P_1, and from D to A the pressure has fallen to P_2, the pressure at exhaust.

The work done during the cycle is represented by the area *ABCDE* for superheated steam, or the modification *ABCG* for saturated steam.

$$\text{Work done} = \text{heat supplied} - \text{heat rejected}$$
$$= \text{total heat at } D - \text{total heat at } E.$$

Total heat supplied during cycle

$$= \text{total heat at } D - \text{total heat at } A.$$

$$\text{Thermal efficiency} = \frac{\text{work done}}{\text{total heat supplied during the cycle}}.$$

THE RANKINE CYCLE ON THE T v. ϕ DIAGRAM

The mathematical expression for the efficiency on the Rankine cycle is both involved and cumbersome. At this stage the efficiency for any given conditions is most easily obtained by tracing the cycle on a temperature *v.* entropy diagram and finding the ratio between the areas *ABCG* (Fig. 235) and *JABCGH*, representing the heat supplied if the steam is saturated, using a planimeter to determine the areas in sq. in. In the case of superheated steam, the areas *ABCDE*, representing work done, and *JABCDEK*, representing the heat supplied, should be found by planimeter and also their ratio, and hence the efficiency calculated.

In actual steam engines the steam cannot be expanded to the full limit owing to practical conditions ; namely, the cylinders would have to be excessively large and the condensation and waste of steam losses would be prohibitive. Before expansion is complete the steam is exhausted from the cylinder at constant volume when the piston is at the end of its stroke and thus is stationary. The work done is represented by the area *ABCLNM* (Fig. 235) ; where *LNM* is a constant volume line on the T *v.* ϕ diagram. The satisfying of the practical limitations mentioned means a loss of work represented by the area *LNMG*.

A comparison of the P *v.* V and T *v.* ϕ diagrams for saturated steam is shown in Fig. 235 for the modified Rankine cycle, and the corresponding points in the cycle bear the same letters in each diagram.

Example. *Use the T v. ϕ diagram to find the thermal efficiency on the Rankine cycle of an engine receiving feed water at* 40° *C. and working at a top pressure of* 140 *lb. per sq. in. with* 30 *Centigrade degrees superheat.*

Consider 1 lb. of feed water.

From the entropy chart the dryness fraction after adiabatic expansion from steam at 140 lb. per sq. in. superheated 30 Cent. degrees to the exhaust temperature of 40° C. is 0·79.

$$\textit{Heat rejected} = h + 0{\cdot}79L,$$

which, when $t = 40°$ C. and $L = 573$ C.H.U. is

$$40 + 0{\cdot}79 \times 573 = 492{\cdot}67 \text{ C.H.U. per lb.}$$

Heat supplied. From tables, total heat at 140 lb. per in.^{2} and 30 Centigrade degrees of superheat = 682·3 C.H.U. per lb.

$$\text{Work done} = \text{total heat of superheated steam} - \text{total heat rejected}$$
$$= 682{\cdot}3 - 492{\cdot}67 = 189{\cdot}63 \text{ C.H.U.}$$

Total heat supplied during cycle

$$= \text{Total heat of supply steam} - \text{total heat of feed water}$$
$$= 682{\cdot}3 - 40 = 642{\cdot}3 \text{ C.H.U.}$$

$$\text{Rankine thermal efficiency} = \frac{\text{work done}}{\text{heat supplied during the cycle}}$$
$$= \frac{189{\cdot}63}{682{\cdot}3} = 29{\cdot}5\%$$

MECHANICAL REFRIGERATION

The processes of mechanical refrigeration constitute the reversal of the heat engine cycle to produce and maintain low temperatures. This mechanical production of cold is achieved by using a working substance which, under favourable conditions, will absorb heat from its surroundings at a rate equal to or greater than the supply of heat to these surroundings. This heat, which is taken from the cold environs, is then transferred to a receiver at a higher temperature.

According to the second law of thermodynamics, heat will flow of its own accord only from a body at a high temperature to one at a lower temperature (p. 368). In order that heat may be transferred to a body at a higher temperature, *external work must be performed,* and this is supplied in the form of heat which in the course of the cycle is derived from mechanical energy.

There are two classes of refrigerator :

(*a*) *Cold air machines,* in which air is the working substance ;

(*b*) *Vapour compression machines,* in which a vapour capable of receiving a high pressure at low temperatures is used as the working substance.

In the **cold air machines,** air is compressed to about four atmospheres and passed through a series of cooling tubes surrounded by water which carries away the heat due to compression. The air is then passed to a series of tubes surrounded by cold air exhausted from the cold chamber, which further cools the air in the tubes. The chilled air is then allowed to expand in a cylinder, thus losing its heat to the performance of mechanical work against a piston, and this cold air is led to the cold chamber.

Vapour compression machines employ as working substance ammonia, carbon dioxide or sulphur dioxide which, in the form of a wet vapour, charges the cylinder of a compressor in which it is compressed. The next stage is to cool and condense this vapour and allow it to expand through a valve to a low pressure and temperature. Then it is passed through tubes surrounded by the freezing agent, generally brine, which in turn is served to the pipes in the cold chamber. For sea-going and large refrigeration plants, the vapour compression type is more generally used.

Fig. 236 shows diagrammatically the action of a vapour compression machine in which ammonia vapour is used as the working sub-

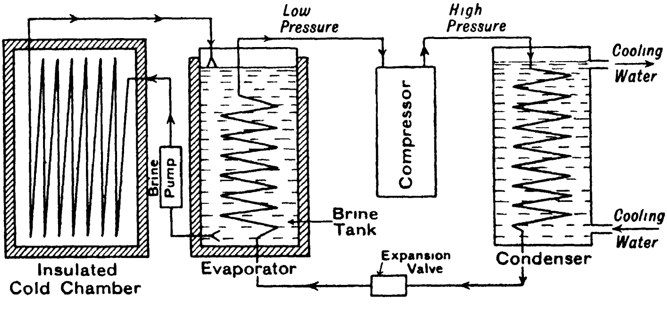


Fig. 236. Diagram of a Vapour Compression Refrigerator

stance. The vapour is compressed in the compressor by mechanical power, and this highly compressed vapour is then taken to the condenser where it is cooled by circulating cooling water. Next it is expanded through the expansion valve into the evaporator, which is a series of tubes immersed in a brine tank. The low pressure, low temperature ammonia is here evaporated and takes up latent heat from the brine, leaving this intensely cold. The ammonia then returns to the compressor to renew the cycle, and a brine pump delivers the cold brine to the service pipes surrounding the cold chamber, which is heat insulated outside the pipes.

Vapour compression cycle. Consider the *PV* diagram for a vapour compression machine (Fig. 237). The horizontal line *ab* represents the suction stage at constant temperature T_2, during which the refrigerant is drawn in at constant pressure and temperature. The adiabatic compression line *bc*, to a temperature T_1, produces in the refrigerant something approximating to a dry saturated condition, and prepares it for condensation at the constant pressure and temperature T_1, when latent heat is given up to the cooling water. This stage is indicated by *cd*, and is followed by the stage *da* when expansion through the reducing valve to the original state brings the refrigerant to a condition suitable for recommencement of the cycle.

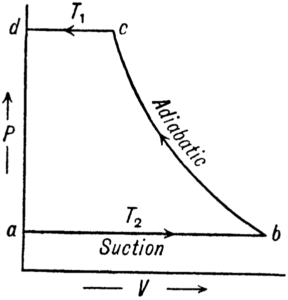


FIG. 237. *PV* DIAGRAM.

The entropy *v.* temperature diagram for this cycle is shown in Fig. 238, in which *da* shows the suction stage at constant temperature, to be followed by the adiabatic compression stage *ab*. The stages *bc* and *cd* indicate condensation at constant temperature T_1 followed by the cooling process in the cooler.

To determine the coefficient of performance of the machine, consider the entropy *v.* temperature diagram (Fig. 238) and one pound of refrigerant:

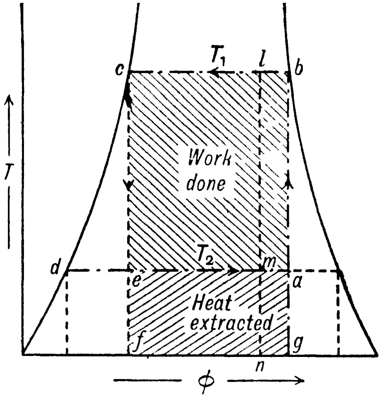


FIG. 238. ENTROPY-TEMPERATURE DIAGRAM.

(1) *ea* is a suction process in which the vapour takes up latent heat at a temperature T_2.

(2) *ab* is adiabatic compression until a dry saturated condition is obtained at T_1.

(3) *bc* is isothermal compression at T_1 during which the vapour discharges latent heat to the cooling water.

(4) *cd*, in which the refrigerant is restored to its original condition or, if a more efficient working is chosen, namely *ce*, involving adiabatic expansion in which the working substance expands in a cylinder and provides motive power to the machine.

Then the coefficient of performance is the ratio

$$\mu = \frac{\textbf{heat extracted}}{\textbf{work done}}.$$

In the more efficient working, if s is the specific heat of the liquid and L its latent heat of evaporation at T_1, then the **heat extracted** from the refrigerant is represented by the area *eafg* and the **work done** on the machine by *abce*.

Hence $$\mu = \frac{\text{heat extracted}}{\text{work done}} = \frac{\text{area } eafg}{\text{area } abce}.$$

Then, $$de = s \log_e \frac{T_1}{T_2} \text{ and } bc = \frac{L}{T_1}.$$

$$\text{Area } eafg \text{ represents heat extracted} = \frac{L}{T_1} \times T_2;$$

$$\text{area } abce \text{ represents work done} = \frac{L}{T_1}(T_1 - T_2).$$

$$\mu = \text{coefficient of performance} = \frac{\text{area } eafg}{\text{area } abce}$$

$$= \frac{\dfrac{L \times T_2}{T_1}}{\dfrac{L}{T_1}(T_1 - T_2)} = \frac{T_2}{T_1 - T_2}.$$

Effect of wetness at the end of compression. When the refrigerant is wet, that is, not dry saturated at the end of compression, the coefficient of performance remains the same, but there is less refrigeration per pound, and thus a larger quantity of liquid will be required for the same amount of refrigeration.

Let x be the dryness fraction after compression.

Then $$x = \frac{cl}{cb}.$$

$$\text{Heat extracted is represented by } emnf = \frac{xL}{T_1} \times T_2.$$

$$\text{Work done is represented by } clme = \frac{xL}{T_1}(T_1 - T_2).$$

$$\mu = \frac{\dfrac{xL}{T_1} \times T_2}{\dfrac{xL}{T_1}(T_1 - T_2)} = \frac{T_2}{T_1 - T_2}.$$

The values of the coefficient of performance calculated by this formula are theoretical values, and the actual values are from 60 to 70% of the theoretical values.

Example 1. *Find the theoretical horse power of a perfect reversed heat engine which will make* 2000 *lb. of ice per hour at* 23° *F. from water at* 65° *F. if* L *for ice is* 140 *B.Th.U. per lb. and the specific heat is* 0·5.

Heat extracted from the water per pound

$$= (65 - 32) + 140 + 0{\cdot}5(32 - 23)$$
$$= 33 + 140 + 4{\cdot}5 = 177{\cdot}5 \text{ B.Th.U.}$$

$$\text{Total heat extracted per hour} = 2000 \times 177{\cdot}5 \text{ B.Th.U.}$$

$$\mu = \text{coefficient of performance} = \frac{T_2}{T_1 - T_2} = \frac{483}{65 - 23}$$
$$= 15{\cdot}1.$$

$$\text{H.P.} = \frac{2000 \times 177{\cdot}5 \times 778}{33{,}000 \times 60 \times 15{\cdot}1} = 9{\cdot}24.$$

Example 2. *An oil engine uses a fuel of calorific value* 19,200 *B.Th.U. per lb. and has a consumption of* 7·4 *lb. of fuel per hour with an overall efficiency of* 30%. *If this engine is used to drive a refrigerator producing ice at* 28° *F. from water at* 65° *F., find the maximum weight of ice which could be produced per hour.* $L = 140$ *B.Th.U. per lb. and* $s = 0{\cdot}5$.

$$\text{Coefficient of performance} = \frac{\text{heat extracted}}{\text{work done}} = \frac{T_2}{T_1 - T_2}$$
$$= \frac{460 + 28}{65 - 28} = 13{\cdot}2.$$

Heat extracted per hour in the formation of W lb. of ice

$$= W\{(65 - 32) + 140 + 0{\cdot}5(32 - 28)\}$$
$$= 175W \text{ B.Th.U.}$$

$$\text{Useful work done per hour} = \tfrac{30}{100} \times 7{\cdot}4 \times 19{,}200 \text{ B.Th.U.}$$
$$= 42{,}624 \text{ B.Th.U.}$$

then
$$175W = 13{\cdot}2 \times 42{,}624.$$
$$W = 3215 \text{ lb. per hour.}$$

Example 3. *An ammonia refrigerator is to produce* 2 *tons of ice per hour at* 28° *F. from water at* 60° *F. If the temperature range in the compressor is between* 78° *F. and* 12° *F., calculate the H.P. required if the efficiency of the machine is* 65%. $L = 140$ *B.Th.U. per lb. and* $s = 0{\cdot}5$.

Heat extracted from the water

$$= (60 - 32) + 140 + 0{\cdot}5(32 - 28)$$
$$= 168 + 2 = 170 \text{ B.Th.U. per lb.}$$

$$\mu = \text{coefficient of performance} = \frac{472}{66} = 7{\cdot}15.$$

$$\text{H.P.} = \frac{100}{65}\left\{\frac{170 \times 2240 \times 2 \times 778}{7{\cdot}15 \times 60 \times 33{,}000}\right\}$$
$$= 64{\cdot}49.$$

EXERCISES ON CHAPTER XVI

1. State, in your own words, the relationship between heat change and entropy change, and show that the entropy change is a function of both heat change and absolute temperature.

2. What is the unit of entropy? Find the quantity of heat supplied to a gas at a temperature of 290° C. in order to produce an entropy change of 0·21 unit.

3. Find the temperature of a gas which suffers a heat content change of 74 C.H.U. per lb. with an entropy change of 0·14 rank.

4. Calculate the change in entropy when one pound of a substance is heated at constant pressure from 170° C. to 290° C. $c_p = 0{\cdot}2$.

5. A gas at 175° C. is heated at constant volume. If the entropy increase per lb. is 0·07 rank and $c_v = 0{\cdot}14$, find the final temperature.

6. Find the change of entropy when superheated steam at 150 lb. per sq. in. abs. is further superheated through 40° C. from 200° C. $c_p = 0{\cdot}59$.

7. Calculate the entropy change in each of the following cases.

(*a*) Water heated from 20° C. to 100° C.

(*b*) Water at 22° C. raised to dry saturated steam at 120 lb. per sq. in. abs.

(*c*) Steam as in (*b*), but 0·85 dryness fraction.

(*d*) Steam as in (*b*), but with 90 Centigrade degrees of superheat if $c_p = 0{\cdot}578$.

8. Calculate the heat drop and entropy change when steam superheated to 300° C. at a pressure of 220 lb. per sq. in. abs. is supplied to a turbine and exhausted at 2 lb. per sq. in. abs. Dryness fraction, 0·85. $c_p = 0{\cdot}62$.

9. Use the temperature-entropy diagram to find the entropy of water steam under each of the following conditions :

(*a*) Water at 160° C.

(*b*) Dry saturated steam at 160° C.

(*c*) Steam at 170° C., 0·9 dryness fraction.

(*d*) Steam at 100 lb. per sq. in. absolute, superheated 40° C.

(*e*) Steam at 1·5 lb. per sq. in. absolute, 0·9 dryness fraction.

10. Find the volume of 1 lb. of steam of dryness fraction 0·85 and at a temperature of 140° C.

11. Find the dryness fraction of steam which has a volume of 4·4 cu. ft. per lb. at 150° C.

12. Mark on a T *v.* ϕ diagram the following processes : (*a*) isothermal expansion at 120° C.; (*b*) adiabatic expansion from 140° C., assuming dry saturated steam.

13. Find the dryness fraction after the expansion adiabatically of 1 lb. of steam from 85 lb. per sq. in. abs. and temperature 190° C. to a pressure of 2 lb. per sq. in. abs.

14. One pound of dry saturated steam is utilised on a Carnot cycle between temperature limits of 200° C. and 30° C. Find the work done per lb.

15. Find the H.P. of an engine working on the Carnot cycle between 170° and 30° C. if 7·4 lb. of steam is used per minute?

16. Find from the T v. ϕ chart the final dryness fraction of steam at 130 lb. per sq. in. abs. with 20 Cent. degrees of superheat if it is expanded adiabatically to 2 lb. per sq. in. abs. What is the heat drop per lb.? If this expansion is through a nozzle of 90% efficiency, find the final velocity of the steam.

17. Find the Rankine efficiency of an engine receiving feed water at 47° C. and working to a top pressure of 120 lb. per sq. in. abs. with 35 Centigrade degrees superheat.

18. What is meant by the entropy of a gas?
Steam at 140 lb. per sq. in. and having 30° C. of superheat, is supplied to a turbine nozzle and there expanded adiabatically to 2 lb. per sq. in. Determine from the entropy chart supplied the final dryness fraction of the steam, and calculate the heat drop per pound of steam during expansion. 90% of this heat drop is converted into kinetic energy of the issuing jet as it leaves the nozzle. Calculate the velocity of the jet. (U.E.I.)

19. Outline the methods of mechanical refrigeration on a reversed heat engine cycle. What are the essential requirements of the refrigerant, and what substances are generally used?

20. Describe, with sketches, a vapour compression machine for use with ammonia gas as the refrigerant.

21. Sketch the P v. V and T v. ϕ diagrams for a vapour compression machine, and explain the stages in the cycle.

22. What is coefficient of performance? Find the coefficient of performance for a reversed heat engine working between temperature ranges of 70° F. and 22° F.

23. Find the theoretical H.P. of a perfect reversed heat engine to produce 1000 lb. of ice per hour at 26° F. from water at 60° F., if L for ice is 140 B.Th.U. per lb. and $s = 0{\cdot}5$.

24. Find the weight of ice produced per hour from a refrigerator which is driven by an oil engine using 10·7 lb. of fuel per hour of calorific value 20,000 B.Th.U. per lb. if the ice is produced at 24° F. from water at 63° F. with an overall efficiency of 34%.

25. An ammonia refrigerator is to be designed to produce 1 ton of ice per hour at 27° F. from water at 65° F. and a temperature range in the compressor between 75° F. and 15° F. Calculate (*a*) the coefficient of performance; (*b*) the H.P. required if the refrigerator efficiency is 70%. $L = 140$ B.Th.U. per lb., $s = 0{\cdot}5$.

26. Calculate the quantity of ice produced by an ammonia refrigerator receiving 20 H.P. and having a compressor range between 76° and 14° F., if the efficiency of the refrigerator is 65% and the ice is produced at 28° F. from water at 68° F. $L = 140$ B.Th.U. per lb., $s = 0{\cdot}5$.

CHAPTER XVII

EXPANSION OF STEAM IN NOZZLES; STEAM TURBINES

THE steam turbine was developed towards the end of the nineteenth century, and provides a fast-running well-balanced steam engine, free from reciprocating parts, and eminently suited to be the prime mover in electrical generating plants. It has been adapted by means of reduction gearing to the propulsion of ships, and it is now common for ocean-going vessels and warships to mount turbines up to 50,000 H.P., and in isolated cases turbines have been installed to develop 160,000 H.P. Turbo-electric installations in various parts of the world generate between 100,000 and 200,000 H.P., which provide speeds of rotation between 2,000 and 3,000 revolutions per minute.

The essential difference between the reciprocating and turbine engine, from the thermodynamical point of view, is that the latter does not utilize the force exerted by expanding steam as the piston effort, but converts a quantity of the heat energy of this steam to kinetic energy which is used to give high velocity to jets of steam led on to vanes attached to the periphery of the rotating portion of the turbine. These high-velocity jets are deflected by the vanes and the change of momentum is employed, either by **impulse** or **reaction**, to drive the rotor, or revolving portion of the turbine.

Expansion of steam in nozzles. The preparation of the steam jet for use in turbine operation of the **impulse type** is achieved by allowing steam to expand through a suitably designed nozzle without performing external work, but with a reduction of its internal energy. This loss of internal energy is converted to kinetic energy and is given to the issuing steam, so that a high-velocity jet results. Nozzles are designed to give a very small frictional resistance to the steam flow and to effect an expansion which is, as nearly as possible, adiabatic. The law for nozzle expansion is of the form $PV^n = C$, where n is a value dependent upon the shape of the nozzle and the pressure range over which expansion takes place.

Limitation of pressure range. A single nozzle is limited in the range of pressures through which it will expand steam, and for turbine work the expansion from high-pressure steam to the very low pressure of final exhaust is undertaken in stages with separate nozzle expansion prior to each stage. These stages are generally arranged for equal pressure drops per stage; the turbine is known as a **multi-stage** turbine when such stage expansion of the steam is used. The ratio between the pressure at discharge from the nozzle and the admission pressure is termed the **pressure ratio**, and the pressure which is the minimum at which the nozzle can discharge for a given admission pressure is called the **critical pressure.**

It has been determined that the pressure ratio for maximum steam delivery through a nozzle is 0·58 when the value of n in $PV^n = C$ is 1·135, and 0·5457 when n is 1·3; thus the critical pressure varies between 0·58 to 0·5457 of the admission pressure, according to the variation of n between 1·135 and 1·3 in the expansion law.

Example. *If the steam supply to a nozzle is at* 200 *lb. per sq. in. absolute and the ratio of expansion, or pressure ratio, is* 0·5457, *find the pressure at exit* (*a*) *for a single nozzle,* (*b*) *for four stages of expansion.*

(*a*) Exit pressure $= 0{\cdot}5457 \times 200 = 109{\cdot}1$ **lb. per sq. in.**

(*b*) After 1st stage, $p_1 = 200 \times 0{\cdot}5457$.
,, 2nd ,, $p_2 = 200 \times 0{\cdot}5457^2$.
,, nth ,, $p_n = 200 \times 0{\cdot}5457^n$.
,, 4*th* ,, $p_4 = 200 \times 0{\cdot}5457^4 = 17{\cdot}74$ **lb. per sq. in.**

Design of nozzles. The fundamental principles in nozzle design are: (*a*) to provide an expansion nozzle free from frictional resistance to steam flow; and (*b*) to allow for free expansion without loss of internal energy due to obstruction. The complete achievement of these two aims would produce expansion on strictly adiabatic lines, but in actual practice there are certain frictional losses and the nozzle has to be designed to meet the pressure ranges required in the particular stage. For given inlet and outlet pressures the throat area of the nozzle determines the weight of steam passing per second.

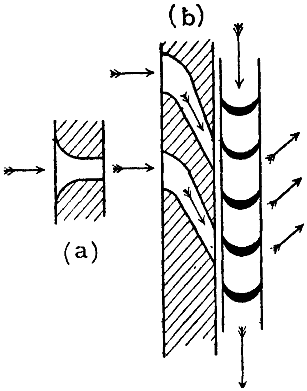


Fig. 239.
Convergent Type.

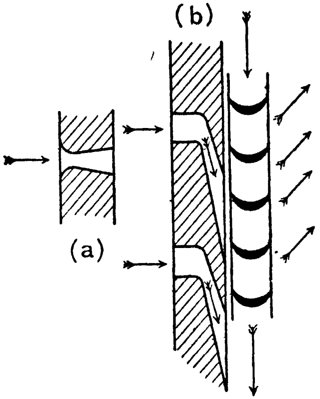


Fig. 240.
Convergent-Divergent Type.

There are two types of nozzle in general use: (1) the *convergent nozzle* and (2) the *convergent-divergent* type.

Convergent nozzles. In this type of nozzle the steam is expanded through a convergent opening (Fig. 239), the theoretical shape of which is shown in (*a*) and its application to the turbine in (*b*). In the turbine diaphragm (Fig. 239(b)) the converging shape of the nozzle is maintained, but it is set at an angle so that the issuing jet strikes the rotor vane at the correct angle. The employment of this type of nozzle is restricted to cases where the pressure ratio is not less than 0·58.

Convergent-divergent nozzles. This type of nozzle is used when the pressure ratio falls between 0·58 and 0·5457, and provides for a rapidly converging diameter to a constriction throat (Fig. 240), followed by a more slowly diverging section. During expansion through the diverging section, the steam reaches a pressure below the critical pressure for the nozzle, with a consequent increase of heat drop, or loss of internal energy, followed by restoration to the critical pressure on discharge to the vanes. Since the delivery velocity depends upon the heat drop, or the quantity of heat energy convertible to kinetic energy, this type of nozzle gives the maximum velocity to the steam jet consistent with a given pressure ratio. The arrangement of nozzles in the turbine rotor is similar to that used for convergent nozzles, but the alteration in steam direction is more marked, with a consequent increase in the energy losses due to this cause.

Summary

Nozzle Shape	Pressure Ratio	Value of n in $PV^n = C$
Convergent - - -	Not less than 0·58	Less than 1·135
Convergent-divergent -	Between 0·58 and 0·5457	Between 1·135 and 1·3

NOTE. *Convergent-divergent nozzles may be used when the pressure ratio is below* 0·5457, *but this is a limiting value for the ratio when a maximum discharge is required, and any reduction in the pressure ratio will affect the efficiency adversely.*

Heat drop and steam velocity. It has been stated that the function of the nozzle is to convert a fall in the internal energy of the steam or *heat drop* to kinetic energy. This heat drop can be determined by measurement of the heat content of the steam at admission and exit, assuming adiabatic expansion over the range of pressures employed and, since it is converted to kinetic energy, the heat drop is equivalent to the kinetic energy of the jet; that is, $\frac{WV^2}{2g}$, if the correct units are employed and losses are neglected.

Let H_1 be the total heat of 1 lb. of steam in C.H.U. at admission, and H_2 the total heat at discharge; then the heat drop is $(H_1 - H_2)$ C.H.U., or $J(H_1 - H_2)$ ft. lb., where J is Joule's equivalent or the mechanical equivalent of heat, namely, 1400 ft. lb. = 1 C.H.U. Upon conversion to kinetic energy this quantity $J(H_1 - H_2)$ becomes $\frac{WV^2}{2g}$, where W is the weight of steam in lb. and V is the velocity of the jet in ft. per sec. Hence, for 1 lb. of steam, $J(H_1 - H_2) = \frac{V^2}{2g}$

$$\text{and} \quad V = \sqrt{2g \, . \, J \, . \, (H_1 - H_2)}.$$

Entropy-temperature aspect of the expansion. Fig. 241 shows the T *v.* ϕ diagram for the expansion of dry saturated steam through a

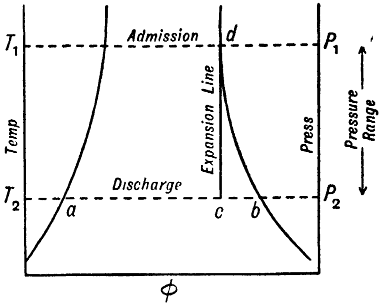


FIG. 241.

nozzle, in which the adiabatic line *dc* indicates the expansion from the dry saturated condition P_1T_1 at *d* to the wet steam condition P_2T_2 at *c*. The dryness fraction x at discharge is given by the ratio $\frac{ac}{ab}$, so that the total heat at discharge H_2 is the sensible heat at T_2 plus x times the latent heat of dry saturated steam at this temperature. The heat drop is therefore $H_1 - H_2$, or the total heat of dry saturated steam at T_1 minus the total heat of wet steam of dryness fraction x at T_2.

Example. *Dry saturated steam at a pressure of* 200 *lb. per sq. in. abs. is expanded through a nozzle with a pressure ratio of* 0·58. *Find* (*a*) *the dryness fraction after expansion,* (*b*) *the heat drop,* (*c*) *the theoretical velocity of the jet.*

Final pressure = 200 × 0·58 = 116 lb. per sq. in.

(*a*) From the T *v.* ϕ diagram, $x = \frac{1·56 - 0·5}{1·59 - 0·5} = \frac{1·06}{1·09} = 0·972$.

(*b*) From steam tables, when $P_1 = 200$ lb. per sq. in. and

$P_2 = 116$ lb. per sq. in.,

$H_1 = 668·6$ C.H.U., and $H_2 = 172·1 + 0·972(491·0)$ C.H.U.

Heat drop $= H_1 - H_2 = 668·6 - 649·3 =$ **19·3 C.H.U. per lb.**

(*c*) $V = \sqrt{2gJ(H_1 - H_2)}$

$= \sqrt{64·4 \times 1400 \times 19·3} =$ **1319 ft. per sec.**

Superheated steam at admission. When superheated steam is employed, the expansion may or may not produce wet steam at discharge according to the degree of superheat and the ratio of

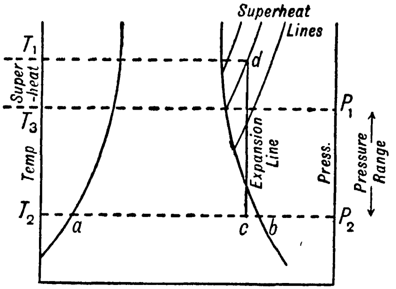


FIG. 242.

pressures during expansion. The T v. ϕ diagram for such a condition is shown in Fig. 242. Steam is admitted at a pressure of P_1 superheated from T_3, the saturation temperature, to T_1, and discharged at a pressure of P_2. The expansion line dc may terminate (1) at c to indicate a discharge either of wet steam, in which case the dryness fraction is $\frac{ac}{ab}$, or (2) at b to indicate a discharge of dry saturated steam, or (3) at a point in the superheat zone indicating a discharge of steam still superheated. The position of c is determined by the amount of initial superheat and the pressure ratio employed. The calculation of heat drop is similar in method to that used for saturated steam, if account is taken of the conditions of admission and discharge.

Example 1. *Steam is admitted to a nozzle at a pressure of* 200 *lb. per sq. in. absolute superheated to* 220° *C. If the pressure ratio is* 0·5457, *calculate* (*a*) *the heat drop,* (*b*) *the theoretical velocity of the jet.*

$$\text{Final pressure} = 0{\cdot}5457 \times 200 = 109{\cdot}14 \text{ lb. per sq. in.}$$

(*a*) From the T v. ϕ diagram, $x = \frac{1{\cdot}576 - 0{\cdot}492}{1{\cdot}6 - 0{\cdot}492} = 98\%$.

From steam tables : $H_1 - H_s + c(T_{\text{sup.}} - T_{\text{sat.}})$, where c is specific heat and $T_{\text{sup.}}$ and $T_{\text{sat.}}$ the temperatures of superheat and saturation respectively.

$$H_1 = 668{\cdot}6 + 0{\cdot}6209(493 - 467{\cdot}4)$$
$$= 668{\cdot}6 + 15{\cdot}9 = 684{\cdot}5 \text{ C.H.U. per lb.}$$
$$H_2 = 169{\cdot}42 + 0{\cdot}98(493{\cdot}0)$$
$$= 652{\cdot}6 \text{ C.H.U. per lb.}$$

$$\text{Heat drop} = H_1 - H_2 = 684{\cdot}5 - 652{\cdot}6$$
$$= \mathbf{31{\cdot}9\ C.H.U.\ per\ lb.}$$

(*b*)
$$V = \sqrt{2gJ(H_1 - H_2)}$$
$$= \sqrt{64{\cdot}4 \times 1400 \times 31{\cdot}9} = \mathbf{1696\ ft.\ per\ sec.}$$

Example 2. *Find the velocity of the jet when steam is expanded through a nozzle on a pressure ratio of* 0·5457 *from an admission condition in which the pressure is* 250 *lb. per sq. in. and there are* 100 *Centigrade degrees of superheat. Take the mean specific heat as* 0·6.

This is a case outside the range of the published T *v.* ϕ diagrams and with a discharge of steam still superheated. Take the expansion law as $PV^{1\cdot3} = C$, corresponding to a pressure ratio of 0·5457.

Final pressure = 250 × 0·5457 = 136·4 lb. per sq. in.

Taking unit volume at admission:

$$P_1V_1^{1\cdot3} = P_2V_2^{1\cdot3},$$
$$250 \times 1^{1\cdot3} = 136{\cdot}4\,V_2^{1\cdot3}.$$
$$V_2 = \sqrt[1\cdot3]{\frac{250}{136{\cdot}4}} = 1{\cdot}593 \text{ units.}$$

Then $$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}; \text{ i.e. } \frac{250 \times 1}{(478 + 100)} = \frac{136{\cdot}4 \times 1{\cdot}593}{T_2}.$$

$$T_2 = \frac{136{\cdot}4 \times 1{\cdot}593 \times 578}{250} = 502 \text{ Centigrade degrees abs.}$$

Thus the final condition of the steam is $P_2 = 136{\cdot}4$ lb. per sq. in. superheated to a temperature of 502 − 273 = 229° C. The saturation temperature at this pressure is 177·2° C.; thus the superheat is

$$229 - 177{\cdot}2 = 51{\cdot}8 \text{ Cent. degrees.}$$

Where $P = 250$ lb. per sq. in. with 100 Centigrade degrees superheat,

$$H_1 = 670{\cdot}6 + 0{\cdot}6 \times 100 = 730{\cdot}6 \text{ C.H.U.}$$

Where $P = 136{\cdot}4$ lb. per sq. in. with 51·8 Centigrade degrees superheat,

$$H_2 = 664{\cdot}8 + 0{\cdot}6 \times 51{\cdot}8 = 695{\cdot}9 \text{ C.H.U.}$$

$$\text{Heat drop} = H_1 - H_2 = 730{\cdot}6 - 695{\cdot}9 = 34{\cdot}7 \text{ C.H.U.}$$

$$V = \sqrt{2g\,.\,J\,.\,(H_1 - H_2)}$$
$$= \sqrt{64{\cdot}4 \times 1400 \times 34{\cdot}74} = \mathbf{1770\ ft.\ per\ sec.}$$

Example 3. *The last stage of a steam turbine is to exhaust at* 2 *lb. per sq. in. absolute,* 0·8 *dryness fraction, with a velocity of* 1000 *ft. per sec.*

Find the condition of the steam at admission to the last stage if the pressure ratio is 0·58.

$$V = 1000 \text{ ft. per sec.}$$

$$\text{Heat drop} = \frac{V^2}{2gJ} = \frac{1{,}000{,}000}{64{\cdot}4 \times 1400} = 11{\cdot}09 \text{ C.H.U.}$$

$$H_2 \text{ at discharge} = 52{\cdot}16 + 0{\cdot}8 \times 566{\cdot}5 = 505{\cdot}4 \text{ C.H.U.}$$

$$H_1 \text{ at admission} = 505{\cdot}4 + 11{\cdot}09 = 516{\cdot}49 \text{ C.H.U.}$$

$$\text{Pressure at admission} = \frac{2}{0{\cdot}58} = 3{\cdot}45 \text{ lb. per sq. in.}$$

Let x be the dryness fraction at admission, then

$$H_1 = h + xL = 63{\cdot}77 + x \times 560{\cdot}1,$$

$$560{\cdot}1x = 516{\cdot}49 - 63{\cdot}77 = 452{\cdot}72.$$

$$\text{Dryness fraction} = x = \frac{452{\cdot}7}{560{\cdot}1} = 0{\cdot}808.$$

Nozzle efficiency. However carefully a nozzle may be designed and manufactured, there will be some losses due to friction and internal resistance which will affect the velocity of discharge. The **coefficient of velocity** is the ratio

$$\frac{\text{actual discharge velocity}}{\text{theoretical discharge velocity}},$$

a quantity denoted by K_v and with a value generally between 0·9 and 0·98. The efficiency of the nozzle is given by the ratio

$$\frac{\text{actual K.E. per lb. at discharge}}{\text{theoretical K.E. per lb. at discharge}},$$

and this can be related to the coefficient of velocity as follows :

Let V be the theoretical velocity of discharge, then K_vV is the actual velocity of discharge.

$$\textbf{Nozzle efficiency } \eta = \frac{(K_vV)^2}{V^2} = (\mathbf{K}_v)^2.$$

Example. *The actual heat drop in a certain nozzle is* 19·4 *C.H.U. per lb. and the actual velocity of the issuing steam is* 1250 *ft. per sec. Calculate* (*a*) *the coefficient of velocity,* (*b*) *the nozzle efficiency.*

$$\text{Theoretical velocity} = V = \sqrt{2gJ(H_1 - H_2)}$$

$$= \sqrt{64{\cdot}4 \times 1400 \times 19{\cdot}4} = 1322 \text{ ft. per sec.}$$

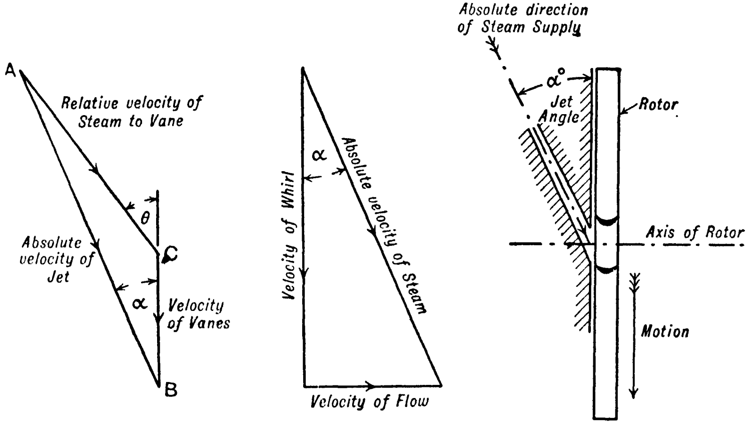
(*a*) Coefficient of velocity $= \dfrac{1250}{1322} = \mathbf{0{\cdot}9452.}$

(*b*) Nozzle efficiency $= 0{\cdot}9452^2 = \mathbf{0{\cdot}8934.}$

The Turbine as a Prime Mover

It has been shown that the purpose of the nozzles is to give the steam jets a high velocity, and it is now necessary to utilize the energy possessed by the jet to exert a rotating force upon the rotor. This is done by directing the steam jets to impinge on correctly designed vanes which divert the direction of the jets, and allow the change of momentum so induced to exert a rotating force on the rotor to which the vanes are attached. (Also see Vol. II, p. 210.)

The steam as directed by the nozzle (Fig. 243) has a certain absolute velocity determined by the heat drop in the nozzle, and is led on to the vanes at an angle α to the direction of rotation of the rotor or vanes. The effective momentum at inlet is determined by the *relative velocity* of the steam to the vanes, and this is the result of the vector subtraction of these two velocities. In the vector diagram, let AB represent the absolute velocity of the steam to scale and at



(a)

Fig. 243. Inlet Vector Diagram.

an angle α to BC, which represents the absolute velocity of the vanes to the same scale. Then AC will represent in direction and magnitude the vector difference of these two velocities, and will be the relative velocity of the steam to the vanes. This will be the effective velocity at which the steam jets reach the blades.

Velocities of whirl and flow. The absolute velocity of the steam discharged from the nozzle may be resolved into two components (Fig. 243, *a*), one parallel to the direction of motion of the vanes called the **velocity of whirl**, and the other parallel to the line in the direction of the rotor axis known as the **velocity of flow.** The velocity of whirl is the factor required when calculating the tangential thrust on the turbine rotor, and the velocity of flow when calculating the end thrust on the turbine axis.

The exit vector diagram is concerned with the velocity of the discharged steam after it has passed through the vanes (Fig. 244). This velocity is the relative velocity of the discharged steam to the vanes, and the problem is generally to find the absolute velocity of discharge, the velocities of whirl and flow and the angle of the discharge steam to the direction of rotation. This becomes a case of vector addition. In the outlet vector diagram, let BC be the relative velocity of the outlet steam to the vane and AB the velocity of the vane; then AC is the vector sum of these velocities and is the absolute velocity of the steam at discharge. AD is the component

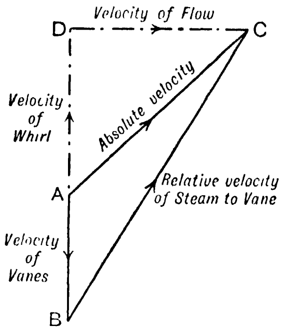


FIG. 244. OUTLET VECTOR DIAGRAM.

of *AC* in the direction of the vane motion or the velocity of whirl, and *DC* the component along the rotor axis, or the velocity of flow.

Design of vanes to eliminate shock. It is essential that the steam entering a turbine vane shall do so without shock, and that it shall be slowly and smoothly deflected by the vane to leave without loss of speed. Actually there are losses within the vane, but the aim in design is to minimize these losses, so that the steam will leave the vanes after having received a change in direction without impact or shock. In order that this may be achieved, the angle of the vane at inlet must be that of the relative velocity between the inlet jet and the vanes, that is, at the angle θ shown in Fig. 243. Similarly, the jet should be taken from the vane in the direction of the absolute velocity of discharge, *AC* in Fig. 244, at an angle *DAC* to the tangent *BAD* at the exit tip of the vanes. If the blades or vanes are designed according to these principles, the inlet and outlet of steam will be free from shock.

Reaction and impulse turbines. The distinction between reaction and impulse turbines may be stated as follows : Reaction turbines are turbines in which the fall of pressure and conversion of heat energy to kinetic energy is performed partly in a series of fixed vanes and partly in the revolving vanes, whereas the impulse turbine operates this change of energy wholly in separate nozzles. In practice, many modern turbines are a combination of the two types, and although turbines exist which are purely impulse or reaction, the two principles are readily overlapped when advantages can be obtained by this course.

The De Laval turbine. This is the best known of the single-stage impulse turbines, in which steam is expanded through a nozzle and the high-velocity steam directed upon vanes. The whole of the expansion is done in one stage, and the resulting velocity of the steam is unusually high. To run the wheel with a blade speed to give maximum efficiency, that is, at about one half of the steam speed, excessive centrifugal forces would be encountered, and for this reason the rotor is run at a comparatively low speed with loss of blade efficiency.

Example. *A De Laval turbine receives saturated steam at a pressure of 200 lb. per sq. in. and exhausts into the atmosphere. The vanes are run* *at a speed of* 1000 *ft. per sec. and the nozzles are set at* 20° *to the plane of the rotor. Calculate* (*a*) *the jet velocity,* (*b*) *the angle of the blade tip at entry.*

From the T *v.* ϕ diagram, dryness fraction after expansion is 0·865.

At inlet, $H_1 = 668{\cdot}6$ C.H.U. per lb.

At outlet, $H_2 = 100 + 0{\cdot}865 \times 539{\cdot}3 = 566{\cdot}5$ C.H.U. per lb.

Heat drop $= H_1 - H_2 = 102{\cdot}1$ C.H.U. per lb.

(*a*) Velocity of jet $= \sqrt{2gJ(H_1 - H_2)} = \sqrt{64{\cdot}4 \times 1400 \times 102{\cdot}1}$

$= \mathbf{3034}$ **ft. per sec.**

(*b*) To find the angle of the blade tip at entry, draw the vector diagram and determine the angle of the relative velocity of jet to vanes. This is the angle at which the blade tip must be made to avoid shock at entry. In Fig. 245 draw V_2 to represent the velocity of the vanes and V_1 that of the steam jet. Complete the vector subtraction to find the relative velocity of jet to vanes., that is, draw V_R. Measure the angle between V_R and V_2, and this is the required angle of the vane tips. **Ans. 30°.**

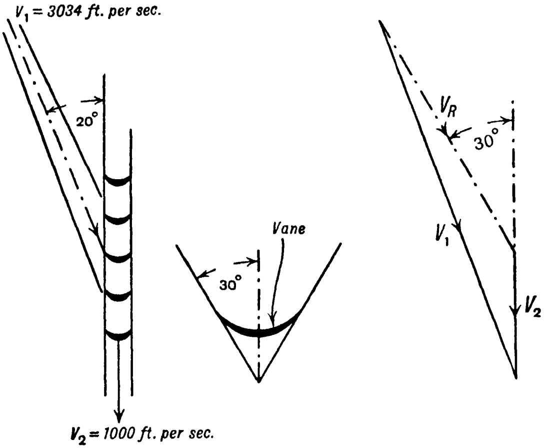


FIG. 245.

A multi-stage reaction turbine. In this class of turbine, notably the **Parsons type,** the nozzles of the impulse turbine are replaced by a ring of stationary vanes through which the steam expands. The steam thus expanded is directed upon a ring of moving blades as illustrated in Fig. 246, and here its direction is altered for the first power stage. The outlet from this stage is then further expanded through a second ring of stationary vanes and prepared for entry to the second power stage, another ring of moving blades. This stage expansion is continued until the exhaust pressure is reached, and the sets of moving blades are coupled by a common shaft to become the rotor of the turbine. The stationary blades are attached to the turbine casing, and the complete arrangement of stationary blades incorporated in the turbine casing is called the **stator.**

The vector diagram for this type of turbine is shown in Fig. 246. *There is a pressure fall across both stationary and rotating vanes, with a consequent release of heat energy. This causes an increase of velocity in the moving blades while the steam is in motion, so that the steam velocity at outlet, relative to the blade speed, is greater than that at inlet.*

In Fig. 246 V_1 represents the absolute velocity of the steam leaving one of the stationary vanes, V_2 the rotor velocity of the rotating vanes and R the relative velocity of V_1 to V_2. The steam is then expanded through the stationary vanes with a relative velocity of entry V_3 and absolute velocity V_4. Thus the vector diagram for the stationary blades is a reversal of that for the rotating blades and the velocity of entry to the next set of rotating blades is V_4, which

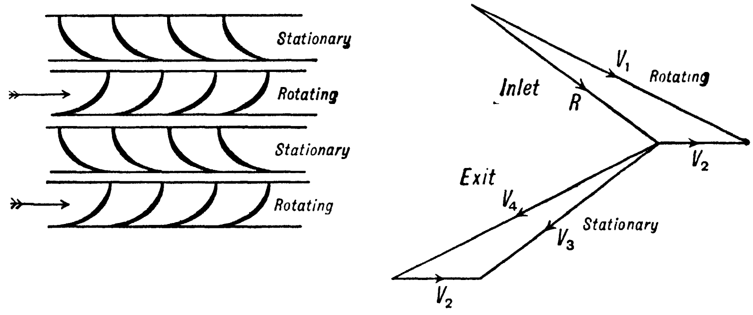


FIG. 246.

DIAGRAM OF BLADING.
MULTI-STAGE REACTION TURBINE.

VECTOR DIAGRAM
FOR ONE STAGE.

is equal to V_1, a condition due to the stationary and moving vanes being the same type. The process is repeated throughout the stages with theoretically no loss of velocity, because the loss in the stationary blades is made up by expansion in the rotating blades.

Force acting on a turbine rotor. The force exerted upon the turbine blades by the steam is the rate of change of momentum in the direction of the motion of the blades, and this is the algebraic difference between the momenta due to the velocity of whirl at entrance and discharge.

Let V_i be the velocity of whirl at inlet in ft. per sec. and V_d be the velocity of whirl at discharge in ft. per sec.

Then if W is the weight of steam flowing per sec. in lb.,
change of momentum in the direction of blade motion

$$=\frac{WV_i}{g}-\frac{WV_d}{g}=\text{Force in lb.}$$

NOTE. Care must be taken to assign to the velocities their correct signs ; thus, if the velocity of whirl at outlet is opposite in direction to that at inlet, the value of V_d is negative, and the change of momentum is

$$\frac{WV_i}{g}-\left(-\frac{WV_d}{g}\right)\text{ lb.}$$

Work done on the rotor. The work done on the rotor is the product of force on the blades and the peripheral speed of the rotor, so that if V is the velocity of the blades in ft. per sec.,

$$\text{Work done}=\left(\frac{WV_i}{g}-\frac{WV_d}{g}\right)V\text{ ft. lb. per sec.}$$

$$\text{Horse power}=\frac{\left(\frac{WV_i}{g}-\frac{WV_d}{g}\right)V}{550}\quad\text{or}\quad\frac{\frac{WV}{g}(V_i-V_d)}{550}.$$

Blade efficiency of the turbine. In common with the efficiency expression for all machines, this is the ratio $\frac{\text{useful work done}}{\text{energy supplied}}$, which becomes

$$\frac{\text{work done on the vanes per lb. of steam}}{\text{kinetic energy of steam per lb. on supply}}.$$

Employing the symbols of the preceding article :

$$\text{Work done on the vanes per lb. of steam} = \frac{V}{g}(V_i - V_d) \text{ ft. lb.}$$

$$\text{K.E. of steam per lb. on supply} = \frac{V_1^2}{2g} \text{ ft. lb.}$$

where V_1 is the absolute velocity of steam at supply.

$$\textbf{Blade efficiency} = \eta = \frac{2V(V_i - V_d)}{V_1^2}.$$

Importance of low exhaust pressure. It is very important that the steam turbine be operated to the lowest possible pressure at exhaust. Since the available energy is related to the heat drop, a slight increase of condenser pressure will materially affect the heat drop and act as an adverse factor on the efficiency. The T *v.* ϕ diagram shows that at low temperatures, where the diagram is wider, and the slope of the curves less steep, any rise in condenser pressure will affect the work done to a relatively large extent.

Example 1. *The peripheral speed of the rotor of a De Laval turbine is* 1000 *ft. per sec. and the absolute velocity of the steam at inlet is* 2400 *ft. per sec. at* 20° *to the plane of the rotor. Find* (*a*) *the absolute velocity of the steam leaving the rotor if it is discharged at* 20° *to the plane of the rotor,* (*b*) *the velocities of whirl at inlet and discharge,* (*c*) *the H.P. of the wheel if the steam consumption is* 6 *lb. per min.,* (*d*) *the blade efficiency,* (*e*) *vane tip angle at entrance and discharge.*

(*a*) Absolute velocity of the steam at discharge = *ab* (Fig. 247) = **860 f. s.**

(*b*) Velocity of whirl at inlet = **2255 f. s.**

,, ,, ,, discharge = **255 f. s. negative.**

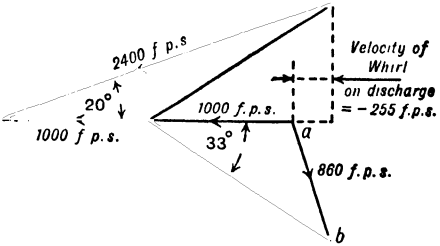


Fig. 247.

(c) H.P. of wheel $= \dfrac{\dfrac{WV}{g}(V_i - V_d)}{550}$.

$$W = \frac{6}{60} = 0{\cdot}1 \text{ lb. per sec.}$$

$$\text{H.P.} = \frac{0{\cdot}1 \times 1000}{32{\cdot}2}\left\{\frac{2255 - (-255)}{550}\right\} = \mathbf{14{\cdot}18.}$$

(d) Blade efficiency $= \dfrac{2V(V_i - V_d)}{V_1^2} = \dfrac{2000(2255 + 255)}{(2400)^2}$
$= 87{\cdot}2\%$

(e) Vane tip angle at entrance and discharge $= 33°$.

Alternative treatment of work done. The work done on a turbine rotor may be obtained direct from the kinetic energy of the inlet and exit jets as follows. Let V_1 be the absolute velocity of the inlet jet and V_2 the absolute velocity of the discharge jet.

Then if V is the velocity of the blades,

$$\text{Loss of K.E.} = \frac{WV_1^2}{2g} - \frac{WV_2^2}{2g} \text{ ft. lb.,}$$

where W is the weight of steam supplied per second, and the quantity $\dfrac{W}{2g}\{V_1^2 - V_2^2\}$ is the work done per sec.,

$$\text{hence} \qquad \textbf{H.P.} = \frac{\mathbf{W(V_1^2 - V_2^2)}}{\mathbf{2g \times 550}}.$$

The blade efficiency will now be $\dfrac{\mathbf{V_1^2 - V_2^2}}{\mathbf{V_1^2}}$.

Example 2. *Apply the alternative H.P. and blade efficiency expressions to parts* (c) *and* (d) *of Example* 1.

V_1 = absolute velocity at supply = 2400 ft. per sec.
V_2 = ,, ,, at discharge = 860 ft. per sec.
$W = 0{\cdot}1$ lb. per sec.

(c) $\text{H.P.} = \dfrac{0{\cdot}1\{2400^2 - 860^2\}}{2g \times 550} = \mathbf{14{\cdot}2.}$

(d) Blade efficiency $= \dfrac{V_1^2 - V_2^2}{V_1^2} = \dfrac{2400^2 - 860^2}{2400^2} = \mathbf{87{\cdot}2\%.}$

NOTE. If Fig. 247 is closely examined, it is not difficult to reconcile the alternative method for blade efficiency with that involving velocity of whirl. The velocity of whirl in each case is the horizontal component of the inlet and discharge absolute velocities, and their vertical components are equal. It follows that the velocities of whirl are related to the absolute velocities, and if this point of view is taken, the two expressions $\frac{2V(V_i - V_d)}{V_1^2}$ and $\frac{V_1^2 - V_2^2}{V_1^2}$ become identical.

Thus since $\sin^2\alpha + \cos^2\alpha = 1$,

$$\begin{aligned} V_1^2 - V_2^2 &= V_1^2\cos^2\alpha + V_1^2\sin^2\alpha - V_2^2\cos^2 B - V_2^2\sin^2 B \\ &= V_1^2\cos^2\alpha - V_2^2\cos^2 B, \text{ because } V_1\sin\alpha = V_2\sin B \\ &= V_i^2 - V_d^2, \text{ because } V_i = V_1\cos\alpha \text{ and } V_d = V_2\cos B \\ &= (V_i + V_d)(V_i - V_d) \\ &= 2V(V_i - V_d) \text{ because } 2V = V_i + V_d. \end{aligned}$$

Example 3. *An impulse turbine with an initial set of nozzles and three rotating rings of blades with two stationary sets of blades intervening, uses superheated steam at* 250 *lb. per sq. in. at a temperature of* 500° *F. Mean specific heat* 0·646. *This steam is expanded through the nozzles to dry saturated steam at* 30 *lb. per sq. in. absolute, and the nozzles are inclined at* 20° *to the plane of the rotor, the vanes on which have a velocity of* 400 *ft. per sec. Calculate* (*a*) *the velocity of exit from the nozzles,* (*b*) *the angles at the tips of the rotating and stationary blades if the rotary blades are each symmetrical,* (*c*) *the absolute velocity of the steam as it leaves the last rotating ring,* (*d*) *the H.P. of the turbine if it uses* 2000 *lb. of steam per hour,* (*e*) *the blade efficiency over all stages.*

$$\begin{aligned} \text{Total heat at entry to nozzles} &= H_1 \\ &= 1207 + c(t_{\text{sup.}} - t_{\text{sat.}}) = 1207 + 0{\cdot}646 \times 99. \\ H_1 &= 1271 \text{ B.Th.U. per lb.} \end{aligned}$$

$$\begin{aligned} \text{Total heat at discharge from nozzles} &= H_2 \\ &= 1165{\cdot}5 \text{ B.Th.U. per lb.} \end{aligned}$$

$$\begin{aligned} \text{Heat drop} = H_1 - H_2 &= 1271 - 1165{\cdot}5 \\ &= 105{\cdot}5 \text{ B.Th.U. per lb.} \end{aligned}$$

(*a*) Velocity at exit from nozzles $= \sqrt{2gJ \times 105{\cdot}55}$

$$= \sqrt{64{\cdot}4 \times 778 \times 105{\cdot}55} = \mathbf{2300 \text{ ft. per sec.}}$$

(*b*) The angles at the blade tips can be determined from the vector diagram (Fig. 248), in which the absolute velocities are shown in broken lines. *ca* is the velocity of the vanes, *ba* that of the steam entering the first rotating set of blades, and *bc* the relative velocity of the steam to the blades. The angle *bcd* is the angle of the blade tips in the first rotating set. Since the rotating blades are symmetrical, the steam moves out of these blades at an angle $\theta = \angle bcd$, and the absolute velocity of entry to the first stationary ring is *ce* and the angle of the entry tip is *ceg*. The absolute velocity of exit from the first stationary ring is *eh* and the vane angle is *geh*. The relative velocity of *eh* to the second rotating set is *ej*, at an angle $\theta = \angle bcd$ to the plane of rotation.

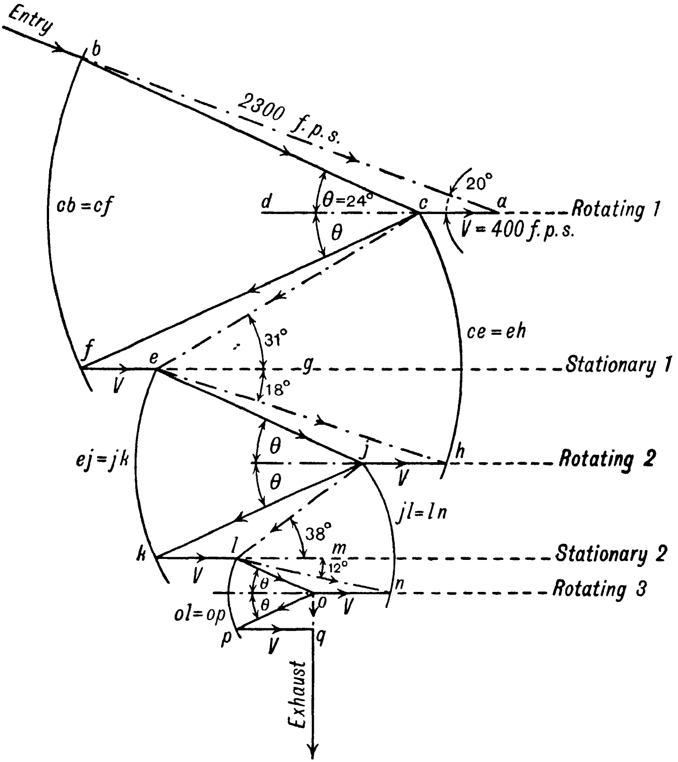


Fig. 248.
Vector Diagram. Absolute Velocities in Chain Lines.

Notice that it is possible to draw the vector triangle *ejh*, because *ej* is drawn parallel to *bc* since the inlet and exit angles of the rotating vanes are to be equal, because the absolute velocity *eh* equals the absolute steam velocity *ce*, and also because *jh* is known and equals *ca*. Similarly, the remaining vector triangles can be drawn.

Again, *jk* is the relative velocity of exit, also at θ to the rotation plane. *jl* is the absolute velocity of inlet to the second stationary set and *jlm* the angle of the blade tips. Similarly, *ln* is the absolute velocity of exit from this stationary set and *lo* the relative velocity of inlet at an angle θ to the plane of rotation. *oq* is the absolute velocity of the final exhaust.

(*b*)

1st Rotating		1st Stationary		2nd Rotating		2nd Stationary		3rd Rotating	
Inlet	Outlet	Inlet	Outlet	Inlet	Outlet	Inlet	Outlet	Inlet	Outlet
24°	24°	31°	18°	24°	24°	38°	12°	24°	24°

(*c*) Absolute velocity of exhaust $= oq =$ **175 ft. per sec.**

(*d*) Weight of steam per sec. $= \frac{2000}{3600} = 0{\cdot}5556$ lb.

$$\text{H.P.} = \frac{W(V_1^2 - V_2^2)}{2g \times 500},$$

where V_1 is absolute velocity at admission and V_2 is absolute velocity at exhaust.

$$\text{H.P.} = \frac{0{\cdot}5556(2300^2 - 175^2)}{2g \times 500} = \mathbf{90{\cdot}7}.$$

(*e*) Blade overall efficiency $= \frac{V_1^2 - V_2^2}{V_1^2}$

$$= \frac{2300^2 - 175^2}{2300^2} = \mathbf{99{\cdot}4\%}.$$

EXERCISES ON CHAPTER XVII

1. Explain the energy processes which take place when steam is expanded through a nozzle. What is the nature of this expansion and what are the limitations to this expansion?

2. State the principles which govern nozzle design and explain the terms " Pressure ratio " and " Critical pressure ".

3. Sketch the two types of nozzle in general use for turbine work, and state clearly the specific conditions governing the use of each type.

4. If the steam supply to a nozzle is at 250 lb. per sq. in. abs. and the pressure ratio is 0·5457, find the pressure at exit (*a*) for a single nozzle, (*b*) for three stages of expansion.

5. Explain the meaning of heat drop as applied to a nozzle, and show how the velocity of the issuing steam may be deduced from the heat drop.

6. Sketch the temperature *v.* entropy diagram for adiabatic expansion of steam through a nozzle, (*a*) when the supply steam is dry saturated, (*b*) when the supply steam is superheated. How is the condition at discharge determined?

7. Dry saturated steam at 180 lb. per sq. in. is expanded through a nozzle with a pressure ratio of 0·58. Find from the T *v.* ϕ diagram the dryness fraction after expansion and calculate (*a*) the heat drop per lb., (*b*) the theoretical jet velocity.

8. Steam is discharged from a nozzle at a pressure of 100 lb. per sq. in. absolute and with a dryness fraction of 0·95. Find from steam tables and the T *v.* ϕ diagram, or by calculation, the condition at entry if the discharge velocity is 1500 ft. per sec. and the entry pressure 200 lb. per sq. in. absolute.

9. Supply steam at 250 lb. per sq. in. absolute superheated to 220° C. is expanded through a nozzle with a pressure ratio of 0·5457. Find (*a*) the condition of the discharge steam, (*b*) the heat drop, (*c*) the jet velocity.

10. Calculate the pressure ratio for a nozzle, which receives steam at 130 lb. per sq. in. absolute and with a superheat of 30 Cen. degrees, in order that it may exhaust in a dry saturated condition.

11. Steam is expanded through a nozzle to the critical pressure according to $PV^{1\cdot3}=C$ from 220 lb. per sq. in. abs. with a superheat of 80 Cen. degrees. Find (*a*) the heat drop, (*b*) the velocity of the jet if the discharge is dry saturated.

12. The last stage but one of a steam turbine is to exhaust at 10 lb. per sq. in. absolute with a dryness fraction of 0·85. Find (*a*) the condition of the steam upon entry to this stage, and (*b*) the exhaust pressure and condition at the last stage if the pressure ratio is 0·5457 for each stage.

13. What is meant by (*a*) coefficient of velocity, (*b*) nozzle efficiency? Calculate each of these values for a heat drop of 35 C.H.U. and an actual nozzle velocity of 1300 ft. per sec.

14. Draw clear diagrams to show the velocities of whirl and flow for a turbine, and explain the importance of these values in design calculations.

15. Steam enters the rotating ring of a turbine at 20° to the plane of the rotor and with a velocity of 2000 ft. per sec. Find the velocities of whirl and flow.

16. How is shock eliminated on the admission and discharge of steam from a rotating blade? Draw a diagram to illustrate the method of determining the angle at the tips of such a blade for entrance and discharge.

17. Distinguish between reaction and impulse turbines, and state clearly the points in design which make many modern steam turbines a combination of the two principles.

18. Why is the speed of a De Laval turbine generally lower than the best blade speed? What effect does this have on (*a*) the angle of the blade tips, (*b*) the efficiency of the turbine? What is the best vane speed in relation to the velocity of the jet?

19. A De Laval turbine receives dry saturated steam at 220 lb. per sq. in. abs. and exhausts at 8 lb. per sq. in. abs. with a blade speed of 1200 ft. per sec. Find the jet velocity, the relative velocity of jet to blades, and angle of blades at entry, if the nozzles are set at 20° to the rotor plane.

20. Deduce expressions for the work done per lb. of steam in terms of the velocities of whirl at inlet and exit, and show how this expression may be re-expressed in terms of the absolute velocities of the steam at inlet and discharge.

21. Find the H.P. and blade efficiency of a turbine receiving steam jets at 20° to the rotor plane with a velocity of 2000 ft. per sec. if the peripheral speed of the rotor is 900 ft. per sec. Assume that the rotating blades are symmetrical, and the quantity of steam used per hour is 600 lb.

22. A turbine has an initial set of nozzles set at 20° and two rotating rings of blades with one intervening stationary ring. If the rotating blades are symmetrical, calculate (*a*) the velocity of discharge from the nozzles if the supply steam is at 200 lb. per sq. in. and 420° F. and the pressure ratio 0·5457 ; (*b*) the angles at the tips of both rotating and stationary blades if the peripheral velocity of the rotor is 450 ft. per sec. ; (*c*) the H.P. of the turbine if the steam consumption is 1800 lb. of steam per hour ; (*d*) the overall blade efficiency.

23. In an impulse steam turbine in which there is a single row of blades, steam issues from nozzles the axes of which are inclined at 20° to the plane of the wheel with a velocity of 3600 ft. per sec. The inlet and outlet angles of the blades are 34°. Find the correct blade speed.

If the steam loses 20 per cent. of its relative velocity while passing through the blade passages, find the work done per pound of steam. (U.E.I.)

24. In a single wheel impulse turbine, the nozzle is inclined at 20° to the direction of the motion of the wheel and the steam issues from the nozzle at 3600 ft. per sec. The blade speed is 1400 ft. per sec. Determine graphically the velocity of the entering steam relative to the blade. What should be the angle for the blades at inlet if the steam enters without shock? If the blade angle at exit is the same as that at entrance, and frictional losses between the steam and the blades be neglected, find the magnitude and direction of the absolute velocity of the steam as it leaves the turbine. (U.E.I.

25. Explain briefly the essential differences between steam turbines of the reaction and impulse types. What is the theoretical horse power of one stage of a turbine under the following conditions?

Steam used, 80 lb. per minute.
Total heat of steam entering turbine, 1378 B.Th.U. per lb.
,, ,, ,, leaving ,, 1328 ,, ,,
Heat lost by radiation, etc., 100 B.Th.U. per min. (U.E.I.)

26. Obtain an expression for the velocity of steam through a nozzle, due to adiabatic expansion. Friction may be neglected. Dry saturated steam at 140 lb. per sq. in. absolute is expanded adiabatically through a nozzle to a pressure of 70 lb. per sq. in. absolute. Find the velocity through the nozzle if the flow is frictionless. (U.E.I.)

CHAPTER XVIII

FUELS AND COMBUSTION, EFFICIENCY OF ENGINES AND BOILERS, BOILER TRIALS

MODERN heat engine practice brings into service a considerable variety of fuels, some selected for their high energy content and others because they are available in certain geographical areas as the waste products of an industry or as an economically produced material which can be used at its source. In the class of fuels with high energy content, the superior coals and petroleum products find special favour; whereas in tropical areas where coal fuel is at a premium the waste products of sugar- and rice-growing are used in specially designed furnaces.

High-pressure boilers can be operated, after adaptation, to the consumption of timber, peat, low-quality coal and the waste products of any industry where such material is combustible. In spite of the use of such poor materials in certain areas, it is still general to burn in boilers either good-quality coal or oil fuel in order to obtain the greatest possible heat exchange for every pound of fuel consumed.

Essential qualities of a fuel. Since the purpose of combustion is to convert the stored chemical energy in the fuel to heat energy, it is essential that a fuel must be :

(*a*) combustible at easily obtained temperatures ;

(*b*) of rich hydrocarbon content in order that combustion may take place in a free supply of air, with the oxygen of the air forming the other element in the chemical process of combustion ;

(*c*) able to give a high exchange of chemical to heat energy for every pound of fuel consumed ;

(*d*) of a nature which allows of ready storage and easy handling without undue danger of pre-ignition.

The fuels which conform most closely to these general requirements are coal and the petroleum products, either refined or crude, and for this reason it is still general to employ this type of fuel in the service of modern heat engines. One product of coal may be mentioned as a popular fuel, namely, coal gas, which is still finding considerable favour in town areas where gas is produced as part of the public services. There is also considerable demand for liquid fuels produced during the hydrogenation processes of coal as substitutes for the petroleum products. These fuels, generally classified as benzoles, are used either in their produced state or as a mixture with petroleum oil for internal combustion engines. The use of petroleum varies considerably, from the highly-refined type of petroleum spirit used in aero or automobile engines to the crude product or one of elementary refinement used in crude oil and Diesel engines. Internal combustion engines are designed, and carburation arranged, to suit a definite grade of oil refinement, and their efficient operation depends to a large extent upon the use of a fuel of the appropriate grade of refinement.

Calorific value of a fuel. The processes of combustion demand that a fuel shall receive during its combustion an adequate quantity of air from which oxygen may be taken to complete the chemical processes involved in combustion. If this condition is satisfied, the burning of one pound of fuel will supply a certain quantity of heat, dependent upon the nature of the fuel, and this quantity of heat obtained from one pound of fuel is called the **calorific value** of the fuel. The determination of calorific value for solid, liquid and gaseous fuels has been described in Vol. II (Exp. 14, p. 42), in which the methods used are consistent with the commercial determination of this quantity by fuel technicians.

Gross and net calorific values. In practice, a boiler does not completely convert the heat energy resulting from combustion to useful purposes. There are losses due to heat being carried away by flue gases, ash and radiation, so that the calorific value of a fuel obtained by experiment in a well-designed apparatus does not agree with the boiler performance when this fuel is used. Furthermore, the hydrogen content of the fuel burns to form water vapour, which has a certain sensible heat in addition to the latent heat of vaporisation, and this heat is carried away in the flue gases. The **gross or higher calorific value** of the fuel is obtained by an expériment which condenses this steam and credits the fuel with the heat given up by condensation, whereas the **net or lower calorific value**, which is the maximum value obtainable under working conditions, assumes this heat to be lost. If there is w lb. of steam formed when 1 lb. of fuel is burned :

$$\text{Net calorific value} = \text{gross value} - \text{losses}.$$

If L_a is the latent heat of steam at atmospheric pressure and h is sensible heat at 100° C., the losses become

$$w\{L_a + (h - h_t)\} \text{ C.H.U. per lb.},$$

where h_t is the sensible heat at 15° C., the normal atmospheric temperature. Hence

$$\mathbf{Net\ C.V. = Gross\ C.V. - w\{L_a + (h - h_t)\}\ C.H.U.\ per\ lb.}$$

Combustion. The composition of fuels is generally confined to hydrogen, carbon, sulphur and nitrogen, of which the first three are combustible and combine with oxygen from the air supply to the furnace or combustion chamber. Air is a mixture of oxygen and nitrogen with small proportions of non-combustible gases, which for the purposes of this work may be neglected. It follows that the complete combustion of a fuel depends upon the supply of a minimum quantity of air, and in actual practice the air supply is generally in excess of this minimum.

One pound of hydrogen has a volume, measured at 0° C. and 14·7 lb. per sq. inch pressure (normal temperature and pressure, N.T.P.), of 178·2 cu. ft., and this is known as a **standard volume** of hydrogen. All substances are characterised by their **atomic weights**, or weights which each atom of the substance possesses when compared with that of one atom of hydrogen. The substances exist in **molecules**, the name given to the smallest possible part of the substance which can have a separate existence, and these molecules consist of two or more atoms inseparable by ordinary means. The **molecular weight** of the substance is the weight of one molecule, and when the substance is a gas the molecular weight in grams is called a **mol**.

Substance	Atomic weight	Molecular weight
Hydrogen - -	1	2
Oxygen - -	16	32
Nitrogen - -	14	28
Carbon - -	12	24
Sulphur - -	32	—

The molecular weight of sulphur is not given because it may exist in molecules of a varying number of atoms.

Avogadro's Law states that **equal volumes of gases at the same temperature and pressure contain the same number of molecules.**

This law enunciated by Avogadro in 1811 has an important bearing upon the theory of combustion; during combustion all fuels become gaseous, hence *the molecules of every gas taking part in the combustion have the same volume at the same pressure and temperature.*

When combustion takes place, two molecules of hydrogen combine with one of oxygen to form two molecules of water gas or steam. This may be expressed by chemical symbols as a combustion equation thus :

$$2H_2 + O_2 = 2H_2O,$$

where H_2O represents one molecule of steam.

Substituting the atomic weights (hydrogen) $H = 1$, (oxygen) $O = 16$,

$$2 \times 2 \times 1 + 2 \times 16 = 2 \times \{2 + 16\}.$$

These numbers represent the relative weights of the participants ; thus 4 lb. of hydrogen combines with 32 lb. of oxygen to form 36 lb. of steam. Hence :

1 lb. of hydrogen combines with 8 lb. of oxygen to form 9 lb. of steam.

If similar equations are formed for the combustion of carbon and sulphur, we obtain the following :

$$C + O_2 = CO_2 \text{ (carbon dioxide)}$$
$$12 + 32 = 44$$

1 lb. of carbon combines with $2\frac{2}{3}$ lb. of oxygen to form $3\frac{2}{3}$ lb. of carbon dioxide.

$$S + O_2 = SO_2 \text{ (sulphur dioxide)}$$
$$32 + 32 = 64$$

1 lb. of sulphur combines with 1 lb. of oxygen to form 2 lb. of sulphur dioxide.

Incomplete combustion of carbon. When carbon is burnt with insufficient air, the resulting product of combustion is carbon monoxide (CO), a poisonous gas readily convertible on explosion or combustion to carbon dioxide.

Heat generated during combustion. It is found that 1 lb. of hydrogen on combustion generates 33,830 C.H.U. of heat, which is known as its **higher calorific value.** Since steam is the product of combustion, a certain portion of this heat is carried away in the form of latent heat given to the steam. The effective, or **lower calorific value,** is equal to the higher value less this latent heat, and is 29,000 C.H.U. per lb.

Similar values for the calorific values of carbon and sulphur are as follows :

Carbon burnt to	carbon dioxide :	8140	C.H.U. per lb.
,, ,,	carbon monoxide :	2450	,, ,,
Sulphur ,,	sulphur dioxide :	2200	,, ,,

It follows from the figures for carbon that in any heat production plant it is essential that the carbon be completely burnt to carbon dioxide, and this gives the principal reason for the supply of air in excess of the minimum quantity required.

	Hydrogen	Carbon	Sulphur
Higher calorific value -	to H_2O 34,300 C.H.U.	to CO_2 8140 C.H.U.	to SO_2 2200 C.H.U.
Lower calorific value -	to H_2O 29,000 C.H.U.	to CO 2450 C.H.U.	—

Minimum air supply for combustion. The composition of air by weight is 23% oxygen and 77% nitrogen, and by volume 21% oxygen and 79% nitrogen. From these figures 1 lb. of air will contain 0·23 lb. of oxygen, and it is possible to use the combustion equations to determine the minimum quantity of air required for any process involving combustion.

Summary

Combustion process		Product of combustion			
Fuel	Oxygen required	Water H_2O	Carbon dioxide CO_2	Carbon monoxide CO	Sulphur dioxide SO_2
1 lb. of hydrogen	8 lb.	9 lb.			
1 lb. of carbon to CO	0·5 lb.			1·5 lb.	
1 lb. of carbon to CO_2	$2\frac{2}{3}$ lb.		$3\frac{2}{3}$ lb.		
1 lb. of sulphur	1 lb.				2 lb.

Example 1. *A sample of fuel oil is submitted to chemical analysis and calorific test with the following results : Chemical composition* 85% *carbon,* 13% *hydrogen,* 1·4% *oxygen and* 0·6% *nitrogen. Calorific test : Weight of sample* 0·93 *gm. : weight of water* 2,170 *gm. ; water equivalent of apparatus* 480 *gm. ; temperature of atmosphere* 14·4° *C. ; initial temperature of water* 12·9° *C. ; final temperature of water* 16·85° *C. Calculate the gross and net calorific values.*

Gross calorific value from calorific test.

$$\text{Equivalent weight of water} = 2170 + 480 = 2650 \text{ gm.}$$

$$\text{Rise of temperature} = 16{\cdot}85 - 12{\cdot}9 = 3{\cdot}95^\circ \text{ C.}$$

Heat absorbed by water and apparatus

$$= 2650 \times 3{\cdot}95 \text{ or } 10{,}468 \text{ calories.}$$

$$\text{Gross calorific value} = \frac{\text{heat to water and apparatus}}{\text{mass of fuel}}$$

$$= \frac{10{,}468}{0{\cdot}93} \text{ or } 11{,}256 \text{ calories.}$$

Since 1 lb. = 453·6 gm. and 1 C.H.U. = 453·6 calories, the temperature unit in the calorie being the centigrade degree.

Gross C.V. = 11,256 C.H.U. per lb. or 20,261 B.Th.U. per lb.

Calorific value from chemical analysis.

Since 1 lb. of C produces 14,650 B.Th.U.,

1 lb. of H produces 61,750 B.Th.U.,

and 1 lb. of H produces 9 lb. of steam (Vol. II, pp. 37–9),

$$\textbf{Gross C.V.} = 14{,}650 \times 0{\cdot}85 + 61{,}750\left(0{\cdot}13 - \frac{0{\cdot}014}{8}\right).$$

where $\frac{0{\cdot}014}{8}$ is the oxygen equivalent which combines with the hydrogen.

$= 12{,}452 + 7919$

$=$ **20,371 B.Th.U. per lb.**

Weight of steam per lb. of fuel = 0·13 × 9 = 1·17 lb.

$$\text{Heat given to steam} = w\,\{L_a + (h - h_a)\}$$

$$= 1{\cdot}17\,\{539{\cdot}3 + (100 - 14{\cdot}4)\}$$

$$= 1{\cdot}17\,\{539{\cdot}3 + 85{\cdot}6\} = 731{\cdot}1 \text{ C.H.U.}$$

$$\textbf{Net C.V.} = 20{,}371 - 731{\cdot}1 \times \frac{1400}{778}$$

$$= 20{,}371 - 1315$$

$=$ **19,056 B.Th.U. per lb.**

Example 2. *A sample of anthracite fuel gave the following constitution on analysis : Carbon* 91·7%, *hydrogen* 4%, *oxygen* 2·9%, *residue of non-combustibles* 1·4%. *Find* (*a*) *the minimum air required per lb. of fuel,* (*b*) *the volume of air per lb. of fuel if the excess air is* 30%, (*c*) *the calorific value.*

0·917 lb. of carbon requires 0·917 × $2\frac{2}{3}$ = 2·445 lb. of oxygen.

0·04 lb. of hydrogen requires 0·04 × 8 = 0·32 „

Total oxygen = 2·765 lb.

Less 0·029 lb. oxygen in the fuel = 2·736 lb.

Since 1 lb. of air contains 0·23 lb. of oxygen,

$$\text{minimum weight of air} = \frac{2{\cdot}736}{0{\cdot}23} = 11{\cdot}9 \text{ lb.}$$

and 1 lb. of air at N.T.P. has a volume of 12·38 cu. ft.

(*a*) Minimum volume of air per lb. of fuel = 11·9 × 12·38 = **147·3 cu. ft.**

(*b*) Volume of air with 30% excess = **191·52 cu. ft.**

(*c*) 0·917 lb. of carbon yields 0·917 × 140 = 7464 C.H.U.

0·04 lb. of hydrogen yields 0·04 × 29,000 = 1160·0 C.H.U.

Total yield = 8569 C.H.U.

Heat of 0·029 lb. of oxygen combining with the hydrogen of the fuel

$$= \frac{0{\cdot}029}{8} \times 29{,}000 = 105{\cdot}1 \text{ C.H.U.}$$

Calorific value of fuel = 8569 − 105·1

$=$ **8464 C.H.U. per lb.**

Example 3. *Find the weight of air required to burn* 1 *lb. of carbon to carbon dioxide and* 1 *lb. of hydrogen to water. Also calculate the weight of air theoretically required to burn completely* 1 *lb. of oil composed of* 0·85 *carbon and* 0·15 *hydrogen.*

If this oil were used in a boiler furnace and the air supply was 50% *in excess of that absolutely necessary for combustion, how much heat would the excess air carry away up the chimney per lb. of oil burned? Temperature of air entering the furnace* 18° *C. and temperature of gases entering the chimney* 290° *C. Given that air contains* 23% *of oxygen by weight; mean specific heat of air* 0·24. (U.L.C.I.)

(*a*) 1 lb. of carbon requires $2\frac{2}{3}$ lb. of oxygen,

$$\text{or} \quad \frac{100}{23} \times 2\tfrac{2}{3} \text{ lb. of air} = \mathbf{11{\cdot}6\dot{1}\ lb.}$$

(*b*) 1 lb. of hydrogen requires 8 lb. of oxygen,

$$\text{or} \quad \frac{100 \times 8}{23} \text{ lb. of air} = \mathbf{34{\cdot}8\ lb.}$$

(*c*) 0·85 lb. carbon requires 0·85 × 11·61 lb. of air = 9·87 lb.

0·15 lb. hydrogen requires 0·15 × 34·8 lb. of air = 5·22 lb.

Wt. of excess air per lb. of oil = 0·5 × 15·09
= 7·545 lb.

Heat given to excess air = 7·545 × 0·24 {290 − 18}
= 7·545 × 0·24 × 272 = **492·5 C.H.U.**

EFFICIENCY

The efficiency of any machine, engine or boiler, is generally expressed as the ratio between the energy output and the energy input. Such a general expression is often too broad in the consideration of engine and boiler performance to indicate to the engineer the information which he requires, and the term efficiency is qualified to meet the particular criterion which he desires to investigate.

For example: **Mechanical efficiency** is a term applied to a machine, or an engine, and expresses the efficiency of conversion of the mechanical energy supplied to that available for useful work. For an engine this is the ratio between the work available at the output coupling of the engine and that actually done on the engine pistons; or it may be expressed thus:

$$\textbf{Mechanical efficiency} = \frac{\textbf{brake horse power}}{\textbf{indicated horse power}}.$$

Thermal efficiency is an absolute efficiency and may be applied to an engine or a boiler. In the case of a boiler it is the ratio between the heat available for steam raising and the heat supplied by, or calorific value of, each pound of fuel. Thus:

$$\text{Thermal efficiency of a boiler} = \frac{\text{heat available for steam raising}}{\text{heat supplied or C.V. of fuel}},$$

each quantity taken for 1 lb. of fuel consumed.

The **indicated thermal efficiency of an engine** has a slightly different interpretation, inasmuch that the energy input is the heat supplied to the engine per 1 lb. of working substance, while the energy output is the work done by this 1 lb. of working substance in the cylinder. Thus:

$$\text{Thermal efficiency of an engine} = \frac{\text{heat equivalent of the work done in the cylinder}}{\text{heat supplied to the engine}}.$$

In each case 1 lb. of working substance is taken.

Comparative thermal efficiencies. The comparative thermal efficiency of an engine is its thermal efficiency compared with that of a certain standard cycle. This standard cycle for steam engines and turbines is the Rankine Cycle, and for the internal combustion engine the Air Standard Cycle. Thus the comparative thermal efficiency of an engine is the ratio

$$\frac{\text{thermal efficiency}}{\text{efficiency on standard cycle}}.$$

Actually the criterion for steam engines and turbines is termed the **efficiency ratio** and is given by the ratio of the theoretical steam consumption of the ideal engine working on the Rankine Cycle per horse-power hour to the actual steam consumption per horse-power hour.

The departures from the Rankine Cycle which characterise modern steam practice with both steam engines and turbines have meant that this efficiency ratio is slowly falling into disuse, and steam turbines especially are usually compared on the basis of their total heat consumption per kW. per hour for power stations, or per B.H.P. per hour for ships. These expressions are usually abbreviated to steam or fuel consumption per kW. hour or per B.H.P. hour.

With internal combustion engines there are other standard cycles besides the one mentioned which conform more closely with the working of the various types of engine, but these cycles cannot be considered at this stage.

Brake thermal efficiency. This is a very informative efficiency inasmuch that it expresses the ratio between the mechanical output of the engine and the actual heat supplied. If the thermal efficiency of an engine is regarded, as it often is, as the indicated thermal efficiency, the brake thermal efficiency can be related to the mechanical efficiency thus :

$$\text{Brake thermal efficiency} = \frac{\text{work output}}{\text{heat supplied}}.$$

$$\text{(Indicated) thermal efficiency} = \frac{\text{work done in cylinder}}{\text{heat supplied}}.$$

$$\text{Mechanical efficiency} = \frac{\text{work output}}{\text{work done in cylinders}}.$$

$\therefore$ **Brake thermal efficiency**

$$= \text{thermal efficiency} \times \text{mechanical efficiency}$$

$$= \textbf{thermal efficiency} \times \frac{\textbf{B.H.P.}}{\textbf{I.H.P.}}.$$

Consumption rate of an engine. The consumption rate of an engine is expressed in lb. or cu. ft. of working substance per horsepower hour, and reference may be made in this respect to either the Indicated H.P. or Brake H.P., the former being the *indicated consumption rate* and the latter the *brake consumption rate.*

Summary of efficiencies

NOTE. In each case the heat is reckoned per lb. of fuel consumed.

Mechanical efficiency	$\frac{\text{Work done at output coupling}}{\text{Work done in cylinders}} = \frac{\text{B.H.P.}}{\text{I.H.P.}}$
(Indicated) thermal efficiency	Boiler : $\frac{\text{Heat available for steam raising}}{\text{Heat supplied by fuel}}$
	Engine : $\frac{\text{Work done in cylinders}}{\text{Heat supplied to engine}}$
Comparative thermal efficiency	$\frac{\text{Indicated thermal efficiency}}{\text{Efficiency on standard cycle}}$

Brake thermal efficiency -	$\dfrac{\text{Work done at output coupling}}{\text{Heat supplied to engine}}$ = Thermal efficiency × mechanical efficiency $= \text{Thermal efficiency} \times \dfrac{\text{B.H.P.}}{\text{I.H.P.}}$
Consumption rate - -	Weight in pounds or Volume in cu. ft. of working substance per I.H.P. or B.H.P. hr.

NOTE. One horse-power hour is equal to 33,000 × 60 ft. lb. of work per hour, which is equivalent to **1414 C.H.U.** Thus the consumption rate of an engine is the number of lb. or cu. ft. of the working substance per 1414 C.H.U. produced, taken on the I.H.P. or B.H.P. basis as required.

Example 1. *During an engine trial the following data and observations were obtained: (a) for B.H.P., dead weight on the dynamometer rope* 102 *lb., spring balance reading* 14 *lb., mean diameter of brake wheel* 4 *ft., engine speed* 184 *r.p.m. (b) for I.H.P., mean effective pressure from the indicator diagram* 58 *lb. per sq. in., stroke* 9 *in., area of piston* 35 *sq. in., engine single acting. Calculate the B.H.P., I.H.P. and Mechanical efficiency.*

$$\text{B.H.P.} = \frac{\pi D(W-S)N}{33{,}000} = \frac{\pi \times 4(102-14)184}{33{,}000} = \mathbf{6{\cdot}17}.$$

$$\text{I.H.P.} = \frac{P\,.\,L\,.\,A\,.\,N}{33{,}000} = \frac{58 \times 9 \times 35 \times 184}{12 \times 33{,}000} = \mathbf{8{\cdot}49}.$$

$$\text{Mechanical efficiency} = \frac{\text{B.H.P.}}{\text{I.H.P.}} = \frac{6{\cdot}17}{8{\cdot}49} = \mathbf{72{\cdot}6\%}.$$

Example 2. *An engine of I.H.P.* 8·2 *and B.H.P.* 6·7 *consumes* 3·9 *lb. of oil fuel with a calorific value of* 18,000 *B.Th.U. per lb. in one hour. Calculate the mechanical and indicated thermal efficiencies, the consumption rates per I.H.P. and B.H.P. hour, and the brake thermal efficiency.*

$$\text{Mechanical efficiency} = \frac{\text{B.H.P.}}{\text{I.H.P.}} = \frac{6{\cdot}7}{8{\cdot}2} = \mathbf{81{\cdot}7\%}.$$

$$\text{(Indicated) thermal efficiency} = \frac{\text{work done on pistons}}{\text{heat supplied}}$$

$$= \frac{8{\cdot}2 \times 33{,}000 \times 60}{3{\cdot}9 \times 18{,}000 \times 778} = \mathbf{29{\cdot}73\%}.$$

$$\text{Consumption rate per I.H.P. hour} = \frac{3{\cdot}9}{8{\cdot}2} = \mathbf{0{\cdot}475\ lb.\ per\ I.H.P.\ hour.}$$

$$\text{,, ,, ,, B.H.P. hour} = \frac{3{\cdot}9}{6{\cdot}7} = \mathbf{0{\cdot}582\ lb.\ per\ B.H.P.\ hour.}$$

$$\text{Brake thermal efficiency} = \frac{6{\cdot}7 \times 33{,}000 \times 60}{3{\cdot}9 \times 18{,}000 \times 778} = \mathbf{24{\cdot}29\%}.$$

Example 3. *A turbine is supplied with steam at* 130 *lb. per sq. in. with* 40 *centigrade degrees of superheat, and this is exhausted at a pressure of* 2 *lb. per sq. in. Calculate* (*a*) *the condition of the steam at exhaust,* (*b*) *the Rankine efficiency,* (*c*) *the comparative efficiency if the thermal efficiency is* 20·8% *taken on test.* [*Take the temperature of the feed water as* 50° *C.*]

(*a*) Condition after expansion from T *v.* ϕ diagram $=$ **0·83** approx.

(*b*) Heat drop in expansion $= 686{\cdot}9 - \{0{\cdot}83L + h\}$

$$= 686{\cdot}9 - \{0{\cdot}83 \times 566{\cdot}51 + 52{\cdot}16\}$$

$$= 686{\cdot}9 - 522{\cdot}36 = 164{\cdot}54 \text{ C.H.U.}$$

$$\text{Rankine efficiency} = \frac{164{\cdot}54}{686{\cdot}9 - 50} = \mathbf{25{\cdot}83\%}.$$

$$\text{(c) Comparative efficiency} = \frac{\text{thermal efficiency}}{\text{Rankine efficiency}} = \frac{20{\cdot}8}{25{\cdot}83} = \mathbf{80{\cdot}6\%}.$$

Example 4. *A motor-car engine working on a four-stroke cycle and having four cylinders* 3·5 *in. bore and* 4·5 *in. stroke consumes* 0·53 *pints of petrol per I.H.P. per hour. The speed is* 1312 *r.p.m. and I.H.P.* 24·1. *Find the mean effective pressure and the indicated thermal efficiency if the petrol has a calorific value of* 19,000 *B.Th.U. per lb. and weighs* 7·3 *lb. per gallon.* (U.E.I.)

$$\text{Mean effective pressure} = \frac{\text{I.H.P.} \times 33{,}000}{L\,.\,A\,.\,N \times 4} = \frac{24{\cdot}1 \times 33{,}000}{0{\cdot}375 \times \pi \times 1{\cdot}75^2 \times 656 \times 4}$$

$$= \mathbf{84{\cdot}03\ lb.\ per\ sq.\ in.}$$

$$\text{Consumption rate per I.H.P. hour} = \frac{0{\cdot}53 \times 7{\cdot}3}{8} = 0{\cdot}484 \text{ lb.}$$

$$\text{Indicated thermal efficiency} = \frac{33{,}000 \times 60}{778 \times 0{\cdot}484 \times 19{,}000}$$

$$= \mathbf{27{\cdot}68\%}.$$

Example 5. *A gas engine works on a four-stroke cycle with a stroke of* 21 *in. and a cylinder diameter of* 12 *in. The speed is* 280 *r.p.m. and the effective explosions average* 120 *per minute. If the engine consumes*

878 *cu. ft. of gas, with certified calorific value of* 475 *B.Th.U. per cu. ft. in one hour and the indicator diagram has an area of* 0·71 *sq. in. with a base of* 3 *in., find* (*a*) *the mean effective pressure if a* 300 *spring is used in the indicator,* (*b*) *I.H.P.,* (*c*) *consumption rate in cu. ft. per I.H.P. hour,* (*d*) *indicated thermal efficiency. If the makers claim a* 75% *mechanical efficiency, what B.H.P. may be anticipated?*

(*a*) Mean effective pressure $= \dfrac{0{\cdot}71 \times 300}{3} = \mathbf{71\ lb.\ per\ sq.\ in.}$

(*b*) $\text{I.H.P.} = \dfrac{P\,.\,L\,.\,A\,.\,N}{33{,}000} = \dfrac{71 \times 1{\cdot}75 \times \pi \times 36 \times 120}{33{,}000}$

$= \mathbf{51{\cdot}1}.$

(*c*) Consumption rate per I.H.P. hour $= \dfrac{878}{51{\cdot}1}$

$= \mathbf{17{\cdot}2\ cu.\ ft.\ per\ I.H.P.\ hr.}$

(*d*) Indicated thermal efficiency $= \dfrac{51{\cdot}1 \times 33{,}000 \times 60}{778 \times 878 \times 475}$

$= \mathbf{31{\cdot}2\%}.$

$\text{B.H.P.} = 0{\cdot}75 \times 51{\cdot}1 = \mathbf{38{\cdot}3}.$

Example 6. *A boiler produces* 2300 *lb. of steam per hour at a pressure of* 200 *lb. per sq. in. absolute superheated* 50 *centigrade degrees from feed water supplied at* 26° *C. Find the thermal efficiency of this boiler if it consumes* 371 *lb. of coal per hour of net calorific value* 14,200 *B.Th.U. per lb.*

$$
\begin{aligned}
\text{Calorific value} &= 14{,}200 \text{ B.Th.U. per lb.}\\
&= \frac{14{,}200 \times 778}{1400}\\
&= 7893 \text{ C.H.U. per lb.}\\
\text{Heat supplied for steam raising} &= H_{\text{sup.}} - h_{\text{feed}}\\
&= 698 - 26\\
&= 672 \text{ C.H.U. per lb.}\\
\text{Total heat given to steam} &= 672 \times 2300 \text{ C.H.U.}\\
\text{Heat available in fuel} &= 371 \times 7893 \text{ C.H.U.}\\
\text{Thermal efficiency} &= \frac{672 \times 2300}{371 \times 7893}\\
&= \mathbf{52{\cdot}8\%}.
\end{aligned}
$$

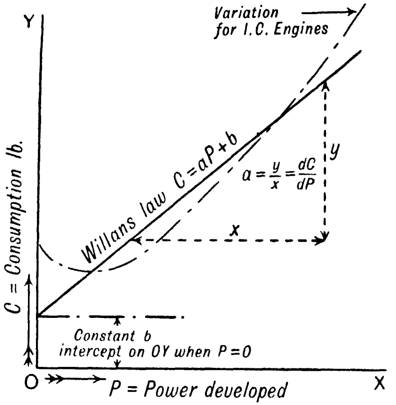

Variation for I.C. Engines
Y
C = Consumption lb.
Willans law $C = aP + b$
$a = \frac{y}{x} = \frac{dC}{dP}$
y
x
Constant b intercept on OY when P = 0
O
P = Power developed
X

FIG. 249.

Willans' Law. In all engines, whether steam, internal combustion, or steam turbines, there is a relationship between the consumption and the horse power, *providing the engine is quantity governed,* that is, governed through the throttle, which controls the amount of working substance supplied. This relationship follows a law illustrated by a straight-line graph for steam engines and turbines and approximately a straight line for other engines. The association was first pointed out by Willans and the law is known as Willans' Law.

Consider the graph shown in Fig. 249 between steam consumption and power output. The general equation to a straight-line graph is $y = mx + c$, which in this case may be interpreted as $C = aP + b$, where C is the steam consumption in lb. per hour, P is the I.H.P. developed in the cylinders, a and b constants.

The constant b is the intercept on the Y-axis, that is, the value of C when P is zero, and may be regarded as a constant loss when no power is developed.

The constant a is the slope of the graph and gives the relationship between the increase of consumption and the increase of power. In the notation of the calculus, $a = \frac{dC}{dP}$.

Willans' Law may be determined by experiment for any engine and when obtained may be applied to find the consumption for any power, or vice versa.

Example 1. *A steam engine is tested for power and consumption and the following results obtained :*

(*a*) *power* 200 *H.P., consumption* 4200 *lb.*
(*b*) *power* 120 *H.P., consumption* 2600 *lb.*

Calculate the values of the constants in Willans' Law, state the law and find the consumption when the power is 170 *H.P.*

Willans' Law : $C = aP + b.$

$$\begin{aligned} &(a) \quad 4200 = a \times 200 + b \\ &(b) \quad \underline{2600 = a \times 120 + b} \\ &\qquad\; 1600 = 80a \\ &\qquad\quad \mathbf{a = 20.} \end{aligned}$$

In (a), $4200 = 20 \times 200 + b.$

$$\mathbf{b = 200.}$$

Willans' Law: $\mathbf{C = 20P + 200.}$

Consumption when the power is 170 H.P.:

$$\begin{aligned} C &= 20 \times 170 + 200 \\ &= \mathbf{3600\ lb.} \end{aligned}$$

Example 2. *The approximation to Willans' Law for an oil engine is* $C = 0{\cdot}52P + 6{\cdot}2$. *Find the power developed for a consumption of* 31 *lb. of oil per hour, and the consumption at* 40 *H.P.*

$$\begin{aligned} C &= 0{\cdot}52P + 6{\cdot}2, \\ 31 &= 0{\cdot}52P + 6{\cdot}2, \\ 0{\cdot}52P &= 24{\cdot}8, \qquad \mathbf{P = 47{\cdot}69\ H.P.} \\ C &= 0{\cdot}52 \times 40 + 6{\cdot}2 \\ &= 20{\cdot}8 + 6{\cdot}2, \qquad \mathbf{C = 27\ lb.\ per\ hr.} \end{aligned}$$

Example 3. *The consumption rate for an oil engine developing* 80 *H.P. is* 0·31 *lb. per I.H.P. hour, and at* 60 *H.P. this rate is reduced to* 0·304 *lb. per I.H.P. hour. Determine Willans' Law for this engine.*

$$C = aP + b$$

$$\begin{aligned} &(a) \qquad 80 \times 0{\cdot}31 = a \times 80 + b. \\ &(b) \qquad 60 \times 0{\cdot}304 = a \times 60 + b \\ &\qquad\qquad 24{\cdot}8 = 80a + b \\ &\qquad\qquad \underline{18{\cdot}24 = 60a + b} \\ &\qquad\qquad 6{\cdot}56 = 20a, \qquad a = 0{\cdot}328. \\ &\qquad\qquad 24{\cdot}8 = 26{\cdot}24 + b, \quad b = -1{\cdot}44. \\ &\qquad\qquad \text{Law } \mathbf{C = 0{\cdot}328P - 1{\cdot}44.} \end{aligned}$$

Note. The negative value of b is due to the variation from a straight line obtained in I.C. engines and indicates an upward trend of the curve at or about zero power. This variation is indicated by the broken line in Fig. 249.

Wall action and cylinder condensation. The main improvements in engine efficiencies have been brought about by the elimination of water and its effects from the cylinder. Water becomes deposited upon and is re-evaporated from the cylinder walls as the temperature of these walls fluctuates during the cycle of operations in the cylinder (pp. 185 *et seq.*, Vol. II).

Calculation of indicated weight of steam at any point on the P v. V diagram. Assuming valve perfection and no piston leakage, the weight of steam in the cylinder during the expansion process should remain constant.

Let W = weight of steam in lb. at any point X on expansion curve ;
V = the total volume of steam in cu. ft. per lb. at X
= clearance volume + volume swept by piston to X ;
P = pressure in lb. per sq. ft. at X ;
V_s = specific volume in cu. ft. per lb. corresponding to P.

Then, $$W \times V_s = V \quad \text{or} \quad W = \frac{V}{V_s}.$$

W is called the **indicated weight** of steam because it is calculated from the indicator diagram and assumes that the steam is dry.

Indicated cylinder feed. This is the weight of steam taken from the boiler per stroke, as calculated from the indicator diagram.

Thus, **indicated cylinder feed** is equal to indicated weight of steam at cut-off minus indicated weight just after compression begins.

It must be remembered that the weight of steam in the cylinder at cut-off includes both fresh boiler steam and the " cushion " steam, previously compressed into the clearance space.

Missing quantity. This is the term used to denote the weight of steam which actually passes through the engine cylinder without being indicated on the indicator diagram. When an engine is tested, the exhaust steam is condensed and measured, when it is always found to be in excess of the indicated cylinder feed as calculated. The missing quantity is measured either as a percentage of the feed or as the weight in lb. per stroke or per unit of time. It is due to two causes, namely, cylinder condensation and leakage past valves and piston.

BOILER TRIALS

The extent to which a boiler trial is conducted depends upon the requirements of the engineer ordering the test. In such a trial taken by a large, steam user, the tests may be continued for a week over varying power loads, in which case the efficiency of every auxiliary is incorporated into the test and frequent representative fuel and flue gas analyses taken. Such a trial will give results which will reflect the quality of the firing on different shifts and the relation between consumption and power output throughout the operation of the steam plant. If conducted with periods under different fuels, or with changing draught conditions, it will determine the best possible conditions and fuel for steam raising. Generally, the trial does not extend to this detail ; the fuel and working conditions of the boiler have been predetermined, and the purpose of the test is to obtain the performance of the boiler under these fixed conditions.

Boiler performance. The actual performance of a boiler can only be measured by its evaporating power, that is, the weight of water evaporated per lb. of fuel consumed. The following terms used in reference to boiler trials are defined as follows :

Boiler capacity is the weight of steam produced by the boiler under full load in one hour ; this is sometimes referred to as the **evaporation.**

The **standard evaporation unit** is the heat required to convert 1 lb. of feed water at atmospheric pressure and a standard temperature of 100° C. or 212° F. into steam at the same temperature and pressure. It is equal to the latent heat of formation of steam at this pressure and temperature ; that is, **539·3 C.H.U. per lb.** or **970·7 B.Th.U. per lb.**

Equivalent evaporation. The feed-water temperature, pressure and other conditions of working of boilers vary considerably, and the performance of a boiler must be reduced to some comparative standard before it can be considered against another boiler. This standard of comparison is the **equivalent evaporation** of the boiler, and is the weight of steam which would be produced by the heat generated if this steam were formed at 100° C. from water at 100° C.

and atmospheric pressure. This quantity is referred to as the **equivalent evaporation from, and at, 100° C.**

The heat given to the formation of steam in a boiler may be calculated as follows :

Let t_1 be temperature of feed water in centigrade degrees,
h_1 the sensible heat of this water in C.H.U.,
t_2 the generation temperature in centigrade degrees,
W the weight of water evaporated in lb.,
L the latent heat of formation in C.H.U. per lb.,
x the dryness fraction,
h_2 the sensible heat at generation in C.H.U.

Then total heat of feed water $= Wh_1$ C.H.U.
,, ,, ,, formed steam $= W\{h_2 + xL\}$ C.H.U.
Heat given to steam $= W\{h_2 - h_1 + xL\}$ C.H.U.

Since the standard evaporation unit is 539·3 C.H.U. per lb., then,
Equivalent evaporation from and at 100° C.,

$$\mathbf{E} = \frac{\mathbf{W}\{\mathbf{h}_2 - \mathbf{h}_1 + \mathbf{xL}\}}{\mathbf{539{\cdot}3}} \text{ lb.}$$

When the steam is superheated in formation,

$$E = \frac{W\{heat_{(\text{super})} - h_1\}}{539{\cdot}3},$$

where $heat_{(\text{super})}$ is the total heat of superheated steam as formed.

Interpretation of equivalent evaporation. If the equivalent evaporation is used to compare two boilers, **it is essential that both boilers use the same fuel and evaporate under the same atmospheric conditions.**

If this is so, the equivalent evaporation may be regarded as a measure of performance ; if otherwise, the comparison must be made on efficiencies.

Example 1. *A boiler produces* 800 *lb. of steam per hour,* 0·95 *dryness fraction, at a pressure of* 160 *lb. per sq. in. gauge from feed water at* 20° *C. Find the equivalent evaporation of this boiler.*

Total heat per lb. of formed steam $= 190 + 0{\cdot}95 \times 476$
$= 642{\cdot}2$ C.H.U.

Total heat of feed water $= 20$ C.H.U. per lb.

$$\text{Heat given to steam per lb.} = 642{\cdot}2 - 20$$
$$= 622{\cdot}2 \text{ C.H.U.}$$

$$\text{Equivalent evaporation} = \frac{622{\cdot}2 \times 800}{539{\cdot}3}$$
$$= \mathbf{923 \text{ lb. per hr. from and at } 100° \text{ C.}}$$

Example 2. *A boiler normally produces* 1260 *lb. of steam per hour at a pressure of* 200 *lb. per sq. in. abs. and with* 60 *centigrade degrees of superheat from feed water at* 28° *C. An economiser is installed which has the effect of raising the feed-water temperature to* 79° *C. Find the equivalent evaporation, the evaporation with the economiser, and the percentage possible increase in evaporation with the economiser.*

Case 1. Heat used for steam raising (from steam tables)

$$= 1260 \times \{703{\cdot}6 - 28\}$$
$$= 1260 \times 675{\cdot}6 = 851{,}256 \text{ C.H.U.}$$

$$\textbf{Equivalent evaporation} = \frac{851{,}256}{539{\cdot}3} = \mathbf{1579 \text{ lb. per hr.}}$$

Case 2. The heat available for steam raising in the boiler is unaltered ; therefore if W is the weight of steam produced per hour with the economiser,

$$W \times \{703{\cdot}6 - 79\} = 851{,}256,$$
$$W \times 624{\cdot}6 = 851{,}256,$$
$$W = \mathbf{1363 \text{ lb. per hour.}}$$

$$\textbf{Percentage increase} = \frac{(1363 - 1260)\,100}{1260} = \mathbf{8{\cdot}17\%}.$$

Example 3. *A selection has to be made from two boilers which gave the following results under test with the same fuelling conditions and quality of fuel. No.* 1 : 1300 *lb. of steam per hour at* 200 *lb. per sq. in. abs.,* 0·94 *dry, from feed water at* 20° *C. No.* 2 : 1200 *lb. of steam per hour, at* 200 *lb. per sq. in. absolute,* 0·97 *dry, fitted with a flue gas feed-water heater receiving water at* 20° *C. and feeding to the boiler at* 68° *C. State which boiler is preferable.*

No. 1. $$\text{Equivalent evaporation} = \frac{1300\{197{\cdot}5 + 0{\cdot}94 \times 471{\cdot}2 - 20\}}{539{\cdot}3}$$

$$= \frac{1300(620{\cdot}4)}{539{\cdot}3} = \mathbf{1500 \text{ lb. per hr.}}$$

No. 2.

$$E = \frac{1200\{197\cdot5 + 0\cdot97 \times 471\cdot2 - 20\}}{539\cdot3}$$

$$= \frac{1200(634\cdot6)}{539\cdot3} = 1413 \text{ lb. per hr.}$$

The equivalent evaporation in No. 1 is greater and this boiler is preferable.

NOTE. The auxiliary feed-water heater is incorporated in the boiler and has the effect of providing heat to the feed water through heat which would otherwise be wasted in the escaping flue gases. It must be realised by the student that the addition of a feed-water heater increases the efficiency of a particular boiler, although in this case the increase of efficiency is insufficient to raise the performance of No. 2 above that of No. 1.

Boiler efficiency and the heat balance. The efficiency of any steam-raising plant seldom exceeds 80% ; hence a considerable portion of the heat energy available in the fuel is unconverted from the view-point of steam-raising. These losses may be traced to various causes, the principal of which are : (1) *heat carried away by flue gases*, (2) *heat lost by radiation and conduction*, (3) *heat rejected with ash.*

The first quantity may be fairly accurately determined by taking a chemical analysis of the flue gases and their temperature, but Nos. 2 and 3 are, to a great extent, indeterminate. It is general to bulk all the losses in a boiler and calculate the efficiency on the following basis.

$$\text{Efficiency} = \frac{\text{heat used for steam raising}}{\text{heat available in the furnace}},$$

the calculations being based on the heat available and the heat in the steam produced per lb. of fuel.

A heat balance may be struck for the performance of any boiler, with or without an economiser or feed-water heater, as follows :

Let W be wt. of fuel supplied per hour (b.)

C the calorific value of fuel (C.H.U. per lb.).

w the wt. of steam raised per hour (lb.).

h_1 the sensible heat at t_1 the temperature of feed water (°C.).

h_2 the ,, ,, t_2 ,, ,, ,, economiser discharge (°C.).

h_3 the ,, ,, t_3 ,, ,, ,, steam formation (°C.).

h_4 the sensible heat at t_4 the temperature after superheat (°C.), if any.

H_4 the total heat of superheated steam at t_4.

s the mean specific heat of superheat.

x the dryness fraction.

L the latent heat at formation (C.H.U. per lb.).

Heat balance for wet steam.

	Heat supplied (C.H.U.)	Heat utilised or lost (C.H.U.)
From fuel - -	$W.C$	
In economiser -		$w(h_2 - h_1)$
In boiler - -		$w(h_3 + xL - h_2)$
In waste - -		$W.C - w(h_3 + xL - h_2 + h_2 - h_1)$
Total - -	$W.C$	$W.C$

Heat balance for superheated steam.

From fuel - -	$W.C$		
In economiser -		$w(h_2 - h_1)$	
In boiler - -		$w(h_3 + L - h_2)$	Total A
In superheater -		$w(H_4 - h_3 - L)$	
In waste - -		$W.C - w(H_4 - h_1)$	
Total - -	$W.C$	$W.C$	

From the heat balance the efficiency of the boiler may be determined (*a*) for wet steam :

$$\text{efficiency} = \frac{w(h_3 + xL - h_1)}{W.C},$$

or approximately $$\frac{w(t_3 + xL - t_1)}{W.C};$$

and (*b*) for superheated steam :

$$\text{efficiency} = \frac{w(H_4 - h_1)}{W.C},$$

or approximately $$\frac{w\{t_3 + L - t_1 + s(t_4 - t_3)\}}{W.C},$$

if s can be relied upon.

Example 1. *A boiler consumes 385 lb. per hour of coal of calorific value 7200 C.H.U. per lb. The steam production is 2180 lb. per hour at 175 lb. per sq. in. gauge, superheated 60 centigrade degrees, the temperature of the feed water at entry to the economiser 19° C. and at discharge 71° C., and the mean specific heat of superheat 0·61. Find the equivalent evaporation per lb. of coal, and the efficiency of the boiler, and prepare a heat balance for its performance indicating the percentage heat given to each stage of the steam raising.*

Heat supplied by fuel $= 385 \times 7200 = \mathbf{2{,}772{,}000}$ **C.H.U.**

Heat supplied in economiser $= 2180 \times (71 - 19) = \mathbf{113{,}360}$ **C.H.U.**

Heat supplied in boiler $= 2180(668{\cdot}1 - 71)$

$= \mathbf{1{,}301{,}678}$ **C.H.U.**

Heat used in superheater $= 2180(0{\cdot}61 \times 60)$

$= \mathbf{79{,}930}$ **C.H.U.**

Total heat utilised $= 1{,}494{,}968$ C.H.U.

$$\text{Efficiency} = \frac{1{,}494{,}968}{2{,}772{,}000} = \mathbf{53{\cdot}84\%}.$$

Stage	Heat per hr. supplied C.H.U.	Heat per hr. utilised C.H.U.	% supply	% used
Combustion in furnace -	2,772,000		100%	
Economiser - - -		113,360		4·07%
Boiler - - - -		1,301,678		46·9%
Superheater - - -		79,930		2·87%
Waste heat - - -		1,277,032		46·16%
Totals - - -	2,772,000	2,772,000	100%	100%

Efficiency of the boiler = sum of % available in the economiser, boiler and superheater = **53·84%.**

Equivalent evaporation per lb. of coal :

$$\text{Evaporation per lb.} = \frac{2180}{385} = 5{\cdot}66 \text{ lb.}$$

$$\text{Heat given to water} = 5{\cdot}66\,\{668{\cdot}1 - 19 + 0{\cdot}61 \times 60\}$$

$$= 5{\cdot}66(685{\cdot}7) = 3881 \text{ C.H.U.}$$

Equivalent evaporation per lb. of coal from and at 100%

$$= \frac{3881}{539{\cdot}3} = \mathbf{7{\cdot}2 \text{ lb. per lb. of fuel.}}$$

Example 2. *A steam boiler was found to consume 5620 lb. of coal of calorific value 7900 C.H.U. per lb. during a 4-hour test. The boiler pressure was 180 lb. per sq. in. absolute, superheat 40 centigrade degrees, feed water 50,580 lb. at temperature 21° C. Find the boiler efficiency.*

An economiser was fitted and the feed-water temperature was thereby raised to 93° C. Find, for the same evaporation, the saving of coal in lb. per hour due to the addition of the economiser. [Specific heat of superheated steam = 0·5.] (U.E.I.)

(*a*) Heat supplied by fuel per hr. $= \dfrac{5620 \times 7900}{4}$ C.H.U.

$= 11{,}099{,}500$ C.H.U.

$$\text{Heat to steam production} = \frac{50{,}580}{4}\{667\cdot6 + 0\cdot5 \times 40 - 21\}$$

$$= 12{,}645\{646\cdot6 + 20\}$$

$$= 8{,}430{,}000 \text{ C.H.U.}$$

$$\text{Efficiency} = \frac{8{,}430{,}000}{11{,}099{,}500} = \mathbf{75\cdot95\%}.$$

(*b*) Useful heat abstracted from flue gases in the economiser

$$= 12{,}645 \times (93 - 21).$$

Coal saved, assuming a constant boiler efficiency,

$$= \frac{12{,}645 \times 72}{7900 \times 0\cdot7595}$$

$$= \mathbf{152 \text{ lb. per hr.}}$$

Example 3. *In a test on a plant the following figures were obtained: Coal fired per hour 95 lb., calorific value of coal 8340 C.H.U. per lb., feed water per hour 880 lb., temperature of feed 89° C., pressure of steam 260 lb. per sq. in. absolute, temperature of steam 277° C. Calculate the thermal efficiency of the boiler and the equivalent evaporation from and at 100° C. per lb. of coal. [Specific heat of superheated steam = 0·56.]* (U.E.I.)

$$\text{Heat supplied} = 95 \times 8340 \text{ C.H.U.}$$

$$\text{Heat in steam raising} = \{670\cdot8 + 0\cdot56 \times 70 - 89\}\,880$$

$$= 621 \times 880 \text{ C.H.U.}$$

$$\text{Efficiency} = \frac{621 \times 880}{95 \times 8340} = \mathbf{68\cdot98\%}.$$

$$\text{Equivalent evaporation} = \frac{621 \times 880}{95 \times 539\cdot3}$$

$$= \mathbf{10\cdot67 \text{ lb. per lb. of coal.}}$$

Laboratory trial of a gas-fired boiler.

SPECIFICATION OF TRIAL. A vertical tubular boiler working at 150 lb. per sq. in. gauge, pump fed without feed-water heater or superheater and serving a horizontal condensing steam engine. The feed-water supply is measured through a water meter and the gas supply through a large dial gas meter. The gas fuel is from town supply of a previously determined calorific value. Quality of steam is found by throttling, and, if necessary, combined throttling and separating during the trial.

Calorific value of town gas at N.T.P. = 475 B.Th.U. per cu. ft.

$$=\frac{475\times 778}{1400}=\mathbf{264\ C.H.U.\ per\ cu.\ ft.}$$

OBSERVATIONS.

Duration of trial = 30 minutes.

$\begin{cases}\text{Initial water meter reading} = 0786\cdot 5 \text{ gallons.}\\ \text{Final} \quad ,, \quad ,, \quad ,, \quad = 0797\cdot 25 \quad ,,\end{cases}$

W = Water consumption = 10·75 gallons = **107·5 lb.**

$\begin{cases}\text{Initial gas meter reading} = 001{,}627 \text{ cu. ft.}\\ \text{Final} \quad ,, \quad ,, \quad ,, \quad = 002{,}009 \quad ,,\end{cases}$

C_1 = Gas consumption at 16° C. = **382 cu. ft.**

Gas pressure = 14·7 lb. per sq. in. abs.

t_1 = Temperature of feed water = **16° C.**

Average boiler pressure = **150 lb. per sq. in. gauge.**

Temperature of steam at formation = **188° C.**

Dryness fraction test with throttling calorimeter.

Temperature of steam at 150 lb. per square in. gauge = 188° C.

,, ,, ,, after throttling = 103° C.

Pressure ,, ,, ,, ,, = 14·7 lb. per sq. in. abs.

Heat in steam before throttling $= 188 + x \times 479$ C.H.U. per lb.

,, ,, after ,, $= 100 + 539\cdot 3 + 0\cdot 48(103 - 100)$

$= 640\cdot 74$ C.H.U. per lb.

then $188 + 479x = 640\cdot 74.$

Dryness fraction x $= \mathbf{94\cdot 5\%}.$

C = Gas consumption at N.T.P. $=\dfrac{273C_1}{289}=$ **361 cu. ft.**

Heat balance and derived results.

Heat given to steam raising $= W\{640{\cdot}74 - t_1\}$.

$$H_1 = 107{\cdot}5 \times 624{\cdot}74 = 67{,}155 \text{ C.H.U.}$$

Heat supplied by fuel $= C \times 264 = 361 \times 264$.

$$H_2 = 95{,}300 \text{ C.H.U.}$$

$$\textbf{Efficiency} = \frac{\mathrm{H}_1}{\mathrm{H}_2} = 70{\cdot}4\%.$$

$$\textbf{Equivalent evaporation} = \frac{67{,}155}{361 \times 539{\cdot}3}$$

$= 0{\cdot}345$ **lb. per cu. ft. at and from 100 °C.**

Stage	Heat supplied C.H.U.	Heat utilised C.H.U.	% supply	% used
Combustion in furnace -	95,300		100%	
Boiler - - - - -		67,155		70·4%
Waste heat - - -		28,145		29·6%
Totals - -	95,300	95,300	100%	100%

TRIAL OF A STEAM-RAISING PLANT

The complete trial of a steam-raising plant is carried out in considerably more detail than the general efficiency test. The preparation of the heat account indicates to the engineer, not only if there is inefficiency in operation, but also just where any inefficiency may be remedied and to what extent the remedy should be applied. The routine testing of steam plant confirms, or otherwise, its efficient operation and is performed in conjunction with laboratory tests of fuel samples, ash and flue gases. The following results have been obtained for such a trial, and the heat balance represents the performance of the plant.

OBSERVATIONS.

Duration of trial = 8 hours.
Weight of feed water = 80,520 lb.
Temperature of feed to economiser = 27° C.
" " outlet from economiser = 126° C.

Steam pressure by gauge = 175 lb. per sq. in.
Atmospheric pressure in boiler house = 15 lb. per sq. in.
Dryness fraction of steam = 0·94.
Weight of coal fired = 9270 lb.
Temperature of air in boiler house = 18° C.
,, ,, flue gases = 325° C.
Weight of ash = 320 lb.

LABORATORY TESTS.

Percentage moisture in coal as fired = 1·7%.

Analysis of dry coal: C 88·5%, H 3·7%, ash 4·7%, other materials 3·1%.

Residual heat value of ash per lb. = 420 C.H.U.

Flue gas analysis by volume :

CO_2 11·1%, CO 1·1%, O 6·8%, N 81%.

Lower calorific value of fuel.

0·885 C yields 0·885 × 8140 = 7200 C.H.U. per lb.
0·037 H ,, 0·037 × 32,100 = 1188 ,,
Calorific value = 8388 C.H.U. per lb.

Weight analysis of flue gases.

CO_2, Volume × molecular wt. = 0·111 × 44 = 4·884
CO ,, ,, ,, = 0·011 × 28 = 0·308
O ,, ,, ,, = 0·068 × 32 = 2·176
N ,, ,, ,, = 0·81 × 28 = 22·68
Total = 30·048

Weight anaylsis CO_2 = 16·26%, CO = 1·03%, O = 7·24%, N = 75·47%.

Weight of flue gases per 1 lb. of dry fuel.

Since the fuel contains 88·5% C per lb. and the flue gases 16·26% CO_2 and 1·03% CO,

$$\text{Carbon present in 1 lb. of flue gases} = 0{\cdot}163 \times \tfrac{12}{44} + 0{\cdot}01 \times \tfrac{12}{28}$$
$$= 0{\cdot}0488 \text{ lb.}$$

$$\text{Weight of flue gases (dry) to 1 lb. of fuel} = \frac{0{\cdot}885}{0{\cdot}0488} = 18{\cdot}13 \text{ lb.}$$

In addition to this quantity, the 0·037 lb. of hydrogen in the fuel produces $0{\cdot}037 \times 9 = 0{\cdot}333$ lb. of steam.

∴ **Weight of flue gases as formed = 18·463 lb. per lb. of fuel burned.**

Mean specific heat of flue gases.

Since the specific heats of the constituent gases at constant volume are $CO_2 = 0{\cdot}216$, $CO = 0{\cdot}245$, $O = 0{\cdot}218$, $N = 0{\cdot}244$ and steam $= 0{\cdot}48$, the mean specific heat at constant volume

$$= \overset{CO_2}{(0{\cdot}163 \times 0{\cdot}216)} + \overset{CO}{(0{\cdot}01 \times 0{\cdot}245)} + \overset{O}{(0{\cdot}072 \times 0{\cdot}218)}$$

$$+ \overset{N}{(0{\cdot}755 \times 0{\cdot}244)} + \overset{\text{steam}}{(0{\cdot}333 \times 0{\cdot}48/18{\cdot}46)} = \mathbf{0{\cdot}246}.$$

HEAT BALANCE.

Heat supplied.

Heat given by fuel $= 9270 \times$ calorific value

$= 9270 \times 8388 =$ **77,756,760 C.H.U.**

Heat utilised for steam raising.

Heat to water in economiser

$= 80{,}520\{126 - 27\}$

= **7,971,480 C.H.U.**

Heat to steam raising in boiler

$= 80{,}520\{194{\cdot}9 + 0{\cdot}94 \times 473{\cdot}2 - 126\}$

$= 80{,}520 \times 513{\cdot}71 =$ **41,363,929 C.H.U.**

Heat to waste.

Heat to ash $= 320 \times 420 =$ **134,400 C.H.U.**

Heat to flue gases $= 9270 \times 18{\cdot}463 \times 0{\cdot}246\{325 - 18\}$

= **12,926,000 C.H.U.**

Heat to moisture in coal $= 9270 \times \frac{1{\cdot}7}{100}\{(100 - 18) + 539{\cdot}3\}$

$= 157{\cdot}59 \times 621{\cdot}3 =$ **97,911 C.H.U.**

Heat unaccounted for = Residue

= **13,733,390 C.H.U.**

	Stage	Heat supplied C.H.U.	Heat used C.H.U.	Percentage	
A	Heat supplied by fuel -	77,756,760		100	
	,, to ash - - -		134,400		0·174
	,, ,, flue gases - -		12,926,000		16·63
	,, ,, moisture in coal -		97,911		0·126
B	,, ,, water in economiser		7,971,480		10·25
C	,, ,, ,, boiler -		41,363,929		53·2
	,, unaccounted for -		15,263,030		19·62
	Totals - - -	77,756,760	77,756,750	100	100

$$\textbf{Efficiency} = \frac{B+C}{A} = \frac{49{,}335{,}409}{77{,}756{,}750} = 64\cdot46\%.$$

Equivalent evaporation from and at 100° C.

$$= \frac{49{,}335{,}409}{539\cdot3} = 91{,}480 \text{ lb.}$$

or, **9·864 lb. of water per lb. of coal.**

EXERCISES ON CHAPTER XVIII

1. State the essential qualities of a good fuel and distinguish between the gross and net calorific values for a fuel.

2. A sample of coal is tested for calorific value and the following results obtained: weight of sample 0·79 gm., weight of water 2008 gm., water equivalent of apparatus 420 gm., rise of temperature of water $2\cdot61^\circ$ C. Calculate the calorific value in C.H.U. per lb.

3. A sample of fuel oil gives a chemical analysis as follows: carbon 87%, hydrogen 12%, nitrogen 1%. Calculate the gross and net calorific values for this fuel.

4. Explain how you would determine experimentally the quality of the steam entering a steam engine cylinder, *or* the calorific value of coal gas or paraffin used in internal combustion engines. You should include in your answer a sketch and description of the apparatus used, an explanation of the method of conducting the experiment and, using symbols, show how the required result is obtained. (U.E.I.)

5. In a boiler plant, apart from radiation losses and those inherent to the plant, there are certain heat losses which can be apportioned to (*a*) the grate and furnace, and (*b*) the fuel. Describe briefly the losses under these two heads.

6. A fuel has an ash content of 4·4%. An analysis of the cinders shows a carbon content of 21%. Assuming that no ash goes up the stack, calculate the percentage of the heating value of the fuel which is wasted in the cinders, assuming for simplicity that carbon is the only combustible in the fuel. (U.E.I.)

7. Define in general terms the meaning of efficiency as applied to any machine, and then show how various types of efficiency are determined in heat engine and boiler tests.

8. An engine is stated to have a mechanical efficiency of 78%. During a test the following observations were made on a rope brake : dead weight 80 lb., spring balance reading 16 lb., mean diameter of rope brake drum 4 ft. 3½ in., speed 178 r.p.m. Calculate the B.H.P. and probable I.H.P.

9. An engine trial gave the following results : dead weight on rope brake 140 lb., spring balance reading 24 lb., mean diameter of brake 4 ft. 6 in., engine speed 145 r.p.m., mean effective pressure from indicator card 35·8 lb. per sq. in., stroke 9 in., area of piston 42 sq. in., engine double acting. Find the I.H.P., B.H.P., and mechanical efficiency.

10. An engine of I.H.P. 11·4, B.H.P. 9·7 consumes 5·1 lb. of oil of calorific value 9600 C.H.U. per lb. in each hour of running. Find the mechanical efficiency, indicated and brake thermal efficiencies and the consumption rate per I.H.P. hour.

11. A turbine is supplied with steam at 150 lb. per sq. in. with 30 centigrade degrees of superheat. This is finally exhausted at 1·5 lb. per sq. in. Find from the $T\,v.\,\phi$ diagram the condition of the steam at exhaust, and calculate (*a*) the Rankine efficiency, (*b*) the thermal efficiency, (*c*) the comparative efficiency.

12. The indicated thermal efficiency of an oil engine is 26·4%, and it uses 0·7 pints of oil, of specific gravity 0·74, calorific value 19,000 B.Th.U. per lb. in one hour's run. Calculate the indicated horse power of the engine.

13. A gas engine makes 130 explosions per minute and consumes 916 cu. ft. of gas of calorific value 450 B.Th.U. per cu. ft. in the hour. If the stroke is 18 in., cylinder diameter 12 in. and mean effective pressure from the indicated card 77 lb. per sq. in., calculate the I.H.P. and indicated thermal efficiency.

14. The stated thermal efficiency of a boiler is 67·4%, and it produces 700 lb. of steam per hour at a pressure of 250 lb. per sq. in. abs. with 70 centigrade degrees of superheat. Calculate the probable hourly fuel consumption if the coal has a calorific value of 8000 C.H.U. per lb., and the feed temperature is 84° C.

15. State Willans' Law and explain the meaning of the terms used in its expression as $C=aP+b$. Draw a diagram showing the application of Willans' Law to (*a*) steam engines, (*b*) I.C. engines. In what way does the law alter for the latter engines?

16. Determine Willans' Law for a steam engine which had a consumption of 3220 lb. at 180 H.P. and 2430 lb. at 110 H.P. Find the power available when the consumption is 2800 lb.

17. When C is in lb., Willans' Law for a certain engine is

$$C = 0{\cdot}462P - 1{\cdot}78.$$

Explain the meaning of the negative value of one constant and find the consumption for a power of 75 H.P. What type of engine does this particular law indicate?

18. Give a clear outline of the purpose of a boiler trial and indicate the observations necessary during the trial.

19. Define the terms : boiler performance, boiler capacity, standard evaporation unit, equivalent evaporation. Also state the conditions which govern the term equivalent evaporation.

20. If you had to compare the specifications of two competitive boilers, what figures would you require and to what extent would the equivalent evaporation data guide you in your selection?

21. Calculate the equivalent evaporation from and at 100° C. for a boiler which produces 620 lb. of steam per hour at a pressure of 200 lb. per sq. in. absolute with a dryness fraction of 0·96 from feed water at 27° C.

22. A boiler produces 520 lb. of steam per hour at a pressure of 175 lb. per sq. in. gauge with atmospheric pressure at 14·7 lb. per sq. in. abs. This steam is found to be 0·94 dry and the feed-water temperature is 29° C. Find the equivalent evaporation from and at 100° C. What would be the percentage increase of steam raised with the same quantity of fuel when an economiser is fitted which raises the temperature of the feed water to 89° C.?

23. Outline the heat balance for a boiler producing superheated steam with an economiser. What observations would have to be made in order to obtain the heat balance for this boiler?

24. A boiler coal has the following analysis by weight : C 81%, H 6%, O 4%. An exhaust gas analysis shows that 23 lb. of air have been supplied per lb. of coal fired. Find (*a*) the weight of excess air supplied per lb. of coal ; (*b*) the boiler efficiency, taking the average specific heat of the gases as 0·26, the air temperature as 22° C., and the flue gas temperature leaving the boiler as 350° C. ; assume that 500 C.H.U. are lost per lb. of fuel in radiation, convection, etc. ; (*c*) the heat lost in the excess air expressed as a percentage of the heat produced by the combustion of the fuel. Specific heat of air, 0·24. (U.E.I.)

25. A steam boiler is found to evaporate 8·4 lb. of water at 20° C. into steam 95% dry at 215 lb. per sq. in. absolute for each pound of coal consumed. The calorific value of the coal is 8020 C.H.U. per lb. ; 18 lb. of air enter the furnace for every pound of coal burned. The temperature of the boiler room is 20° C. and of the gases at the base of the chimney 260° C.

Determine (*a*) the efficiency of the boiler ; (*b*) the equivalent evaporation per lb. of coal ; (*c*) the proportion of the heat of the coal passing up the chimney.

You may assume the mean specific heat of the flue gases to be 0·24. (U.E.I.)

26. The following data were recorded during a boiler test :

Duration of trial	8 hours.
Boiler pressure in lb. per sq. in. abs.	183.
Temperature of air in boiler house	31° C.
Temperature of gases in chimney	272° C.
Temperature of feed water	15·5° C.
Total feed water	102,600 lb.
Total coal	10,738 lb.
Contents of ash pans	148 lb.
Combustibles percentage of ash pans' contents	42·7.
Air used per lb. of coal	17 lb.

Determine :—

(*a*) The heat units used in generating steam from feed water per lb. of coal.

(*b*) The number of heat units passing away up the chimney per lb. of coal.

(*c*) The heat units represented by unburnt fuel per lb. of coal.

(*d*) The thermal efficiency per cent.

(*e*) By difference, the heat units lost by radiation, imperfect combustion, etc., per lb. of coal.

Given : The calorific value of the coal 7,944 C.H.U. and the specific heat of the products of combustion 0·24. (U.L.C.I.)

27. What is meant by the efficiency of a boiler? A boiler raises 9½ lb. of steam per lb. of coal burned. The composition of the coal is C 82·5% ; O 1·6% ; and H 4·8%. The steam is produced at a pressure of 170 lb. per sq. in. abs., 90% dry from feed water at 68° C. What is the efficiency of the boiler?

Calorific value of carbon = 8080 C.H.U. per lb.

Lower calorific value of hydrogen = 29,000 C.H.U. per lb. (U.E.I.)

28. Distinguish between "higher" and "lower" calorific values of a fuel. A sample of oil contains 84% carbon and 16% hydrogen by weight. Find the higher and lower calorific values and the minimum air required for complete combustion of 1 lb. of this fuel.

Calorific value of carbon = 8,080 C.H.U. per lb.
Higher calorific value of hydrogen = 34,000 ,, ,,
Lower ,, ,, ,, ,, = 29,000 ,, ,,

Air contains 23% by weight of oxygen. (U.E.I.)

29. During a boiler test the following data were observed :

Duration of test	2 hr.
Average steam pressure by gauge	175 lb. per sq. in.
Temperature of steam leaving boiler	320° C.
Total steam generated	18,200 lb.
Calorific value of coal as fired	7,029 C.H.U. per lb.
Coal fired	2,260 lb.
Average feed water temperature	60° C.
Total heating surface	2,320 sq. ft.
Total grate surface	73·2 sq. ft.

Find (*a*) the efficiency of the boiler ; (*b*) the coal fired per sq. ft. of grate area per hour ; (*c*) the steam generated per sq. ft. of heating surface per hour.

The specific heat of superheated steam is to be taken as 0·56.

(U.L.C.I.)

30. A test of a steam generating plant gave the following data :

Total weight of steam generated per hour, 12,000 lb.
Absolute pressure of steam generated, 170 lb. per sq. in.
Temperature of feed water entering economiser, 65° C.
Temperature of feed water leaving economiser, 166° C.

Find (*a*) the percentage saving of heat due to the economiser ; (*b*) the evaporation of the boiler in lb. per hour at 170 lb. per sq. in. abs. under the same conditions of firing but without the economiser ; (*c*) the equivalent of the economiser in percentage increase of boiler power.

(U.L.C.I.)

CHAPTER XIX

STEAM AND INTERNAL COMBUSTION ENGINE TRIALS

THE running tests of an engine may be, in the same manner as for a boiler, of a superficial or commercial type, or they may involve the preparation of a heat balance to show the heat distribution at all stages of operation. The former test is generally satisfied by the determination of the brake and indicated horse powers and the consumption per horse-power hour, whereas the latter will show thermal and comparative thermal efficiencies and a heat balance for the engine trial.

The general procedure for running a trial is to bring the engine and its accessories to mechanical perfection and to run the engine until it has reached its best running condition and adjustment. The necessary observations are then taken over an adequate period, with 'calls' at reasonable intervals to ensure that mean and representative values are obtained. The test can then be divided into the determination of three factors :

(1) *Mechanical efficiency.*

(2) *Thermal and comparative thermal efficiency.*

(3) *Heat distribution and losses.*

Mechanical efficiency. The mechanical efficiency of an engine is essentially a power relationship and calls for the measurement of the *indicated horse power*, or power developed in the cylinders, and the *brake horse power*, or power available at the output coupling. The indicated horse power is determined by employing a suitable indicator (*Engineering Science,* vol. II, pp. 191–202), and using it to obtain the work diagram for the engine. The choice of indicator is important, especially in internal combustion engines, as the high speed and almost instantaneous explosion produces unreliable cards with an unsuitable indicator. The brake horse power is determined by the use of some form of absorption dynamometer (*Engineering Science*, vol. II, pp. 202–4), with the engine uncoupled from its normal load. Mechanical efficiency is then the relation between B.H.P. and I.H.P., and the difference between the two values of horse power may be regarded as the mechanical loss.

Thermal and comparative thermal efficiency. The *thermal efficiency* is the relationship between the heat equivalent of the indicated horse power and the heat supplied to the engine. In a steam engine this heat supplied is a measure of the heat content of the steam upon delivery to the engine, a quantity which may be obtained from steam tables after the *condition* or dryness fraction has been obtained by throttling, or the combined throttling and separating, of a sample of steam before delivery. This value, when the pressure is known, enables the heat content of the steam supplied per hour to be calculated. The indicated horse power is determined in connection with the mechanical efficiency test and when expressed as a heat equivalent forms the numerator of the thermal efficiency ratio.

The *comparative thermal efficiency* is the indicated thermal efficiency compared with that of a similar engine operating on one of the standard cycles, such as the *Rankine for a steam engine or turbine*, and the *air standard cycle for internal combustion engines.*

Heat distribution. The purpose of this investigation is to trace the distribution of heat energy over the engine and its auxiliaries. To achieve this a number of thermometer pockets and pressure gauges are fitted in places where temperature and pressure readings may be profitably taken, and certain measurements are obtained applicable to the particular engine under test. These provisions may be enumerated as follows :

Steam engines.

(1) Pressure at stop valve or entry.
(2) Vacuum in condenser.
(3) Weight of condensate, which is equal to that of supply steam.
(4) Temperature at entry of cooling water to condenser.
(5) Temperature at outlet of cooling water from condenser.
(6) Weight of cooling water passed.

Internal combustion engines.

(1) Volume, or weight, of fuel supplied.
(2) Calorific value of fuel.
(3) Inlet temperature of cooling water.
(4) Outlet temperature of cooling water.
(5) Compression ratio of engine.
(6) Weight of cooling water.
(7) Chemical analysis of exhaust gases.
(8) Mean specific heat of gases from (7).
(9) Temperature of exhaust gases.
(10) Temperature of supply for fuel.

To these, for the purposes of a complete thermodynamic test must be added the observations required for the determination of mechanical efficiency, namely:

(1) Dimensions of engine: stroke and cylinder diameter, tail rod diameter if necessary.

(2) Number of effective strokes or explosions per minute.

(3) Diameter of brake-wheel.

(4) Dead weight and spring balance readings on the brake, or other facts for the particular type of brake.

(5) Revolutions per minute.

(6) Indicator spring scale.

When these observations have been taken, the trial of the engine and the associated results can be followed through on the lines suggested by the following examples.

TRIAL OF A 5 B.H.P. DOUBLE ACTING CONDENSING STEAM ENGINE

LOG OF THE TRIAL.

Engine dimensions and running observations.

L = stroke = 10 in. = **0·833 ft.**
Dia. of piston = 6 in.
Dia. of piston rod = $1\frac{1}{4}$ in.
A_1 = area of piston (crank end) $= \pi \times 3^2 - \pi \times 5/8^2 =$ **27·03 sq. in.**
A_2 = area of piston (cover end) $= \pi \times 3^2 =$ **28·26 sq. in.**
N = mean number of revolutions per minute on a twenty minutes' trial = **116.**
Scale of indicator spring = **80.**

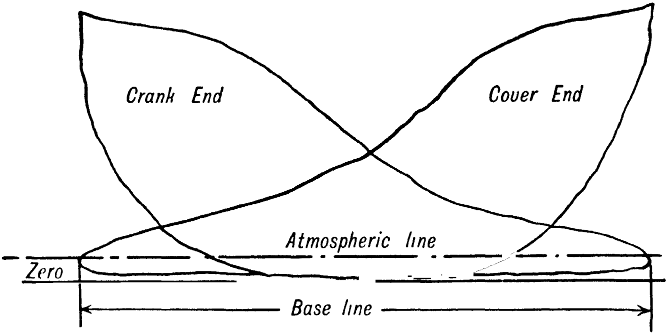


FIG. 250.

D = mean diameter of brake wheel = **4·5 ft.**
W = dead load = **130 lb.**
S = mean spring balance reading = **18 lb.**
Duration of trial = 20 min.
Weight of condensate = 48·8 lb. or 2·44 lb. per min.
Boiler pressure = 135 lb. per sq. in. gauge.
Vacuum in condenser = 26 in.
Barometer reading = 29·98 in.

$$\text{Pressure in condenser} = \frac{3\cdot98}{29\cdot98} \times 14\cdot7 = 1\cdot95 \text{ lb. per sq. in.}$$

Temperature of inlet of cooling water = 12° C.
,, ,, outlet ,, ,, = 30° C.
Quantity of cooling water = 104 gallons = 1040 lb.

NOTE. *The duration of the trial should be as long as is possible, and will depend upon the time available to conduct the test.*

MECHANICAL EFFICIENCY TEST.

Indicated horse power.

Area of crank end diagram = 1·751 sq. in.
,, cover ,, ,, = 1·69 ,,
Length of base line = 3 in.

$$\text{Mean effective pressure (crank end)} = \frac{1\cdot751 \times 80}{3},$$

$$P_1 = 46\cdot69 \text{ lb. per sq. in.}$$

$$\text{Mean effective pressure (cover end)} = \frac{1\cdot69 \times 80}{3},$$

$$P_2 = 45\cdot06 \text{ lb. per sq. in.}$$

$$\text{I.H.P. (crank end)} = \frac{P_1 . L . A_1 . N}{33{,}000}$$

$$= \frac{46{\cdot}69 \times 0{\cdot}833 \times 27{\cdot}03 \times 116}{33{,}000}$$

$$= 3{\cdot}696.$$

$$\text{I.H.P. (cover end)} = \frac{P_2 . L . A_2 . N}{33{,}000}$$

$$= \frac{45{\cdot}06 \times 0{\cdot}833 \times 28{\cdot}26 \times 116}{33{,}000}$$

$$= 3{\cdot}728.$$

Total I.H.P. = **7·42.**

Brake horse power.

$$\text{B.H.P.} = \frac{\pi D(W - S)N}{33{,}000}$$

$$= \frac{\pi \times 4{\cdot}5 \times 112 \times 116}{33{,}000},$$

B.H.P. = **5·56.**

$$\textbf{Mechanical efficiency} = \frac{\text{B.H.P.}}{\text{I.H.P.}} = \frac{5{\cdot}565}{7{\cdot}42} = 75\%.$$

THERMAL EFFICIENCY TEST.

Condition of steam at entry. (Throttling test.)

Temperature corresponding to 135 lb. per sq. in. gauge = 181° C.

Pressure after throttling = 14·7 lb. per sq. in.

Temperature ,, ,, = 104° C.

Then if the specific heat of superheated steam is 0·48 :

$$183{\cdot}5 + x \times 482{\cdot}3 = 639{\cdot}3 + 0{\cdot}48(104 - 100),$$

$$x = \frac{457{\cdot}72}{482{\cdot}3} = 94{\cdot}8\% \text{ dry.}$$

Heat given to engine per minute

$$= 2{\cdot}44\{183{\cdot}5 + 0{\cdot}948 \times 482{\cdot}3\}$$

$$= 1563{\cdot}3 \text{ C.H.U.}$$

Heat rejected by condensate per minute

= Heat of water at 1·95 lb. per sq. in. abs.

$$= 2{\cdot}44 \times 51{\cdot}64 = 126{\cdot}0 \text{ C.H.U.}$$

Net heat supplied = Heat supplied − heat rejected

$$= 1563{\cdot}3 - 126 = 1437{\cdot}3 \text{ C.H.U.}$$

Indicated thermal efficiency:

$$\mu = \frac{\text{Heat equivalent of I.H.P.}}{\text{Net heat supplied}}$$

$$= \frac{7{\cdot}42 \times 33{,}000}{1437{\cdot}3 \times 1400} = 12{\cdot}17\%.$$

Steam consumption per I.H.P. per hour

$$= \frac{2{\cdot}44 \times 60}{7{\cdot}42} = 19{\cdot}73 \text{ lb.}$$

HEAT DISTRIBUTION AND HEAT BALANCE.

Heat given to cooling water

$$= \text{weight of water per minute} \times \text{temperature rise}$$

$$= \frac{1040}{20} \times 18^\circ \text{ C.} = 936 \text{ C.H.U.}$$

Stage	Heat supplied C.H.U. per min.	Heat utilised C.H.U. per min.	% supplied	% utilised
Heat supplied to engine	1563·3		100%	
Heat rejected by condenser		126·0		8·1%
Heat equivalent of B.H.P.		131·2		8·4%
Heat equivalent of friction losses		43·7		2·8%
Heat given to cooling water		936·0		59·9%
Other losses		326·4		20·8%
Total	1563·3	1563·3	100%	100%

Summary.

Brake horse power	5·6
Indicated horse power	7·4
Mechanical efficiency	75%
Indicated thermal efficiency	12·2%
Consumption per I.H.P. hour	19·7 lb.

Note on thermal efficiency. The general definition of thermal efficiency is the ratio between the heat equivalent of the indicated horse power and the heat supplied. In the case of a steam engine, the heat supplied is, by convention, taken as *the heat content of the supply steam less the heat possessed by the water composing the condensate.* Actually this heat possessed by the water of the condensate is of no value, unless the condensate becomes a constituent of the feed water. In some tests made by steam users, a second thermal efficiency is computed in which the water heat of the condensate is neglected; but this second thermal efficiency is not regarded as the standard for comparison with the efficiency under the Rankine cycle or an even more popular mode of comparison, namely, steam consumption per kW. per hour or per H.P. per hour.

TRIAL OF A 25 B.H.P. GAS ENGINE

The fuel supply is from town mains with a calorific value of 475 B.Th.U. per cu. ft. determined by a Boys' calorimeter test and reduced to N.T.P. The gas consumed is measured by a gas meter and the consumption reduced to N.T.P. Jacket cooling water is measured by a calibrated orifice and B.H.P. by a rope brake.

LOG OF THE TRIAL.

Engine dimensions. Dia. of cylinders = 12 in.
Stroke = 18 in.
Compression ratio = 3·75.

Running observations. Duration of trial = 40 minutes.
Total gas consumed (by meter) = 863 cu. ft.
Meter pressure (by manometer) = 770 mm.
Meter temperature = 17° C.

$$C = \text{Gas consumed per min. at N.T.P.} = \frac{273 \times 863 \times 770}{290 \times 760 \times 40} = \mathbf{20{\cdot}58 \text{ cu. ft.}}$$

N = Average speed over trial = **228 r.p.m.**
N_1 = Average number of explosions per min. = **102.**
D = Diameter of brake wheel + rope = **6 ft. 1½ in.**
W = Dead weight on rope = **220 lb.**
S = spring balance reading = **31 lb.**
Total quantity of cooling water = 23·9 cu. ft.
J = Cooling water per min. = **37·2 lb.**
Inlet temperature of cooling water = 16° C.
Outlet ,, ,, ,, ,, = 59·5° C.
t = Rise in temperature of cooling water = **43·5° C.**

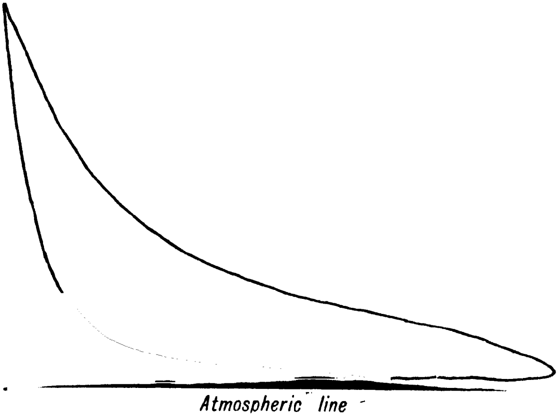


FIG. 251.

MECHANICAL EFFICIENCY TEST.

Scale of spring = 150.

Area of diagram = 1·36 sq. in.

Mean effective pressure = 77 lb. per sq. in.

$$\text{Indicated horse power} = \frac{P\,.\,L\,.\,A\,.\,N_1}{33{,}000}$$

$$= \frac{77 \times 1{\cdot}5 \times \pi \times 6^2 \times 102}{33{,}000}$$

$$= 40{\cdot}37 \text{ H.P.}$$

$$\text{Brake horse power} = \frac{\pi D(W - S)N}{33{,}000}$$

$$= \frac{\pi \times 6{\cdot}125 \times 189 \times 228}{33{,}000}$$

$$= 25{\cdot}13 \text{ H.P.}$$

$$\text{Mechanical efficiency} = \frac{\text{B.H.P.}}{\text{I.H.P.}}$$

$$= \frac{25{\cdot}13}{40{\cdot}37} = 62{\cdot}24\%.$$

THERMAL EFFICIENCY TEST.

Calorific value of fuel = 475 B.Th.U. per cu. ft.
= 264 C.H.U. per cu. ft.

$$\text{Heat equivalent of I.H.P. per min.} = H_1 = \frac{40{\cdot}37 \times 33{,}000}{1400}$$

$$= 951{\cdot}7 \text{ C.H.U.}$$

$$\text{Heat supplied per min.} = C \times 264$$
$$= H_2 = 20{\cdot}58 \times 264 = 5433 \text{ C.H.U.}$$

$$\textbf{Indicated thermal efficiency} = \frac{H_1}{H_2} = \frac{951{\cdot}7}{5433}$$
$$= 17{\cdot}52\%.$$

$$\textbf{Efficiency on air standard cycle} = 1 - \frac{1}{r^{(\gamma-1)}},$$

where r = ratio of compression = 3·75,
$\gamma = 1{\cdot}4$.

$$\text{Efficiency} = 1 - \frac{1}{(3{\cdot}75)^{0{\cdot}4}} = 1 - 0{\cdot}5893$$
$$= 41{\cdot}07\%.$$

$$\textbf{Comparative thermal efficiency} = \frac{17{\cdot}5}{41{\cdot}07} = 42{\cdot}6\%.$$

HEAT DISTRIBUTION TEST.

Heat given to cooling water per min. = Jt
= 37·2 × 43·5 = 1618 C.H.U.
Heat supplied per min. = 5433 C.H.U.
Heat equivalent of I.H.P. = 951·7 C.H.U.
,, ,, ,, B.H.P. $= \dfrac{25{\cdot}13 \times 33{,}000}{1400}$
= 592·3 C.H.U.
Heat to friction = 951·7 − 592·3 = 359·4 C.H.U.
Heat to exhaust, etc. = 5433 − {1618 + 951·7}
= 2863 C.H.U.

HEAT BALANCE.

Stage	Heat supplied C.H.U. per min.	Heat utilised C.H.U. per min.	% supplied	% used
Heat supplied by fuel -	5433		100%	
Heat to B.H.P. - -		592·3		11%
,, ,, friction - -		359·4		6·6%
,, ,, jacket water -		1618·0		29·8%
,, ,, exhaust, etc. -		2863		52·6%
Totals - -	5433	5432·7	100%	100%

$$\textbf{Consumption per I.H.P. hour} = \frac{20{\cdot}58 \times 60}{40{\cdot}37} = 30{\cdot}6 \textbf{ cu. ft.}$$

Summary.

Indicated horse power	40·4
Brake horse power	25·1
Mechanical efficiency	62·2%
Indicated thermal efficiency	17·5%
Comparative thermal efficiency	42·6%
Efficiency on air standard cycle	41·1%
Consumption per I.H.P. hr.	30·6 cu. ft.

TYPICAL EXAMPLES ON ENGINE TRIALS

Example 1. *The following observations were made in a test on a six-cylinder four-stroke cycle petrol engine, having cylinders* 5⅝ *in. diameter, stroke* 7½ *in. and clearance volume* 55·9 *cu. in.*

Brake horse power 163.
Fuel consumption 100 *pints per hour.*
Specific gravity of fuel 0·725.
Water flowing through jackets per min. 64 *lb.*
Rise of temperature of jacket water 110° *F.*

Calculate :

(*a*) *The compression ratio.*
(*b*) *The air standard cycle efficiency ;* $\gamma = 1{\cdot}4$.
(*c*) *Fuel consumption in lb. per brake horse power per hour.*
(*d*) *The horse power lost to the jacket water.* (U.E.I.)

(*a*) Compression ratio $$= \frac{\pi \times 2{\cdot}813^2 \times 7{\cdot}5 + 55{\cdot}9}{55{\cdot}9}$$

$$= \frac{242{\cdot}3}{55{\cdot}9} = 4{\cdot}334.$$

(*b*) $$\mu = 1 - \frac{1}{r^{(\gamma-1)}}$$

$$= 1 - \frac{1}{(4{\cdot}334)^{0{\cdot}4}} = 44{\cdot}38\%.$$

(*c*) Fuel consumption $$= \frac{100 \times 10 \times 0{\cdot}725}{8 \times 163}$$

$$= 0{\cdot}556 \text{ lb. per B.H.P. hour.}$$

(*d*) Horse power lost to jackets $$= \frac{64 \times 110 \times 778}{33{,}000} = 166 \text{ H.P.}$$

Example 2. *The following data were recorded during the test of a gas engine : I.H.P.* 560 *; B.H.P.* 447 *; revs. per min.* 300 *; gas consumption* *(reduced to* 15° *C. and* 30 *in. of mercury)* 32,500 *cu. ft. per hour ; lower calorific value of the gas per cu. ft.* 71·2 *C.H.U. ; jacket water used per hour* 2,870 *gallons ; temperature of the jacket water at inlet and outlet respectively* 18·1° *C. and* 54·3° *C. Determine (a) The mechanical efficiency of the engine. (b) The thermal efficiency of the engine. (c) The percentage heat energy of the gas carried away by the jacket water. (d) Neglecting waste of heat by radiation, etc., find the percentage heat energy of the gas carried away by the exhaust gases.* (U.L.C.I.)

(*a*) Mechanical efficiency $=\dfrac{\text{B.H.P.}}{\text{I.H.P.}}=\dfrac{447}{560}=79{\cdot}8\%$.

(*b*) Thermal efficiency $=\dfrac{560\times 33{,}000\times 60}{1400\times 32{,}500\times 71{\cdot}2}=34{\cdot}2\%$.

(*c*) Heat to jacket water $=28{,}700\,\{54{\cdot}3-18{\cdot}1\}$

$=1{,}039{,}000$ C.H.U. per hr.

$$\text{Percentage}=\frac{1{,}039{,}000}{32{,}500\times 71{\cdot}2}=44{\cdot}9\%.$$

(*d*) Heat to exhaust gases

$$=32{,}500\times 71{\cdot}2-\left\{\frac{560\times 33{,}000\times 60}{1400}+1{,}039{,}000\right\}$$

$$=2{,}314{,}000-\{792{,}000+1{,}039{,}000\}$$

$$=483{,}000\ \text{C.H.U. per hour.}$$

$$\text{Percentage}=\frac{483{,}000\times 100}{2{,}314{,}000}=20{\cdot}9\%.$$

Example 3. *The following particulars were obtained during a one-hour trial of a Diesel engine developing* 130 *H.P. : Mechanical efficiency* 70% *; average speed* 160 *revs. per min. ; total oil consumed,* 42·2 *lb. of calorific value,* 10,800 *C.H.U. per lb. ; air supplied per lb. of fuel* 40 *lb. ; specific heat of exhaust gases* 0·25 *; temperature of engine room* 18° *C. ; temperature of exhaust gases* 290° *C. ; cooling water per min.* 44 *lb. with a temperature rise of* 46° *C. Draw up an approximate heat balance sheet for this engine.* (U.E.I.)

Heat equivalent of I.H.P. $=\dfrac{130\times 33{,}000}{1400}=\mathbf{3065}$ **C.H.U. per min.**

Heat equivalent of B.H.P. $=\frac{7}{10}\times 3065=\mathbf{2145{\cdot}5}$ **C.H.U. per min.**

Heat to friction $=3065-2145{\cdot}5=\mathbf{919{\cdot}5}$ **C.H.U. per min.**

Heat supplied $=\dfrac{42{\cdot}2}{60}\times 10{,}800=\mathbf{7596}$ **C.H.U. per min.**

$$\text{Heat to exhaust gases} = \frac{42{\cdot}2}{60}\{1+40\}0{\cdot}25 \times \{290-18\}$$

$$= \mathbf{1961\ C.H.U.\ per\ min.}$$

$$\text{Heat to cooling water} = 44 \times 46 = \mathbf{2024\ C.H.U.\ per\ min.}$$

Heat Balance.

Stage	Heat supplied C.H.U. per min.	Heat utilised C.H.U. per min.	% supplied	% utilised
Heat supplied by fuel	7596		100%	
Heat equivalent of B.H.P.		2145·5		28·24%
Heat to friction		919·5		12·11%
Heat to exhaust gases		1961·0		25·8%
Heat to jacket water		2024·0		26·65%
Heat unaccounted for		546·0		7·2%
Totals	7596	7596	100%	100

Example 4. *The following observations were taken during the trial of a double-acting steam engine : Stroke,* 1 *ft.* 6 *in. ; piston diameter,* 8 *in. ; piston rod,* 1½ *in. dia. ; steam supply pressure,* 180 *lb. per sq. in. abs. ; dryness fraction,* 0·96 *; average speed over trial,* 194 *r.p.m. ; area of indicator diagram crank end,* 2·04 *sq. in.; cover end,* 1·97 *sq. in.; base line,* 3·5 *in.; spring,* 100 *; dia. of brake wheel,* 6 *ft.* 1 *in. with rope ; dead weight,* 356 *lb. ; spring balance reading,* 44 *lb. ; vacuum in condenser,* 27·4 *in. ; barometer,* 30·0 *in. The steam supply is at the rate of* 1158 *lb. per hour. Find the mechanical and thermal efficiencies and the consumption of steam per brake horse-power per hour.*

Area of piston.

$$A_1(\text{crank end}) = \pi\{16 - 3/4^2\} = 48{\cdot}5 \text{ sq. in.}$$

$$A_2(\text{cover end}) = \pi \times 16 = 50{\cdot}24 \text{ sq. in.}$$

Mean effective pressures.

$$P_1(\text{crank end}) = \frac{204}{3{\cdot}5} = 58{\cdot}3 \text{ lb. per sq. in.}$$

$$P_2(\text{cover end}) = \frac{197}{3{\cdot}5} = 56{\cdot}3 \quad ,, \quad ,,$$

$$\textbf{Indicated horse power} = \frac{P_1 . L . A_1 . N}{33{,}000} + \frac{P_2 . L . A_2 . N}{33{,}000}$$

$$= \frac{1{\cdot}5 \times 194}{33{,}000}\{58{\cdot}3 \times 48{\cdot}5 + 56{\cdot}3 \times 50{\cdot}24\}$$

$$= \mathbf{49{\cdot}86.}$$

$$\textbf{Brake horse power} = \frac{\pi \times 6{\cdot}083 \times (356 - 44)194}{33{,}000} = \mathbf{35{\cdot}05.}$$

$$\textbf{Mechanical efficiency} = \frac{\text{B.H.P.}}{\text{I.H.P.}} = \frac{35{\cdot}05}{49{\cdot}86} = \mathbf{70{\cdot}3\%.}$$

Thermal efficiency.

Heat content of steam per lb. on supply

$$= 192{\cdot}3 + 0{\cdot}96 \times 475{\cdot}3 = 648{\cdot}59 \text{ C.H.U.}$$

$$\text{Vacuum pressure} = \frac{2{\cdot}6}{30} \times 14{\cdot}7 = 1{\cdot}274 \text{ lb. per sq. in.}$$

Water heat of condensate = 43·22 C.H.U. per lb.

Net heat supplied = 648·59 − 43·22

= 605·37 C.H.U. per lb.

Heat equivalent of I.H.P. per lb. of steam used

$$= \frac{1}{1158} \times \frac{49{\cdot}86 \times 33{,}000 \times 60}{1400} = 60{\cdot}9 \text{ C.H.U.}$$

$$\textbf{Indicated thermal efficiency} = \frac{\text{heat equivalent of I.H.P.}}{\text{net heat supplied}}$$

$$= \frac{60{\cdot}9}{605{\cdot}37} = \mathbf{10{\cdot}1\%.}$$

Steam consumption per B.H.P. per hour

$$= \frac{1158}{35{\cdot}05} = \mathbf{33 \text{ lb.}}$$

Answers. **Mechanical efficiency = 70·3%.**

Indicated thermal efficiency = 10·1%.

Steam consumption per B.H.P. hr. = 33 lb.

Example 5. *The following data relate to a very compact* 550 *kW. turbo-generator made by Messrs. Belliss and Morcom at their Birmingham works. The turbine is of the impulse type, compounded for velocity and pressure with thirteen stages. The turbine has a speed of* 7000 *r.p.m. geared down to* 1000 *r.p.m. for an electrical generator ; and the observations and derived results for eight tests at different mean loads are tabulated on p.* 494.

			1	2	3	4	5	6	7	8
a	Load (kW.)		529·5	552·5	545	561	552	417·5	280	0
b	Steam consumption (lb./hr.)		7466	7652	7544	7941	7571	5914	4207	1018
c	Steam per kW. per hr.		14·1	13·85	13·84	14·15	13·72	14·17	15·02	—
d	Steam Supply (Boiler)	Absolute pressure (lb. in.^{2})	431	442	465	450	451	477·5	497·5	447·5
e		Temperature (° F.)	655	659	682	688	698	687	660	563
f		Total Heat (B.Th.U./lb.)	1339	1341	1353	1356	1362	1343	1338	1285
g	Condenser	Back pressure (inches of mercury)	1·68	1·68	1·705	1·71	1·61	1·51	0·99	0·74
h		Total Heat (B.Th.U./lb.)	895	893	896	896	893	886	856	821
i	Available supply of energy heat drop (B.Th.U./lb.)		444	448	457	460	469	457	482	464
j	Overall efficiency (%)		54·5	55	53·9	52·4	53·3	41·1	47·1	—
k	Brake thermal efficiency turbine (%)			60·3	(2) GEARING EFFICIENCY 98%. GENERATOR EFFICIENCY 93%.					

TURBO-GENERATOR TEST.

Observations.

The principal observations, and also derived results, for each of the eight tests at different loads are tabulated on the opposite page.

Line *a*, average load in kW.
,, *b*, total steam consumption per hour.
,, *d*, steam supply absolute pressure in lb./in.^{2}
,, *e*, steam supply temperature in Fahrenheit degrees.
,, *g*, back pressure on the turbine in inches of mercury, as measured by the absolute pressure in the condenser.
,, *k*, for test (2) the gearing efficiency is 98% and the generator efficiency is 93%.

Derived results. From the above observations the following results are calculated :

Line *c*, steam per kW. per hour $= \dfrac{\text{Steam consumption in lb./hr.}}{\text{Load in kW.}}$

,, *f*, Total heat of steam of boiler supply in B.Th.U. per lb., obtained from steam tables and charts from observations in lines *d* and *e*.

,, *h*, total heat of exhaust steam to condenser in B.Th.U. per lb., obtained from the Total Heat v. Entropy chart, assuming adiabatic expansion through the turbine.

,, *i*, heat drop in B.Th.U. per lb., that is, total heat of boiler supply steam per lb. (line *f*) minus total heat per lb. of exhaust steam (line *h*). This is the heat energy available for conversion to mechanical and electrical energy by the turbo-generator.

,, *j*, overall efficiency $= \dfrac{\text{Load in kW. converted to B.Th.U.}}{\text{Heat drop in B.Th.U.}} \times 100.$

This efficiency includes the effect of all losses in the turbo-generator, including those in the turbine, gearing and generator, based on the heat drop.

,, *k*, Since for the second test the gearing efficiency and generator efficiency are 98% and 93% respectively the brake thermal efficiency for the turbine alone, based on the heat drop for the 552 kW. load, is

$$55\% \times \frac{1}{0{\cdot}98} \times \frac{1}{0{\cdot}93} = 60{\cdot}3\%.$$

A Willans line has been drawn as shown in the graph of Fig. 252, in which the load in kW. is plotted against steam consumption in lb. per

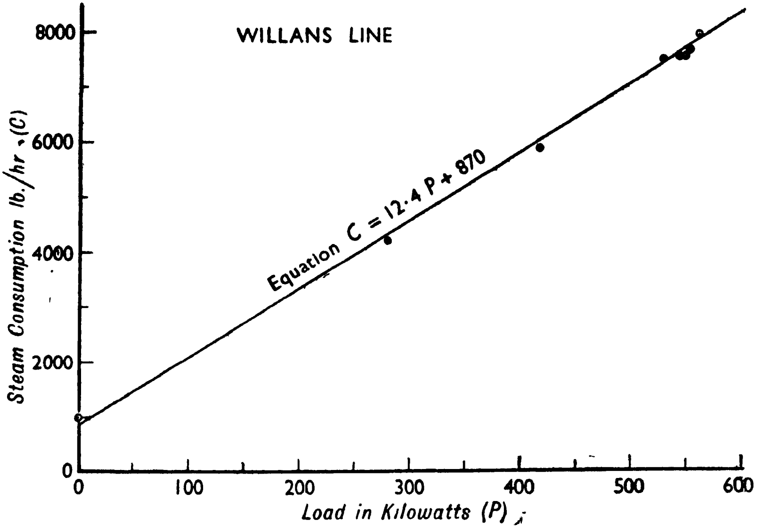


FIG. 252.

hour. This gives an approximate straight line following the law

$$C = 12{\cdot}4P + 870.$$

Example 6. *A Diesel-electric installation is to serve a* 230-*volt equipment. A test taken gives the following observations : I.H.P.,* 59·4 ; *B.H.P. of engine uncoupled from generator,* 42·4 ; *calorific value of fuel,* 9000 *C.H.U. per lb. ; output amperage of generator,* 110 *on full load.*

Find (*a*) *the mechanical efficiency of the engine ;*
(*b*) *the output of the generator in B.T.U. ;*
(*c*) *the efficiency of the generator;*
(*d*) *the overall efficiency of the installation ;*
(*e*) *the indicated thermal efficiency of the engine ;*
(*f*) *the consumption rate of the engine in lb. of fuel per B.T.U. generated.*

Fuel consumption on a one-hour trial = 22·8 *lb.*

Find the cost of generation if the fuel costs 9 *pence per gallon and its specific gravity is* 0·78.

(*a*) Mechanical efficiency of the engine $= \frac{\text{B.H.P.}}{\text{I.H.P.}} = \frac{42{\cdot}4}{59{\cdot}4} = 71{\cdot}4\%$.

(*b*) Output of generator $= 230 \times 110$ watts $= 25{\cdot}3$ **B.T.U.**

(*c*) Efficiency of generator $= \dfrac{\text{output}}{\text{B.H.P. of engine}}$.

Since 1 H.P. = 746 watts,

$$\mu = \frac{230 \times 110}{42{\cdot}4 \times 746} = 80\%.$$

(*d*) Overall efficiency $= \dfrac{\text{output of generator}}{\text{heat value of fuel}}$

$$= \left\{\frac{230 \times 110 \times 33{,}000}{746}\right\} \div \left\{\frac{22{\cdot}8}{60} \times \frac{9000}{1} \times \frac{1400}{1}\right\}$$

$$= \frac{230 \times 110 \times 33{,}000 \times 60}{746 \times 22{\cdot}8 \times 9000 \times 1400} = 23{\cdot}4\%.$$

(*e*) Indicated thermal efficiency of engine

$$= \frac{59{\cdot}4 \times 33{,}000 \times 60}{22{\cdot}8 \times 9000 \times 1400} = 41\%.$$

(*f*) Consumption rate $= \dfrac{22{\cdot}8 \times 1000}{230 \times 110} = 0{\cdot}9$ **lb. per B.T.U**

Cost per unit of electricity generated

$$= 0{\cdot}9 \times \frac{9}{7{\cdot}8} \text{ per B.T.U.} = 1{\cdot}04 \text{ pence per unit}$$

Answers. (*a*) **71·4% ;** (*b*) **25·3 B.T.U. ;** (*c*) **80% ;**
(*d*) **23·4% ;** (*e*) **41% ;** (*f*) **0·9 lb. per B.T.U.**
Cost: 1·04 pence per B.T.U.

EXERCISES ON CHAPTER XIX

1. Outline the objects of a steam or internal combustion engine trial, and state the nature of the observations you would take when conducting a trial.

2. How are the component parts of a heat balance obtained, and what is the purpose of this heat account?

3. The trial of a single-acting steam engine of dimensions, piston area 30·1 sq. in., stroke 10½ in., gave the following results: mean effective pressure, 52·4 lb. per sq. in.; torque on brake wheel, 140 lb. ft.; speed, 110 r.p.m.; condenser pressure, 2 lb. per sq. in abs.; boiler pressure, 120 lb. per sq. in. abs. The condenser passed cooling water at the rate of 22 lb. per min. with a temperature rise of 15° C., and the weight of steam condensed is 1·8 lb. per minute. Prepare a heat balance for this engine if the boiler produces steam 0·97 dry.

4. The results of a gas engine test gave the following results: I.H.P., 7·9; B.H.P., 6·3; cooling water, 740 lb. per hour with a tem-

perature rise of 34° C. ; consumption, 180 cu. ft. of gas per hour of calorific value 450 B.Th.U. per cu. ft. Prepare the heat balance for this engine.

5. In a one-hour trial of a Diesel engine the I.H.P. was 60·2 and the mechanical efficiency 70%. The cooling water entered the jacket at 10° C. and left at 50° C., and 141 gallons per hour were circulated. The calorific value of the fuel was 9100 C.H.U. per lb. and 22·1 lb. were used. Calculate the brake thermal efficiency ; the heat given to the jacket water ; the heat carried off by the exhaust gases and otherwise ; and draw up a heat balance sheet. (U.E.I.)

6. Describe, with the aid of sketches of apparatus used, the method of determining the mechanical efficiency of an engine. (U.L.C.I.)

7. The mean effective pressure in an internal combustion engine working on the two-stroke cycle was 91·4 lb. per sq. in. The total fuel consumption per hour was 126·7 lb. and the calorific value of the fuel was 10,120 C.H.U. per lb. The B.H.P. was 327·3 and the speed 125 r.p.m. Diameter of cylinder, 22 in.; stroke, 36 in. Determine the mechanical efficiency, the indicated thermal efficiency and the brake thermal efficiency. (U.L.C.I.)

8. The results of a gas engine test gave the following : I.H.P., 7·6 ; B.H.P., 6·1 ; jacket water, 700 lb. per hour ; mean temperature rise, 35° C. ; gas used, 175 cu. ft. per hour ; calorific value of gas, 344 C.H.U. per cu. ft. Draw up a heat balance sheet, taking quantities in C.H.U. per hour, and state the indicated and brake thermal efficiencies. (U.E.I.)

9. You are required to carry out tests for power, fuel consumption and thermal efficiency on (*a*) a high-speed petrol engine developing about 40 B.H.P. ; *or* (*b*) a gas engine developing about 10 B.H.P. at 300 r.p.m.

Make a neat diagrammatic sketch of the layout of the plant you would use, explaining, briefly, the purpose of each item. Draw up a probable heat balance sheet for the test at maximum power. (U.E.I.)

10. A four-stroke cycle gas engine has a cylinder 9 in. diameter and 16 in. stroke and it is fitted with a brake wheel 4½ ft. in diameter. The indicator diagram shown [Fig. 253] was obtained during a test on the engine. Prick the diagram through on to your examination paper and determine the mean pressure on the engine piston.

During the test the following results were obtained : Revolutions per minute, 250 ; explosions per minute, 66 ; effective load on brake, 82 lb. ; gas consumed, 60 cu. ft. in 15 minutes ; calorific value of gas, 280 C.H.U. per cu. ft. Determine the indicated horse power, mechanical efficiency, and the thermal efficiency on a brake horse power basis. (U.E.I.)

11. Draw up the heat balance sheet for an oil engine from the following data, taking quantities in C.H.U. per hour:

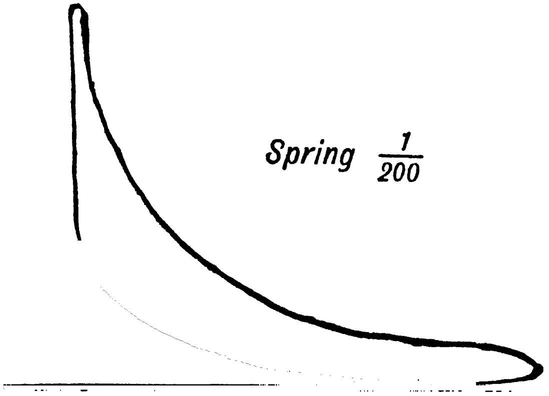


FIG 253.

Oil per hour, 3·08 lb.; calorific value, 10,200 C.H.U. per lb.; indicated horse power, 5·8, brake horse power, 4·4; cooling water per hour, 385 lb; rise in temperature of cooling water, 36° C. (U.E.I.)

12. The following particulars were obtained from a test of a four-stroke cycle gas engine: Cylinder diameter, 6½ in.; stroke, 12 in.; dia. of brake wheel, 5½ ft.; effective brake load, 60 lb; revolutions per min., 306; explosions per min., 128; area of indicator card, 0·68 sq. in.; length, 2·25 in., spring, $\frac{1}{300}$; gas used per hour, 453 cu. ft.; calorific value, 530 B.Th.U. per cu. ft. Calculate the I.H.P., B.H.P., mechanical efficiency, gas consumption per B.H.P. per hour, and the brake thermal efficiency (U.E.I.)

13. The area of an indicator diagram taken from a double-acting steam engine is 2·1 sq. in. and its length 3·5 in. The strength of spring used is 60 lb. per sq. in. of height. The diameter of the cylinder is 18 in. and the piston stroke 26 in. Speed is 150 revolutions per minute. Find the I.H.P. of the engine and also the B.H.P., the mechanical efficiency being 78 per cent. (U.L.C.I.)

14. A gas engine governed on the "hit and miss" principle gave the following results on test: Revolutions per minute, 230; explosions per minute, 68, net brake load, 47·5 lb.; diameter of brake drum, 3 ft. 6⅜ in; area of indicator diagram, 1·38 sq. in; length of diagram, 3·6 in.; indicator spring rate, 1 in. = 200 lb.; cylinder diameter, 6½ in.; stroke, 14 in. Find the indicated and brake horse powers. If 126·5 cubic feet of gas of calorific value 472 B.Th.U. per cu. ft. are used per hour, what will be the brake thermal efficiency of the engine at this load? (U.E.I.)

15. The stroke and bore of a gas engine are 12 in. and 6·5 in. respectively. When the connecting rod is 3 ft. long the clearance volume is

124 cu. in. When a packing piece is inserted in the big end the connecting rod is increased in length by 0·75 in. Determine the compression ratio for each length of connecting rod.

Discuss the effect of increasing the compression ratio upon : (*a*) the thermodynamic cycle of the engine, (*b*) its working as a practical machine.

What considerations limit the compression ratio which can be used in practice? On testing the engine without the packing piece, it was found that the gas used was 318·5 cu. ft. per hour. If the diameter of the brake wheel was 5·5 ft., effective brake load 60 lb., revolutions of crank shaft per minute 300, calorific value of gas 250 C.H.U. per cu. ft., calculate the gas consumption per brake horse power per hour and the brake thermal efficiency. (U.E.I.)

16. A ship is driven by two sets of eight-cylinder internal combustion engines working on the four-stroke cycle with an impulse stroke every cycle. Diameter of cylinders, 19·7 in. ; stroke, 26 in. ; revs. per min., 145·3 ; average mean effective pressure for all cylinders, 74 lb. per sq. in. The thermal efficiency of the engines is 44 per cent.

Find the total I.H.P. driving the ship and the weight of oil used per I.H.P. hour and per day's run of 24 hours. The calorific value of the oil is 9890 C.H.U. per lb. (U.L.C.I.)

17. A Diesel-electric set produces 145 amperes at 230 volts. The efficiency of the generator is 89%, the mechanical efficiency of the engine 71% and the calorific value of the fuel 9020 C.H.U. per lb.

Find (*a*) the brake horse power of the engine ; (*b*) the indicated horse power of the engine ; (*c*) the consumption rate in lb. per electrical horse power per hour if the overall thermal efficiency on the electric output basis is 25% ; (*d*) the indicated thermal efficiency of the engine.

18. What current output may be expected on a 200-volt service from a petrol-electric set using 6 pints of petrol of calorific value 70,000 C.H.U. per gallon in a one-hour run if the indicated thermal efficiency of the engine is 37% and the overall thermal efficiency on the electrical output 28%. Find the indicated horse power of the engine and the consumption rate per B.T.U.

19. An engine is coupled to a pump delivering 10,000 gallons of water per hour to a height of 100 ft. with a pump efficiency of 76%. Find the brake horse power of the engine. If the brake thermal efficiency is 31%, find the consumption of fuel per hour if the calorific value is 8720 C.H.U. per lb.

20. A ship develops 20,000 shaft horse power at 60 r.p.m. If the mechanical efficiency of engine and transmission is 52%, find the indicated horse power of the engines. Assuming an indicated thermal efficiency of 28% for the engines, calculate the oil consumption per hour for a fuel of calorific value 9020 C.H.U. per lb. What is the loss of horse power between that available from the fuel and that given to the propeller shaft?

MATHEMATICAL TABLES.

TABLE I. USEFUL CONVERSION CONSTANTS.

Linear. 1 inch = 25·4 millimetres, 1 mm. = 0·03937 in.
1 foot = 30·48 centimetres, 1 metre = 39·37 in.

Superficial. 1 sq. in. = 6·452 sq. cm., 1 sq. cm. = 0·1557 sq. in.

Volume. 1 cu. in. = 16·39 cu. cm., 1 cu. cm. = 0·06011 cu. in.

Capacity. 1 gallon = 0·1604 cu. ft. = 10 lb. of water at 62° F.

Weight. 1 pound avoirdupois = 453·6 grammes.

Density. 1 cu. ft. of water weighs 62·3 lb.
1 cu. ft. of air at 0° C. and 1 atmos. weighs 0·0807 lb.
1 cu. ft. of hydrogen at 0° C. and 1 atmos. weighs 0·00559 lb.

Pressure. 1 atmosphere = 14·7 lb. per sq. in. = 2116 lb. per sq. ft. = 760 mm. of mercury.
Gauge Pressure = Absolute Pressure − 14·7 lb. per sq. in.

Velocity. 1 knot = 6080 feet per hour.
60 miles per hour = 88 feet per second.

Power. 1 Horse Power = 33,000 ft. lb. per minute = 550 ft. lb. per second = 746 watts.
1 watt = 1 volt × 1 ampere.

Energy. 1 British Thermal Unit = 778 ft. lb.
1 Centigrade Heat Unit = 1400 ft. lb.

Angular. 1 radian = 57·3 degrees.

Logarithms. To convert common into Napierian Logarithms multiply by 2·3026. Base of Napierian Logarithm = e = 2·7183.

Acceleration due to earth's pull. Value of "g" in London = 32·182 ft. per sec. per sec.

TABLE II. LOGARITHMS.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170						5	9	13	17	21	26	30	34	38
						0212	0253	0294	0334	0374	4	8	12	16	20	24	28	32	36
11	0414	0453	0492	0531	0569						4	8	12	16	20	23	27	31	35
						0607	0645	0682	0719	0755	4	7	11	15	18	22	26	29	33
12	0792	0828	0864	0899	0934						3	7	11	14	18	21	25	28	32
						0969	1004	1038	1072	1106	3	7	10	14	17	20	24	27	31
13	1139	1173	1206	1239	1271						3	6	10	13	16	19	23	26	29
						1303	1335	1367	1399	1430	3	7	10	13	16	19	22	25	29
14	1461	1492	1523	1553	1584						3	6	9	12	15	19	22	25	28
						1614	1644	1673	1703	1732	3	6	9	12	14	17	20	23	26
15	1761	1790	1818	1847	1875						3	6	9	11	14	17	20	23	26
						1903	1931	1959	1987	2014	3	6	8	11	14	17	19	22	25
16	2041	2068	2095	2122	2148						3	6	8	11	14	16	19	22	24
						2175	2201	2227	2253	2279	3	5	8	10	13	16	18	21	23
17	2304	2330	2355	2380	2405						3	5	8	10	13	15	18	20	23
						2430	2455	2480	2504	2529	3	5	8	10	12	15	17	20	22
18	2553	2577	2601	2625	2648						2	5	7	9	12	14	17	19	21
						2672	2695	2718	2742	2765	2	4	7	9	11	14	16	18	21
19	2788	2810	2833	2856	2878						2	4	7	9	11	13	16	18	20
						2900	2923	2945	2967	2989	2	4	6	8	11	13	15	17	19
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13	14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	8	10	11	12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10	11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	7	8	9	10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7	8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7	8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8

TABLE II. LOGARITHMS.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	3	3	4	5	6	7	8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	2	3	4	5	6	7	7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4	5	6	6	7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	2	3	4	5	6	6	7
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4	5	5	6	7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4	5	5	6	7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4	5	5	6	7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4	4	5	6	7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4	4	5	6	7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1	1	2	3	4	4	5	6	6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	4	4	5	6	6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1	1	2	3	3	4	5	6	6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3	4	5	5	6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1	1	2	3	3	4	5	5	6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1	1	2	3	3	4	5	5	6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1	1	2	3	3	4	5	5	6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1	1	2	3	3	4	5	5	6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1	1	2	3	3	4	4	5	6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1	1	2	2	3	4	4	5	6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1	1	2	2	3	4	4	5	6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1	1	2	2	3	4	4	5	5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1	1	2	2	3	4	4	5	5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1	1	2	2	3	4	4	5	5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1	1	2	2	3	4	4	5	5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1	1	2	2	3	3	4	5	5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1	1	2	2	3	3	4	5	5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1	1	2	2	3	3	4	4	5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1	1	2	2	3	3	4	4	5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1	1	2	2	3	3	4	4	5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1	1	2	2	3	3	4	4	5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1	1	2	2	3	3	4	4	5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1	1	2	2	3	3	4	4	5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1	1	2	2	3	3	4	4	5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1	1	2	2	3	3	4	4	5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1	1	2	2	3	3	4	4	5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1	1	2	2	3	3	4	4	5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0	1	1	2	2	3	3	4	4
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0	1	1	2	2	3	3	4	4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2	3	3	4	4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0	1	1	2	2	3	3	4	4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0	1	1	2	2	3	3	4	4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0	1	1	2	2	3	3	4	4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0	1	1	2	2	3	3	4	4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0	1	1	2	2	3	3	4	4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0	1	1	2	2	3	3	4	4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0	1	1	2	2	3	3	4	4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0	1	1	2	2	3	3	4	4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0	1	1	2	2	3	3	4	4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0	1	1	2	2	3	3	3	4

TABLE III. ANTILOGARITHMS

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
·00	1000	1002	1005	1007	1009	1012	1014	1016	1019	1021	0	0	1	1	1	1	2	2	2
·01	1023	1026	1028	1030	1033	1035	1038	1040	1042	1045	0	0	1	1	1	1	2	2	2
·02	1047	1050	1052	1054	1057	1059	1062	1064	1067	1069	0	0	1	1	1	1	2	2	2
·03	1072	1074	1076	1079	1081	1084	1086	1089	1091	1094	0	0	1	1	1	1	2	2	2
·04	1096	1099	1102	1104	1107	1109	1112	1114	1117	1119	0	1	1	1	1	2	2	2	2
·05	1122	1125	1127	1130	1132	1135	1138	1140	1143	1146	0	1	1	1	1	2	2	2	2
·06	1148	1151	1153	1156	1159	1161	1164	1167	1169	1172	0	1	1	1	1	2	2	2	2
·07	1175	1178	1180	1183	1186	1189	1191	1194	1197	1199	0	1	1	1	1	2	2	2	2
·08	1202	1205	1208	1211	1213	1216	1219	1222	1225	1227	0	1	1	1	1	2	2	2	3
·09	1230	1233	1236	1239	1242	1245	1247	1250	1253	1256	0	1	1	1	1	2	2	2	3
·10	1259	1262	1265	1268	1271	1274	1276	1279	1282	1285	0	1	1	1	1	2	2	2	3
·11	1288	1291	1294	1297	1300	1303	1306	1309	1312	1315	0	1	1	1	2	2	2	2	3
·12	1318	1321	1324	1327	1330	1334	1337	1340	1343	1346	0	1	1	1	2	2	2	2	3
·13	1349	1352	1355	1358	1361	1365	1368	1371	1374	1377	0	1	1	1	2	2	2	3	3
·14	1380	1384	1387	1390	1393	1396	1400	1403	1406	1409	0	1	1	1	2	2	2	3	3
·15	1413	1416	1419	1422	1426	1429	1432	1435	1439	1442	0	1	1	1	2	2	2	3	3
·16	1445	1449	1452	1455	1459	1462	1466	1469	1472	1476	0	1	1	1	2	2	2	3	3
·17	1479	1483	1486	1489	1493	1496	1500	1503	1507	1510	0	1	1	1	2	2	2	3	3
·18	1514	1517	1521	1524	1528	1531	1535	1538	1542	1545	0	1	1	1	2	2	2	3	3
·19	1549	1552	1556	1560	1563	1567	1570	1574	1578	1581	0	1	1	1	2	2	3	3	3
·20	1585	1589	1592	1596	1600	1603	1607	1611	1614	1618	0	1	1	1	2	2	3	3	3
·21	1622	1626	1629	1633	1637	1641	1644	1648	1652	1656	0	1	1	2	2	2	3	3	3
·22	1660	1663	1667	1671	1675	1679	1683	1687	1690	1694	0	1	1	2	2	2	3	3	3
·23	1698	1702	1706	1710	1714	1718	1722	1726	1730	1734	0	1	1	2	2	2	3	3	4
·24	1738	1742	1746	1750	1754	1758	1762	1766	1770	1774	0	1	1	2	2	2	3	3	4
·25	1778	1782	1786	1791	1795	1799	1803	1807	1811	1816	0	1	1	2	2	2	3	3	4
·26	1820	1824	1828	1832	1837	1841	1845	1849	1854	1858	0	1	1	2	2	3	3	3	4
·27	1862	1866	1871	1875	1879	1884	1888	1892	1897	1901	0	1	1	2	2	3	3	3	4
·28	1905	1910	1914	1919	1923	1928	1932	1936	1941	1945	0	1	1	2	2	3	3	4	4
·29	1950	1954	1959	1963	1968	1972	1977	1982	1986	1991	0	1	1	2	2	3	3	4	4
·30	1995	2000	2004	2009	2014	2018	2023	2028	2032	2037	0	1	1	2	2	3	3	4	4
·31	2042	2046	2051	2056	2061	2065	2070	2075	2080	2084	0	1	1	2	2	3	3	4	4
·32	2089	2094	2099	2104	2109	2113	2118	2123	2128	2133	0	1	1	2	2	3	3	4	4
·33	2138	2143	2148	2153	2158	2163	2168	2173	2178	2183	0	1	1	2	2	3	3	4	4
·34	2188	2193	2198	2203	2208	2213	2218	2223	2228	2234	1	1	2	2	3	3	4	4	5
·35	2239	2244	2249	2254	2259	2265	2270	2275	2280	2286	1	1	2	2	3	3	4	4	5
·36	2291	2296	2301	2307	2312	2317	2323	2328	2333	2339	1	1	2	2	3	3	4	4	5
·37	2344	2350	2355	2360	2366	2371	2377	2382	2388	2393	1	1	2	2	3	3	4	4	5
·38	2399	2404	2410	2415	2421	2427	2432	2438	2443	2449	1	1	2	2	3	3	4	4	5
·39	2455	2460	2466	2472	2477	2483	2489	2495	2500	2506	1	1	2	2	3	3	4	5	5
·40	2512	2518	2523	2529	2535	2541	2547	2553	2559	2564	1	1	2	2	3	4	4	5	5
·41	2570	2576	2582	2588	2594	2600	2606	2612	2618	2624	1	1	2	2	3	4	4	5	5
·42	2630	2636	2642	2649	2655	2661	2667	2673	2679	2685	1	1	2	2	3	4	4	5	6
·43	2692	2698	2704	2710	2716	2723	2729	2735	2742	2748	1	1	2	3	3	4	4	5	6
·44	2754	2761	2767	2773	2780	2786	2793	2799	2805	2812	1	1	2	3	3	4	4	5	6
·45	2818	2825	2831	2838	2844	2851	2858	2864	2871	2877	1	1	2	3	3	4	5	5	6
·46	2884	2891	2897	2904	2911	2917	2924	2931	2938	2944	1	1	2	3	3	4	5	5	6
·47	2951	2958	2965	2972	2979	2985	2992	2999	3006	3013	1	1	2	3	3	4	5	5	6
·48	3020	3027	3034	3041	3048	3055	3062	3069	3076	3083	1	1	2	3	4	4	5	6	6
·49	3090	3097	3105	3112	3119	3126	3133	3141	3148	3155	1	1	2	3	4	4	5	6	6

TABLE III. ANTILOGARITHMS.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	
·50	3162	3170	3177	3184	3192	3199	3206	3214	3221	3228	1	1	2	3	4	4	5	6	7
·51	3236	3243	3251	3258	3266	3273	3281	3289	3296	3304	1	2	2	3	4	5	5	6	7
·52	3311	3319	3327	3334	3342	3350	3357	3365	3373	3381	1	2	2	3	4	5	5	6	7
·53	3388	3396	3404	3412	3420	3428	3436	3443	3451	3459	1	2	2	3	4	5	6	6	7
·54	3467	3475	3483	3491	3499	3508	3516	3524	3532	3540	1	2	2	3	4	5	6	6	7
·55	3548	3556	3565	3573	3581	3589	3597	3606	3614	3622	1	2	2	3	4	5	6	7	7
·56	3631	3639	3648	3656	3664	3673	3681	3690	3698	3707	1	2	3	3	4	5	6	7	8
·57	3715	3724	3733	3741	3750	3758	3767	3776	3784	3793	1	2	3	3	4	5	6	7	8
·58	3802	3811	3819	3828	3837	3846	3855	3864	3873	3882	1	2	3	4	4	5	6	7	8
·59	3890	3899	3908	3917	3926	3936	3945	3954	3963	3972	1	2	3	4	5	5	6	7	8
·60	3981	3990	3999	4009	4018	4027	4036	4046	4055	4064	1	2	3	4	5	6	6	7	8
·61	4074	4083	4093	4102	4111	4121	4130	4140	4150	4159	1	2	3	4	5	6	7	8	9
·62	4169	4178	4188	4198	4207	4217	4227	4236	4246	4256	1	2	3	4	5	6	7	8	9
·63	4266	4276	4285	4295	4305	4315	4325	4335	4345	4355	1	2	3	4	5	6	7	8	9
·64	4365	4375	4385	4395	4406	4416	4426	4436	4446	4457	1	2	3	4	5	6	7	8	9
·65	4467	4477	4487	4498	4508	4519	4529	4539	4550	4560	1	2	3	4	5	6	7	8	9
·66	4571	4581	4592	4603	4613	4624	4634	4645	4656	4667	1	2	3	4	5	6	7	9	10
·67	4677	4688	4699	4710	4721	4732	4742	4753	4764	4775	1	2	3	4	5	7	8	9	10
·68	4786	4797	4808	4819	4831	4842	4853	4864	4875	4887	1	2	3	4	6	7	8	9	10
·69	4898	4909	4920	4932	4943	4955	4966	4977	4989	5000	1	2	3	5	6	7	8	9	10
·70	5012	5023	5035	5047	5058	5070	5082	5093	5105	5117	1	2	4	5	6	7	8	9	11
·71	5129	5140	5152	5164	5176	5188	5200	5212	5224	5236	1	2	4	5	6	7	8	10	11
·72	5248	5260	5272	5284	5297	5309	5321	5333	5346	5358	1	2	4	5	6	7	9	10	11
·73	5370	5383	5395	5408	5420	5433	5445	5458	5470	5483	1	3	4	5	6	8	9	10	11
·74	5495	5508	5521	5534	5546	5559	5572	5585	5598	5610	1	3	4	5	6	8	9	10	12
·75	5623	5636	5649	5662	5675	5689	5702	5715	5728	5741	1	3	4	5	7	8	9	10	12
·76	5754	5768	5781	5794	5808	5821	5834	5848	5861	5875	1	3	4	5	7	8	9	11	12
·77	5888	5902	5916	5929	5943	5957	5970	5984	5998	6012	1	3	4	5	7	8	10	11	12
·78	6026	6039	6053	6067	6081	6095	6109	6124	6138	6152	1	3	4	6	7	8	10	11	13
·79	6166	6180	6194	6209	6223	6237	6252	6266	6281	6295	1	3	4	6	7	9	10	11	13
·80	6310	6324	6339	6353	6368	6383	6397	6412	6427	6442	1	3	4	6	7	9	10	12	13
·81	6457	6471	6486	6501	6516	6531	6546	6561	6577	6592	2	3	5	6	8	9	11	12	14
·82	6607	6622	6637	6653	6668	6683	6699	6714	6730	6745	2	3	5	6	8	9	11	12	14
·83	6761	6776	6792	6808	6823	6839	6855	6871	6887	6902	2	3	5	6	8	9	11	13	14
·84	6918	6934	6950	6966	6982	6998	7015	7031	7047	7063	2	3	5	6	8	10	11	13	15
·85	7079	7096	7112	7129	7145	7161	7178	7194	7211	7228	2	3	5	7	8	10	12	13	15
·86	7244	7261	7278	7295	7311	7328	7345	7362	7379	7396	2	3	5	7	8	10	12	13	15
·87	7413	7430	7447	7464	7482	7499	7516	7534	7551	7568	2	3	5	7	9	10	12	14	16
·88	7586	7603	7621	7638	7656	7674	7691	7709	7727	7745	2	4	5	7	9	11	12	14	16
·89	7762	7780	7798	7816	7834	7852	7870	7889	7907	7925	2	4	5	7	9	11	13	14	16
·90	7943	7962	7980	7998	8017	8035	8054	8072	8091	8110	2	4	6	7	9	11	13	15	17
·91	8128	8147	8166	8185	8204	8222	8241	8260	8279	8299	2	4	6	8	9	11	13	15	17
·92	8318	8337	8356	8375	8395	8414	8433	8453	8472	8492	2	4	6	8	10	12	14	15	17
·93	8511	8531	8551	8570	8590	8610	8630	8650	8670	8690	2	4	6	8	10	12	14	16	18
·94	8710	8730	8750	8770	8790	8810	8831	8851	8872	8892	2	4	6	8	10	12	14	16	18
·95	8913	8933	8954	8974	8995	9016	9036	9057	9078	9099	2	4	6	8	10	12	15	17	19
·96	9120	9141	9162	9183	9204	9226	9247	9268	9290	9311	2	4	6	8	11	13	15	17	19
·97	9333	9354	9376	9397	9419	9441	9462	9484	9506	9528	2	4	7	9	11	13	15	17	20
·98	9550	9572	9594	9616	9638	9661	9683	9705	9727	9750	2	4	7	9	11	13	16	18	20
·99	9772	9795	9817	9840	9863	9886	9908	9931	9954	9977	2	5	7	9	11	14	16	18	20

TABLE IV.

Angle.		Chords.	Sine.	Tangent.	Cotangent.	Cosine.			
Deg.	Radians.								
0°	0	0	0	0	∞	1	1·414	1·5708	**90°**
1	·0175	·017	·0175	·0175	57·2900	·9998	1·402	1·5533	89
2	·0349	·035	·0349	·0349	28·6363	·9994	1·389	1·5359	88
3	·0524	·052	·0523	·0524	19·0811	·9986	1·377	1·5184	87
4	·0698	·070	·0698	·0699	14·3006	·9976	1·364	1·5010	86
5	·0873	·087	·0872	·0875	11·4301	·9962	1·351	1·4835	**85**
6	·1047	·105	·1045	·1051	9·5144	·9945	1·338	1·4661	84
7	·1222	·122	·1219	·1228	8·1443	·9925	1·325	1·4486	83
8	·1396	·139	·1392	·1405	7·1154	·9903	1·312	1·4312	82
9	·1571	·157	·1564	·1584	6·3138	·9877	1·299	1·4137	81
10	·1745	·174	·1736	·1763	5·6713	·9848	1·286	1·3963	**80**
11	·1920	·192	·1908	·1944	5·1446	·9816	1·272	1·3788	79
12	·2094	·209	·2079	·2126	4·7046	·9781	1·259	1·3614	78
13	·2269	·226	·2250	·2309	4·3315	·9744	1·245	1·3439	77
14	·2443	·244	·2419	·2493	4·0108	·9703	1·231	1·3265	76
15	·2618	·261	·2588	·2679	3·7321	·9659	1·217	1·3090	**75**
16	·2793	·278	·2756	·2867	3·4874	·9613	1·204	1·2915	74
17	·2967	·296	·2924	·3057	3·2709	·9563	1·190	1·2741	73
18	·3142	·313	·3090	·3249	3·0777	·9511	1·176	1·2566	72
19	·3316	·330	·3256	·3443	2·9042	·9455	1·161	1·2392	71
20	·3491	·347	·3420	·3640	2·7475	·9397	1·147	1·2217	**70**
21	·3665	·364	·3584	·3839	2·6051	·9336	1·133	1·2043	69
22	·3840	·382	·3746	·4040	2·4751	·9272	1·118	1·1868	68
23	·4014	·399	·3907	·4245	2·3559	·9205	1·104	1·1694	67
24	·4189	·416	·4067	·4452	2·2460	·9135	1·089	1·1519	66
25	·4363	·433	·4226	·4663	2·1445	·9063	1·075	1·1345	**65**
26	·4538	·450	·4384	·4877	2·0503	·8988	1·060	1·1170	64
27	·4712	·467	·4540	·5095	1·9626	·8910	1·045	1·0996	63
28	·4887	·484	·4695	·5317	1·8807	·8829	1·030	1·0821	62
29	·5061	·501	·4848	·5543	1 8040	·8746	1·015	1·0647	61
30	·5236	·518	·5000	·5774	1·7321	·8660	1·000	1·0472	**60**
31	·5411	·534	·5150	·6009	1·6643	·8572	·985	1·0297	59
32	·5585	·551	·5299	·6249	1·6003	·8480	·970	1·0123	58
33	·5760	·568	·5446	·6494	1·5399	·8387	·954	·9948	57
34	·5934	·585	·5592	·6745	1·4826	·8290	·939	·9774	56
35	·6109	·601	·5736	·7002	1·4281	·8192	·923	·9599	**55**
36	·6283	·618	·5878	·7265	1·3764	·8090	·908	·9425	54
37	·6458	·635	·6018	·7536	1·3270	·7986	·892	·9250	53
38	·6632	·651	·6157	·7813	1·2799	·7880	·877	·9076	52
39	·6807	·668	·6293	·8098	1·2349	·7771	·861	·8901	51
40	·6981	·684	·6428	·8391	1·1918	·7660	·845	·8727	**50**
41	·7156	·700	·6561	·8693	1·1504	·7547	·829	·8552	49
42	·7330	·717	·6691	·9004	1·1106	·7431	·813	·8378	48
43	·7505	·733	·6820	·9325	1 0724	·7314	·797	·8203	47
44	·7679	·749	·6947	·9657	1·0355	·7193	·781	·8029	46
45	·7854	·765	·7071	1·0000	1 0000	·7071	·765	·7854	**45**
								Radians.	Deg.
			Cosine.	Cotangent.	Tangent.	Sine.	Chords.	Angle.	

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ANSWERS TO EXERCISES

CHAPTER I (p. 21)

2. (*a*) 13·64 m.p.h. ; (*b*) 67·56 f.s. ; (*c*) 3682 m.p.h. per hour ; (*d*) 251·3 rad./sec. ; (*e*) 1432 revs./min./min. ; (*f*) 0·073 rad./sec./sec.

3. 66 f.s., 0·04 rad./sec., 3·3 f.s.s., 0·002 rad./sec./sec.

4. 0·209 rad./sec./sec., 0·42 f.s.s. **8.** 6928 ft. lb. **9.** 59,013 ft. tons.

10. 5878 ft. lb., 0. **12.** 75,000 ft. tons, 124·2 H.P. **13.** 94·3 H.P.

16. $20\frac{1}{4}$, 2·6 lb. **17.** 14·4, 18 : 5.

18. $7\frac{1}{2}$, 96 r.p.m., 5, *Y* five times as strong as *X*.

19. 70·4 r.p.m., 74·7 lb. ft.

20. 200 r.p.m., $363\frac{7}{11}$ r.p.m., 618 r.p.m., 1000 r.p.m.

21. (*a*) $4\frac{7}{12}$ and $1\frac{4}{11}$; (*b*) 180·1 lb. ; (*c*) 289 lb. **22.** 100 r.p.m., 431 lb.

23. 19·54 lb., 6·51 lb. **24.** (*a*) 28·27 cm. ; (*b*) 942·5 ; (*c*) 0·027 mm.

25. 0·8 in. per inch. **26.** 5·21 ft./min., 3802 lb. **27.** 6480 ; 28,370 lb. ft.

28. (*a*) 960 ; (*b*) 0·64. **29.** 21,710 lb.

30. (*a*) 500,000 f.s.s. ; (*b*) 138,393 tons ; (*c*) 138,393 ft. tons.

32. 0·127, 0·2, 0·54, 0·82, 1·63 H.P. ; 0·035, 0·15, 0·41, 0·61, 1·23 kW. ; 5·39, 8·47, 23·1, 34·65, 69·3 B.Th.U./min.

33. 14·73 ft. tons/min., 883·9 ft. tons/hr.

34. 30,016,800 ft. lb., 992·3 H.P. **35.** 180,000 gals., 250 H.P., 186·5 kW.

CHAPTER II (p. 55)

1. (*a*) 0, 0, 1_{30°. **2.** $2{\cdot}05_{13^\circ\,4'}$. **3.** $4{\cdot}63_{322^\circ\,4'}$.

4. (*a*) $11{\cdot}96_{108^\circ\,45'}$; (*b*) $10{\cdot}87_{184^\circ\,11'}$. **6.** $1{\cdot}68_{217^\circ\,5'}$.

7. 12·95 lb. at an angle of 220° 43′ and acting tangential to a circle of radius 0·025″ about the point of application of the 10 lb. force.

8. 13 lb. at $5\frac{4}{13}$ ft. from the 2 lb. force. **9.** $2{\cdot}83_{135^\circ}$ lb. at 3·54 ft. from *D*.

10. 960_{63° lb., 820_{90° lb.

11. $16_{47^\circ\,56'}$ lb. cutting the line *pq*, 1·13 ft. from *p*.

12. 54_{331° tons, line of action clockwise and 4·8 ft. from outside corner of base of wall ; 259·2 tons ft. about same point.

13. $8{\cdot}6_{107^\circ}$ tons, $2{\cdot}9_{34^\circ}$ tons. **14.** $10{\cdot}26_{2^\circ\,47'}$ tons, $14{\cdot}5_{135^\circ}$ tons.

15. $W = 138{\cdot}6$ lb., $R = 91{\cdot}7_{70^\circ\,54'}$ lb. **16.** 49° 48′.

17. (*a*) Horizontally, 13·23 in. below the bowl centre ; (*b*) at 27° 53′ to the horizontal.

19. Top, 240_{180° lb., Bottom $312{\cdot}4_{39^\circ\,48'}$ lb. **20.** 17·4 lb., 13·5 lb. **21.** 45°.

22. 2·79 ft. horizontally from A and 4·83 ft. vertically below A ; 0·577 ton.
23. 21·92 in.
24. 0·5s from one edge and 0·346s from adjacent edge if s is length of side of square.
25. $7\frac{1}{2}$ in. from centre.
26. 1·69 in. from each outside edge and 3 ft. from either end of bar.
27. 1·32 in. from longer outside edge and 1·82 in. from shorter outside edge.
28. 27° 37′, 209·2 tons. 31. 72 in.
32. $1\frac{7}{8}$ in. above line through axles and 21·34 in. horizontally from back axle.
33. 1565 lb., 2267 lb., 652 lb. ft.
34. (a) 862 lb. ; (b) 10·97 in. from DC and 16·17 in. from DB ; (c) 340·5 lb. (d) $760_{103°}$ lb.

CHAPTER III (p. 78)

3. 50 f.s., $90{\cdot}5_{147°}$. 4. $6\frac{1}{2}$°. 5. $12\frac{3}{11}$ sec. 6. 65 min.
7. 4·13 min. ; 132° 5′ with direction of stream.
8. (a) 27·5 knots ; (b) $43\frac{7}{11}$ min. making an angle of 104° with direction of No. 1.
9. 4·33 min. after noon ; 26·1 and 17·4 miles.
10. W. 30° N., 35 knots. 11. First train must not be more than 240 ft. long.
12. (a) 9·6 f.s. ; (b) 4·8 ft. from C measured at right angles to BC ; (c) 2 rad./sec. ; (d) 0·64 tons ft.
13. Crosshead 10·22 f.s., valve momentarily at rest.
14. I_{CD} at D, I_{AB} at A, I_{BC} 9·7 ft. from D and 6·03 ft. from A.
15. 1·3 f.s. 16. 4·46 and 10π rad./sec., 22·8 f.s.
17. 0·36 f.s., 277 lb. 18. 10·4 f.s., 0·4 f.s.
19. 150 r.p.m., 24 r.p.m. both clockwise.
20. 192, 192, 96 r.p.m. clockwise ; 50 r.p.m. A.C.W., 200 r.p.m. C.W., 200 r.p.m. A.C.W.
21. 1·6 : 1. 22. Reduction from 25·2 to 1, $\frac{1}{7}$.

CHAPTER IV (p. 116)

1. 4·08 tons/in.^{2}. 2. 0·0001 and 98 lb. 3. 0·00006 and 2000 lb./in.^{2}.
4. (a) 10·2 tons/in.^{2} ; (b) 0·091 in. 5. (a) 0·045 in. ; (b) 13·6 tons/in.^{2}.
6. 2·35 in. 7. 4 tons/ins.^{2} and 0·059 in. 8. 9·2 tons.
9. 39 tons, 0·037 in. 10. 6·48 tons. 11. 4123 lb. 12. π : 4.
13. $\frac{7}{8}$ in. 14. 6 in. × 4 in. × $\frac{5}{8}$ in.
15. 34,380 lb./in.^{2}, 5,819 ft. 16. 13,750 tons/in.^{2}.
17. 13,750,000 lb./in.^{2}. 18. 13,180 tons/in.^{2}. 19. 0·29.

20. (*a*) 29,970,000 lb./in.^{2} ; (*b*) 0·25. **21.** 169 lb./in.^{2}, 0·33.
22. (*a*) 29·6 tons and 360 lb./in.^{2} ; (*b*) 30·4 tons and 2·4 tons/in.^{2} ; $8\cdot1 \times 10^{-8}$.
23. Four $1\frac{5}{8}$ in. dia. rods. **24.** 103 lb./in.^{2}, 2070 lb./in.^{2}.
25. 101·7 lb., 198·3 lb. by steel wire. **26.** 12,800,000 lb./in^2.
27. 0·27 in. **28.** 9380 lb./in^2. and 16,580 lb.
29. 0·000547, 7·33 tons/in.^{2}, 23 tons. **30.** 349° C., 50 tons/in.^{2}.
31. $62\frac{1}{2}$° C. increase, 9·75 tons/in.^{2}.
32. Dia. of pin 1·0022 in., Boss dia. is 0·0005 in. greater, 282° F.
33. 2·9838 in. × 1·9892 in., 35·1 tons/in.^{2}. **34.** 1·64 in.
35. 82·4 tons. **36.** 142. **37.** $\frac{7}{8}$ in. **38.** 1·4 in. dia.
39. 0·555 in. tons. **40.** 0·012 and 3·05 in. tons.
41. 0·0038 in., 25·55 in. lb., 6 in. **42.** $34\frac{1}{2}$ ft.
43. 1592 lb./in.^{2}, 2830 lb./in.^{2}, 6 in. lb. **44.** 9,340,00 lb./in.^{2}, 128·3 in. lb.
45. (*a*) 0·00052 ; (*b*) 7·03 tons/in.^{2} ; (*c*) 116·1 in. lb.
46. 0·116 in., 18·67 in. lb. **47.** 0·000273 in., 8150 lb./in.^{2}.
48. 1·75 in. **49.** 0·009 in., 2250 lb./in.^{2}.
50. (*a*) 80 tons ; (*b*) 60 tons ; (*c*) 55 tons ; (*d*) 70 tons ; (*a*) 5·92 in.^{2} ; (*b*) 4·4 in.^{2} ; (*c*) 4·07 in.^{2} ; (*d*) 5·16 in.^{2}.
51. 1 in., 2·5 in., 60%. **52.** $3\frac{5}{16}$ in., $\frac{13}{16}$ in.
53. $91\frac{2}{3}$%, 3·5 tons/in.^{2}, 7·68 tons/in.^{2}. **54.** 1 in., 5 rivets.
55. $\frac{15}{16}$ in. dia., 6 rivets, plate 9 in. wide. **56.** $\frac{3}{4}$ in., 3 in., 75%.
57. $\frac{15}{16}$ in. dia. rivets, $4\frac{1}{2}$ in. pitch. **58.** 7200 lb./in.^{2} ; 3600 lb./in.^{2}.
59. 0·2 in. **60.** $\frac{15}{16}$ in. plate. **61.** $1\frac{1}{4}$ in. plate.

CHAPTER V (p. 161)

16. 128. **17.** 503 kgm.
19. 36·34 tons/in.^{2}, 37·89 tons/in.^{2}, 40·9 tons/in.^{2}.

CHAPTER VI (p. 178)

1. (*a*) 1 ton ; (*b*) 0·39 ton. **2.** 4·52 cwt., 5·09 cwt.
3. 1826 lb. **4.** 127·3 lb., 146 lb., 124·7 lb.
5. $\mu = 0\cdot205$; at an angle of 23° 35′ to the horizontal. **6.** Slide ; 0·5.
7. $E = 0\cdot0089L + 10\cdot3$; 37·45%.
8. (*a*) Efficiency $= 1/(1\cdot62 + 10/L)$; (*b*) 61·7% ; (*c*) 58·1%.
9. 0·707 μWd. **10.** 60·6 lb. **11.** 23·4 lb.
12. 171 lb. ft., 16·4%. **13.** 6702 lb. **14.** 4300 lb.
15. 5·4 lb. **16.** $4\frac{3}{4}$ in. **17.** 2·27 turns. **18.** 2615 H.P.
19. 2912 lb., 249 lb. **20.** 0·87 H.P. **21.** (*a*) 1575 lb. in. ; (*b*) 599 lb.

CHAPTER VII (p. 196)

1. 2·5 tons, 3·5 tons.

Member	*AC*	*BC*	*BE*	*CE*	*DE*	*CD*	*AD*
Force in tons	1·43	2	4	0·6	1·7	0·6	2·9
Nature (Compressive or Tensile)	C	C	T	T	T	C	T

2. Reactions 4500 lb. each.

Member	*AF*	*FE*	*FG*	*BG*	*GH*	*HE*
Force in lb.	16,500	14,900	2600	15,200	7750	8000
Nature	C	T	C	C	T	T

Member	*HJ*	*CJ*	*JK*	*KE*	*KD*
Force in lb.	7750	15,200	2600	14,900	16,500
Nature	T	C	C	T	C

3. $A = 24·4$ tons C, $B = 4·6$ tons T, $C = 19·8$ tons T.

4.

Member	*BC*	*CE*	*ED*	*CA*	*EF*	*FA*
Force in lb.	2080	693	1387	1040	693	1734
Nature	C	T	C	T	C	T

Member	*FH*	*HG*	*HJ*	*JK*	*JA*
Force in lb	693	1387	693	2080	1040
Nature	C	C	T	C	T

5. Fig 118. Reaction 6½ tons.

Member	*AB*	*AD*	*FE*	*EG*	*ED*	*DC*	*CH*	*BC*
Force in tons	7·1	3·1	2·25	1·0	4·3	5·6	4·75	6·9
Nature	T	T	T	C	C	T	C	C

FIG. 119. Reactions 9⅔ tons, 4⅓ tons.

Member - - -	*AB*	*BH*	*BC*	*CD*	*CF*	*DE*	*DH*	*EH*	*EG*
Force (tons) - -	10·8	5·0	5·0	5·9	7·8	0	11·0	11·0	11·8
Nature - - -	C	T	T	C	C	Nil	T	T	C

FIG. 120. Reactions 3·59 tons, 2·91 tons.

Member - -	*AF*	*FG*	*GH*	*BG*	*HC*	*HD*	*FE*
Force (tons) -	4·3	0·12	2·6	3·1	3·75	1·0	3·1
Nature - -	C	C	T	C	C	T	T

6. Equivalent loads at B and $C=2$ tons and 1 ton.
Resultant $=8$ tons at 10½ ft. from A.

Member - -	*UP*	*PQ*	*QR*	*QS*	*RS*	*RT*
Force in tons -	14	11·3	6	6	8·5	6
Nature - -	T	C	T	C	C	T

7. FIG. 122.

Member - -	*AE*	*ED*	*EF*	*BF*	*FG*	*GD*
Force in lb. - -	2050	1660	800	1470	750	830
Nature - -	C	T	C	C	T	T

Member - -	*GH*	*BH*	*HJ*	*JD*	*CJ*
Force in lb. - -	1520	1940	1600	2500	3100
Nature - -	T	C	C	T	C

FIG. 123.

Member - -	*AD*	*ED*	*EF*	*FG*	*AF*	*GD*	*GH*	*BH*
Force in tons -	6·26	15·6	6·7	7·4	16·0	12·2	2·8	11·4
Nature - -	C	C	C	C	T	C	T	T

Member - -	*HJ*	*JM*	*JK*	*KC*	*KL*	*MC*	*LM*
Force in tons -	4·2	13·0	6·6	16·2	7·0	5·74	15·6
Nature - -	T	C	C	T	C	C	C

8. Reactions 236·4 tons, 336·4 tons.

Member - -	*BL*	*AJ*	*BA*	*BC*	*CD*	*CJ*	*DL*
Force in tons -	236·4	336·4	Nil	198	56	380	248
Nature - -	T	C	—	C	T	C	T

Member - -	*DE*	*EJ*	*EF*	*FL*	*FG*	*GL*	*GJ*
Force in tons -	162	365	80	240	120	258	276
Nature - -	C	C	T	T	C	T	C

9. Wind on left. Reactions 3040 lb., 1180 lb.

Member -	*pq*	*qr*	*ur*	*uv*	*qs*	*ps*	*rs*	*st*	*rt*	*ut*	*tv*
Force in lb.	4200	4200	3050	3050	2000	5200	3000	2350	450	0	2700
Nature -	C	C	C	C	C	T	T	T	T	–	T

Wind on right. Reactions 2300 lb., 2320 lb.

Member - -	*pq*	*qr*	*ur*	*uv*	*qs*	*ps*	*rs*	*st*	*rt*	*ut*	*tv*
Force in lb. -	2320	2320	3580	3580	0	0	0	0	2600	2000	2600
Nature - -	C	C	C	C	–	–	–	–	T	C	T

10. Reactions 5·7 tons, 6·4 tons.

Member -	*pq*	*qr*	*rs*	*st*	*uq*	*up*	*ur*	*us*	*ut*
Force (tons)	13·1	11·3	11·9	14·4	2·74	13·1	4·67	2·4	12·7
Nature -	C	C	C	C	C	T	T	C	T

11. $A = 41{\cdot}6$ tons (strut), $B = 20{\cdot}8$ tons (tie), $C = 31{\cdot}6$ tons (tie).

12. $DC = 4{\cdot}92$ tons (strut), $DE = GF = 3{\cdot}6$ tons (ties).

13. $AB = 22{,}600$ lb. (tie), $AC = 8500$ lb. (strut),
$AD = 12{,}100$ lb. (tie), $CD = 22{,}800$ lb. (strut).

14. $BC = 83{\cdot}8$ lb., $AC = 73{\cdot}6$ lb., compressive force $= 198{\cdot}3$ lb.

15. 1·7 tons in longer legs, 2·2 tons in shorter leg.

CHAPTER VIII (p. 218)

1. 180 tons ft. at the 20 tons load. 2· 34·5 tons ft. at the 5 tons load.

3. $13\frac{1}{2}$ tons and $75\frac{1}{4}$ tons ft.

4. S.F. changes abruptly from 4·1 tons positive to 1·9 tons negative. B.M. 49 tons ft.

5. $R_A = 7{\cdot}09$ tons, $R_B = 19{\cdot}91$ tons ; Max$^{\text{m}}$. S.F. 12·91 tons at 5 tons load ; Max$^{\text{m}}$. B.M. 28 tons ft. at R_B ; Points of inflexion 2·2 ft. and 10 ft. from L.H. end.

6. Max$^{\text{m}}$. B.M. 643·75 tons ft. at the 17½ tons load.

7. (*a*) 34·08 and 34·42 tons ; (*b*) 15·87 ft. ; (*c*) 637 tons ft.

8. 4·8 tons/in.^{2}. 9. 306·25 tons in.

10. Centroids (*a*) 3·22 in. ; (*b*) 3·52 in. from base line in each case ; Moments of inertia (*a*) 155·11 in.^{4} ; (*b*) 202·8 in.^{4}.

11. 933⅓ tons in. 12. 937½ ft., 253·2 tons in.

13. 2100 lb./in.^{2}, 214 ft., 896 lb. per ft. run.

14. 964⅓ ft., 292·8 tons in. 15. 1742 lb.

16. 1747 lb./ft. run, 7·8 tons.

17. 0·072 in., $1{\cdot}318 \times 10^{-6}$ in.^{4}, $5{\cdot}128 \times 10^{-4}$ tons in. 18. 15 in.

19. 107 lb. per sq. ft.

20. Tensions 3·33, 10, 16·67 ; Moments 2·22, 15·63, 42. 21. 12·55 ft.

22. 0·08 in., 0·06 in. 23. 17,770,000 lb./in.^{2}. 24. 32,770,000 lb./in.^{2}.

25. (*a*) 900 tons in. ; (*b*) 180 in.^{3} units. 26. 20·51 lb. in., 14·1 tons/in.^{2}.

27. 1·58 tons/in.^{2}.

28. 5·89 tons, 14·45 tons ft. ; (*a*) 1·43 tons per ft. run ; (*b*) 975 ft. ; (*c*) 28·9 in.^{3}

29. (*a*) 10 tons ft., 8·9 tons ; (*b*) 1·61 tons/in.^{2}.

30. Max$^{\text{m}}$. B.M. 72,000 lb. ft., 1044 lb./in.^{2}. 31. 11·57 tons/in.^{2}.

32. (*a*) 10 tons/in.^{2} ; (*b*) 3 tons in. ; (*c*) 12 tons/in.^{2} tensile and 8 tons/in.^{2}. tensile.

33. 0·525 in., 41·47 tons in.

CHAPTER IX (p. 240)

1. 31·67 ft./sec.^{2}. 2. 31·4 m.p.h. ; 15,150 ft.

3. $g \sin \theta$, (*a*) 0·326 ft./sec.^{2} ; (*b*) 1·014 ft./sec.^{2} ; 63·75 lb.

4. 856 H.P. 5. $\mu = 0{\cdot}034$. 6. 4200 lb. 7. 105·7 sec.

8. 0·49 H.P. 9. 8·125 sec.

10. (*a*) 0·105 rad./sec./sec. ; (*b*) 0·0325 tons ft.

11. $Wk^2/g = 2{\cdot}84$ tons and ft. units, 0·99 tons ft.

12. $Wk^2/g = 34{\cdot}9$ lb. and in. units, 31·4 rad./sec./sec., 1150 lb. in.

13. 492 lb. 14. 1876 ft. lb. per rev., 1334 revs.

15. 314·7 secs., 314 revs. $Ww/g = 140$ ton and ft units. 16. 687·7 lb. ft.

17. 74·8 tons ft. 18. 26·8 ft./sec., 161 tons. 19. 41 ft. tons, 1408 ft.

20. 117·6 ft. tons, 263 ft. 21. $Wv/g = 1866$ ft. and lb. units ; 466,000 ft. lb.

22. 15·5 ft./sec. ; 129 : 1.

23. (*a*) 6·28 rad./sec./sec. ; (*b*) 3930 lb. ft. ; (*c*) 21,600 ft. lb.

24. (1) **487 tons ft.** ; (2) **860 ft. tons.** **25.** 52·6 secs., 222·2 ft. tons, 1980 revs.

26. 0·351 tons ft. ; 49,400 ft. lb. **27.** $Wk^2/g = 0{\cdot}667$ ft. and lb. units.

28. $Wk^2/g = 0{\cdot}227$ ft. and lb. units. **29.** 1·944 × 10 ft. lb. ; 19,500 ft. lb.

30. 34 ft./sec. ; reduction 24·2 ft./sec. **31.** (*a*) 1028 lb. in. ; (*b*) 374·4 in. lb.

32. 0·0557 ft./sec., 0·8917 ft./sec. **33.** 4·44 tons.

34. (*a*) $Wk^2/g = 49{,}500$ ft. and lb. units ; (*b*) 7·89 tons.

CHAPTER X (p. 252)

1. 5490 lb. per sq. in. **2.** 1344 H.P. **3.** 2·35°, 2050 lb. per sq. in.

4. 4·5 H.P. **5.** 5850 lb. per sq. in., 0·161° per ft.

6. 3·42 in. say 3½ in. **7.** 1·887 in. say 2 in. **8.** 10·35 in. say 10½ in.

9. (*a*) 216 lb. ft. ; (*b*) 6·18 H.P.

10. (*a*) 4 tons per sq. in. ; (*b*) 785·5 tons in. ; (*c*) 975 H.P.

11. 11·42 in. and 7·61 in. **12.** 150 r.p.m.

13. 71,080 lb. ft. ; 4070 H.P. **14.** 6·5 in. dia. ; 23,500 lb. ft.

15. 1·58 ; $C = 11{,}814.000$ lb. per sq. in. **16.** 11,410,000 lb. per sq. in.

17. 1·56 ; 12,220,000 lb. per sq. in. **18.** 12,060,000 lb. per sq. in.

19. 2·76 in. **20.** 12·25 lb. **21.** 0·256 in.

22. Shaft, 2·41 in. say 2½ in. ; bolts 0·75 in.

23. 1·05 in. ; 11,875 lb. per sq. in.

CHAPTER XI (p. 280)

1. 41·35 ft. per sec. **3.** 13,870 lb., 325 lb. ft.

4. 2·87 ft. per sec. per sec. and 8·91 cm.

5. 12·24 m.p.h., 6013 ft. **6.** 8·49 in.

7. Locomotive will be stable and safe up to a height of 15·66 ft. for its centre of gravity.

8. (*a*) 30 m.p.h. ; (*b*) 39·9 m.p.h. ; (*c*) Overturning speed is 57·85 m.p.h. but sideslip occurs earlier at 39·9 m.p.h. If h were equal to $\frac{5}{3}d$ the condition for both overturning and sideslipping is reached at 39·9 m.p.h.

9. (*a*) Level ; $R_A = 7{\cdot}8$ tons, $R_E = 12{\cdot}2$ tons. Slope ; $R_A = 11{\cdot}2$ tons, $R_E = 8{\cdot}8$ tons : (*b*) 6° 51′ ; (*c*) 66·8 m.p.h.

10. (*a*) 17·73 m.p.h. ; (*b*) 24·3 m.p.h. **11.** 66·4 r.p.m., 93·9 r.p.m.

12. 9·79 in. **13.** Assuming equal arms, 158·8 r.p.m.

14. 202·8 r.p.m., 175·7 r.p.m. **15.** 117·5 tons. **16.** 353·4 lb.

17. 69·5 lb., 17 lb. **18.** 198·6 lb., 6·81 oz.

19. $W = 161$ lb. ; measured anticlockwise from W ; 100 lb., 70° ; 120 lb., 196° ; 116 lb., 209°.

20. 277·5 lb. ; 283° 36′ from 10 lb. mass and 37·7 lb. in magnitude.

21. $W = 30{\cdot}7$ lb. ; angle between W and 20 lb. at 18 in. = 150°.

22. $W = 22{\cdot}1$ lb. ; angular displacements of others from mass in plane A, 166°, 356° and 183° respectively.

23. Balance weights $117\frac{1}{2}$ lb. symmetrically arranged relatively to the cranks. Angular positions measured from one of cranks $113\frac{1}{2}°$ and $156\frac{1}{2}°$.

24. 4 ft. **25.** 0·55 sec. **26.** 2·006 sec. if $g = 32·2$.

27. 0·45 sec., 2·2. **28.** (*a*) 4·9 lb. ; (*b*) 2·43 ft. per sec. ; (*c*) $1\frac{2}{3}$ sec.

30. 0·45sec. **31.** (*a*) 394·8 lb. ; (*b*) 405·2 lb.

32. 88·8 ft. per sec. per sec.

33. π ft. per sec. and 0 ; π^2 ft. per sec. per sec., 0 ; π ft. per sec., 0.

34. 0·45 sec., 32·21 ft. per sec. per sec. **37.** 0·856 sec.

38. (*a*) 4·86 lb. in. ; (*b*) 0·425 in. lb. ; (*c*) 1·03 sec. **39.** 11,945,000 lb. per in.^{2}.

40. Movement of piston 1·527 in. ; 12·3 ft. per sec. ; 533·7 ft. per sec. per sec. ; 197·6 ft. per sec. per sec.

CHAPTER XII (p. 315)

1. 220 ft., 574 ft./sec. **2.** 31·9 ft. **3.** 5·03 ft.

4. 40·1 ft./sec., 28·4 ft./sec. **5.** 48·85 ft./sec.

6. 31·6 ft./sec., 64·9 lb./sq. in. above atmospheric.

7. 126·3 lb. per sq. in. **8.** 50·6 ft./sec.

9. 12·75 lb. per sq. in. **10.** 304 ft. sec., 784 gallons per min.

11. 84·6 ft. **12.** 21,390,000 gallons per day. **13.** 17·1 ft.

14. 59 ft., 3·94 H.P. **15.** 76·1 ft. **16.** 2·25 ft./sec., 1,489,000 gallons per day.

17. 44·8 H.P. **18.** 29 ft./sec., 1891 gallons. **19.** $m = 2·732$ ft., $i = 1/150$.

20. 25,020,000 gallons per day. **21.** 42,600 gallons per hour.

22. 2·264 in. **23.** 52,500 gallons per hour. **24.** $K = 0·625$.

25. 69,100 gallons per hour. **26.** 4·36 in. **27.** 9·93 ft.

28. 3558 gallons per hour. **29.** 76·8 ft.

30. $C' = 0·618$, $C'_v = 0·71$. **31.** 9,103,000 gallons per hour.

32. 22,030 gallons per hour.

33. 20·2 in., 4·7 ft., 2·03 lb. per sq. in.

CHAPTER XIII (p. 332)

1. 18,550 lb. **2.** 3830 lb. **3.** 2217 lb. **4.** 16,290 lb.

5. 71,170 in.^{4} units. **6.** 12,416 in.^{4} units. **7.** $13\frac{1}{3}$ ft. from free surface.

8. 10·1 ft. from free surface.

9. 57,810 lb. ; 29·18 ft. from free surface. **10.** 5·07 ft. from free surface.

11. 46,725 lb. ; $3\frac{8}{9}$ ft. above bottom of gate on centre line.

12. 692·5 lb. **13.** 2572 lb. **14.** 218·6 lb.

15. (*a*) $R = 10,560$ lb. stable ; righting moment 40,542 lb. ft. ; overturning moment 10,380 lb. ft.

(*b*) $R = 39,430$ lb. stable ; righting moment 358,100 lb. ft. ; overturning moment 83,070 lb. ft.

(*c*) $R = 25,500$ lb. stable ; righting moment 109,300 lb. ft. ; overturning moment 19,440 lb. ft.

CHAPTER XIV (p. 359)

2. 1080 lb. **3.** (*a*) 810 lb. ; (*b*) 29·4 H.P. ; (*c*) 37·5%.

5. 478·3 lb. per sec. **7.** (*a*) 135 lb. ; (*b*) 4·9 H.P. ; (*c*) 50%.

8. 708·4 lb. per sec., 3·18 ft. **9.** 18·8 ft. per sec., 5 H.P.

10. 1293 lb. **11.** 226 lbs. per inch. **13.** 25° 55′.

14. 22·6 ft. per sec. at 35° 5′ to the direction of flow.

15. 2864 lb. **16.** 6·94 ft. per sec., 3·23 ft. per sec.

18. (*a*) 28° 2′ to direction of jet ; (*b*) 35·6 ft. per sec. ; (*c*) 93° 27′ to direction of jet at inlet ; (*d*) 25·34 ft. per sec. ; (*e*) 761 lb. ; (*f*) 28° 36′ to the jet at inlet.

21. 63·4 H.P. **22.** 13·25 ft.

24. (*a*) 23° 48′ to jet ; (*b*) 4·96 ft. per sec. ; (*c*) 4·14 ft. per sec.

26. 99·6%, 704 H.P. **27.** 683 H.P., 99·8%. **28.** 149 ft. lb. per lb.

29. 1619 H.P., 97·7%. **30.** 0·63 H.P., 69·5%, 50·1%. **31.** 5 jets.

32. 234 ft. per sec. ; 144,300 lb. per min. ; 86%. **33.** 80·8 H.P., 92·7%.

34. (*a*) 107·4 ft. per sec. ; (*b*) 62,720 ft. lb. ; 0·5 H.P., 26·3%.

35. (*a*) 73·6 ft. per sec. ; (*b*) 0·61 H.P. ; (*c*) 76%.

36. (*a*) 0·78 H.P. ; (*b*) 99·4% ; 7·44, 8·13 lb., 1·09 lb.

CHAPTER XV (p. 395)

3. (*a*) 639·3 ; (*b*) 661·5 ; (*c*) 632·9 ; (*d*) 790·7 ; (*e*) 521·8 C.H.U. per lb.

4. 202·7 C.H.U. per lb., 3023 ft. per sec. **5.** 30 C.H.U. per lb.

6. (*a*) 151·6 C.H.U. per lb. ; (*b*) 2614 ft. per sec. ; (*c*) 810·4 H.P.

7. 829 sq. ft. **8.** 381·5 H.P. **9.** 67,200 lb. per hour.

10. 1099 kW. **11.** 95·74%. **12.** 95·8%.

13. (*a*) 593·6 C.H.U. per lb.

14. 783 lb. per min., 39 tubes. **15.** 15·5 lb. per kWh.

16. 167·7 lb. per sq. in. **17.** 165·3 lb. per sq. in.

18. (*a*) 33,840 ft. lb. ; (*b*) 30,620 ft. lb. **19.** 12,782 ft. lb.

20. 84·2 lb. per sq. in., 279° C. **21.** 103,680 ft. lb., 179 C.H.U.

22. 187 C.H.U. ; 88,900 ft. lb.

23. (*a*) 549 cu. ft. per min. ; (*b*) 4·37 ; (*c*) 9,427, 200 ft. lb. ; (*d*) 175·3° C.

24. 130 cu. ft. ; 10,690,000 ft. lb. ; 9,396,000 ft. lb.

25. 541° C. ; 0·74 lb. ; 13,820 ft. lb. ; 32·9 C.H.U.

26. 13·1 cu. ft. **27.** 61·3 lb. per sq. in. ; 231·7° C.

28. 1·24 cu. ft. ; 473° C. ; 10·3 ; 107,700 ft. lb.

29. (*a*) 3·275 cu. ft. ; (*b*) 4·68 ; (*c*) 694 lb. per sq. in. and 455·5° C. ; (*d*) 80,568 ft. lb. ; 1·43 lb. per cu. ft.

30. 192·6 C.H.U. **31.** 96·07, 24·26 cu. ft., 32·39 cu. ft. **32.** $8{\cdot}02 \times 10^{-4}$ lb.

33. 394 lb. per sq. in,. 518·4° C. and 0·0896 lb. **35.** 42·65%.

38. 78·14%, 35%, 44%.

CHAPTER XVI (p. 424)

2. 118·2 C.H.U. 3. 256° C. 4. 0·048 rank. **5. 465° C.**
6. 0·048 rank.
7. (*a*) 0·2414, ; (*b*) 1·512 ; (*c*) 1·347 ; (*d*) 1·617 ranks.
8. 198·4 C.H.U., 0·0016 rank.
9. (*a*) 0·461 ; (*b*) 1·613 ; (*c*) 1·482 ; (*d*) 1·656 and (*e*) 1·761 ranks.
10. 6·96 cu. ft. per lb. 11. 70·4%. 13. 84%.
14. 166·6 C.H.U. or 233,240 ft. lb. 15. 48·1 H.P.
16. 164·8 C.H.U., 3657 ft. per sec. 17. 27%.
18. 81%, 163·1 C.H.U. per lb., 3700 ft. per sec. 22. 10. 23. 4·7 H.P.
24. 5160 lb. 25. (*a*) 7·92 ; (*b*) 27·8 H.P. 26. 1420 lb. per hour.

CHAPTER XVII (p. 445)

4. (*a*) 136·4 lb. per sq. in. ; (*b*) 40·6 lb. per sq. in.
7. 96·5% ; (*a*) 22·8 C.H.U. per lb. ; (*b*) 1434 ft. per sec. 8. 98·5%.
9. (*a*) 97·5% ; (*b*) 15·2 C.H.U. per lb. ; (*c*) 1170 ft. per sec.
10. 0·67 11. (*a*) 56·4 C.H.U. per lb. ; (*b*) 2250 ft. per sec.
12. (*a*) 18·3 lb. per sq. in. abs., 87·5% dry ; (*b*) 5·46 lb. per sq. in. abs., 82·5% dry.
13. (*a*) 0·732 ; (*b*) 0·535. 15. 1879 ft. per sec., 684 ft. per sec.
19. 3440 ft. per sec., 2380 ft. per sec., 30°. 21. 16·6 H.P., 88·4%.
22. (*a*) 3950 ft. per sec.

(*b*)

1st Rotating		Stationary		2nd Rotating	
Inlet	Exit	Inlet	Exit	Inlet	Exit
22·5°	22·5°	27°	31°	40°	40°

(*c*) 146·1 H.P. (*d*) 80·2%.

23. 1600 ft. per sec. ; 186,100 ft. lb. per lb.
24. 2300 ft. per sec., 32°, 1350 ft. per sec. at 68° to the rotor.
25. 92 H.P., 1584 ft. per sec.

CHAPTER XVIII (p. 476)

2. 8021 C.H.U. per lb.
3. Gross 11,088 C.H.U. per lb. ; net 10,510 C.H.U. per lb. 6. 1·22%.
8. 4·65, 5·97. 9. 7·21, 9·9, 72·7%.
10. 85·1%, 32·9%, 28%, 0·447 lb. per hour.
11. 80%, (*a*) 28·3% ; (*b*) 26·4% ; (*c*) 93%. 12. 1·28 H.P.

13. 51·4 H.P., 31·8%. **14.** 81·6 lb. **16.** $C=11{\cdot}29\,P+1188$; 143 H.P.

17. 32·9 lb. **21.** 716 lb. per hour from and at 100° C.

22. 589 lb. per hour from and at 100° C., 9·83%.

24. (*a*) 11·7 lb. ; (*b*) 70·2% ; (*c*) 10·8%.

25. (*a*) 65·6% ; (*b*) 9·75 lb. from and at 100° C. ; (*c*) 13·6%.

26. (*a*) 6230 C.H.U. per lb ; (*b*) 1041 C.H.U. per lb. ; (*c*) 46·6 C.H.U. per lb. (*d*) 78·4% ; (*e*) 674 C.H.U. per lb.

27. 65%. **28.** 12,230 C.H.U. per lb. ; 11,430 C.H.U. per lb. ; 15·3 lb.

29. (*a*) 77·9% ; (*b*) 15·4 lb. ; (*c*) 3·92 lb.

30. (*a*) 16·8% ; (*b*) 9990 lb. ; (*c*) 20·2%.

CHAPTER XIX (p. 497)

3.

Heat supplied in C.H.U. per min.	1167·8	
Heat to B.H.P. in C.H.U. per min.		69·1
,, ,, Friction ,, ,, ,, ,,		39·3
,, ,, Condensate ,, ,, ,, ,,		93·9
,, ,, Cooling Water ,, ,, ,, ,,		330·0
,, unaccounted for ,, ,, ,, ,,		635·5
Total	1167·8	1167·8

4.

Heat supplied in C.H.U. per min.	1350	
Heat to B.H.P. in C.H.U. per min.		148·5
,, ,, Friction ,, ,, ,, ,,		37·7
,, ,, Cooling water ,, ,, ,, ,,		419·3
,, unaccounted for ,, ,, ,, ,,		744·5
Total	1350	1350

5. 29·6% ; 56,400 C.H.U. ; 59,570 C.H.U.

Heat supplied in C.H.U.	201,110	
Heat to B.H.P. in C.H.U.		59,598
,, ,, Friction ,, ,,		25,542
,, ,, Jacket Water ,, ,,		56,400
,, ,, exhaust gases etc, ,, ,,		59,570
Total	201,110	201,110

7. 82·9%, 43.5%, 36·1%.

8

Heat supplied C H U per hour	60,200	
Heat to B H P C H U per hour		8,627
,, ,, Friction ,, ,, ,,		2,121
,, ,, Jacket Water ,, ,, ,,		24,500
,, unaccounted for ,, ,, ,,		24,952
Total	60,200	60,200

(*a*) 17 85% , (*b*) 14 33%

10 13 57, 64 7%, 18 5%

11

Heat supplied C H U per hour	31,416	
Heat to B H P C H U per hour		6,223
,, Friction ,, ,, ,,		1,980
,, ,, Cooling Water ,, ,, ,,		13,860
,, unaccounted for ,, ,, ,,		9,353
Total	31,416	31,416

12 11 67 , 9 61 , 82 3% , 47 14 cu ft per B H P per hour , 10 2%.

13 180 4, 140 7 14 6 12, 3 69, 15 7%

15 r_1 4 208 r_2 5 014 , 33 81 cu ft , 16 7%

16 1721 H P , 0 325 lb per I H P per hour , 13,420 lb per day

17. 50 23 70 74 0 627 lb , 39 6%

18 38 8 amp , 13 7 H P , 0 774 pint per B T U

19 6 64 H P , 3 48 lb per hour 20 38,460 H P , 21,540 lb , 117,400.

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